INTRODUCTION TO ELECTRIC POWER SYSTEMS

1991 Edition

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INTRODUCTION TO ELECTRIC POWER SYSTEMS

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The area of electric power systems includes a broad range of topics that concerns the electrical engineer, where the fundamental principles that underlie power systems are well within the background of students at the Junior level for which this text has been written. These basic principles are not emphasized elsewhere in the electrical engineering curriculum, and thus a basis is provided from which a clear and sound understanding of electromechanical energy conversion, power flow, and transmission can be obtained. To those electrical engineering students for whom this course is terminal, the basic concepts and devices covered must be well understood, since electric power systems cuts across most of the areas of electrical engineering. To those students who continue in the power area, this text provides an excellent basis for further undergraduate and graduate study.

The emphasis in this text is on the power flow of electric energy, since it must be generated at remote locations, transmitted to load centers, and then converted to useful form. The basic principles of power flow are considered important for two reasons,

Power flow must be controlled, i.e., a watt-var balance must continually be achieved, since energy in an electric power system cannot be stored.

Power flow through an energy-conversion device must be well understood to appreciate the energy irreversibly lost, and the energy stored that is required to transfer or convert electric energy to useful form.

The material in this text is thought-provoking, very practical and logical, and requires all the attributes of a successful engineer, regardless of his or her area of interest, needed to apply physical laws and principles to practical devices. To this end the text is bountifully illustrated with diagrams, charts, figures and schematics that allow an engineer to form a mind-picture associated with a physical law or principle, in a meaningful way, that can be remembered and applied practically.

All of the material in this text is taught in one semester with the first half devoted to fundamentals, and the second half to an application of these fundamentals. The first three chapters are an intensive review of ac, magnetic circuit and transformer principles. The fourth chapter, electromechanical energy conversion, is usually new to electrical engineering students, some of whom have some difficulty grasping mechanical principles and spatial concepts, so these principles and concepts are well illustrated, and repeated, for better understanding. The material in the fourth chapter concerning reluctance force energy conversion involves the principle of virtual displacement and work. This principle is applied to practical devices whose magnetic circuits are invariably linearized so that electrical engineers can measure inductance, which is a linear concept. Since linearity is invariably assumed, coenergy is always equal to the energy stored in the device magnetic field, and for this reason field energy is emphasized, and the concept of coenergy is not mentioned to avoid unnecessary complication of the thought-provoking principle of virtual work.

The remaining four chapters emphasize the basic elements of an electric power system together with their equivalent-circuit models. The last chapter, then, ties together the foregoing chapters with an engineering summary of the basic principles of power flow, and how power-system generation and load impact the watt-var balance that must be continually achieved. Finally, a rigorous and in-depth study of transformers, transmission lines and machines is pointless, at this introductory level, unless their characteristics and role in a complete power system can be shown.

The course based on this text has been taught for many years, and my thanks go to my colleagues, W. D. Stevenson Jr., J. J. Grainger and M. E. Elbuluk, who contributed greatly over the years to its content, to the hundreds of students with their valuable suggestions, and to the forbearance of my wife.

Alfred J. Goetze

CHAPTER 1

ALTERNATING-CURRENT CIRCUITS

Electric power systems and their components - generators, motors, transformers, transmission lines and loads are driven primarily by sinusoidal forcing functions. This chapter will provide a review of linear circuits that are excited by single-phase or three-phase sinusoidal waveforms. The concepts of phasors, impedance, admittance and power flow must be well-understood to set the foundation for the analysis of most of the topics covered in subsequent chapters. Single-phase circuits will be considered first and then extended to an analysis of three-phase circuits.

1-1 TIME-VARYING WAVEFORMS

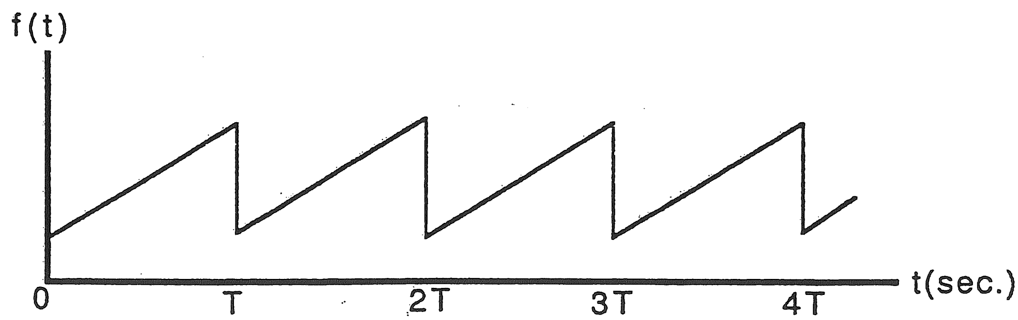


Figure 1.1 Timevarying Periodic Waveform

The waveform in Fig. 1.1 varies periodically with period, T. Any periodic function can be expressed as,

$$f(t) = f(t + nT) \qquad n = 1,2, \ldots \tag{1.1}$$

The frequency, with which this function varies, is given by,

$$f = \frac{1}{T} \qquad \text{(cycles/sec or Hz)} \tag{1.2}$$

The function in Fig. 1.1 is a nonsinusoidal function of time and is equivalent to a Fourier series consisting of an average or dc value plus an infinite number of sinusoidal terms which, when summed, converge to the function, f (t).

Two very important quantities characterize the function in Fig. 1.1.

1. Average (mean) value,

The arithmetic mean of a finite number of discrete quantities is their sum divided by their number. The mean value of a continous function is, then,

$$\langle F \rangle = \frac{1}{T}\int_{0}^{T} f(t)\, dt \tag{1.3}$$

2. Root-Mean-Square (RMS) value,

The root-mean-square value is taken in that order and is defined,

$$|F| = \sqrt{\frac{1}{T}\int_{0}^{T}[f(t)]^2\, dt} \tag{1.4.}$$

The average value is usually referred to as the dc component of a periodic waveform and is, in fact, the constant term in its Fourier-series representation. The rms value, often called the effective value, is very important in power systems because it is the metered value of voltage or current and is the equivalent heating effect to dc in ac power flow.

A sinusoidal function of time is given in Fig. 1.2, and is completely described when its maximum value, phase shift and frequency is known.

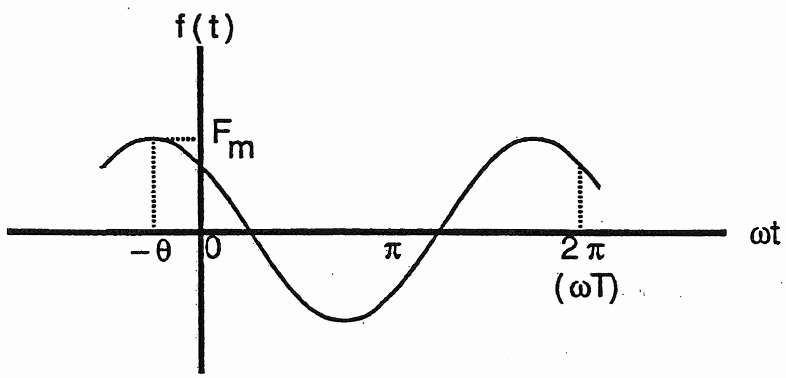


Figure 1.2 A Sinusoidal Function Of Time

The waveform in Fig. 1.2 is given by,

$$f(t) = F_m \cos(\omega t + \theta) \qquad (1.5)$$

where,

F_m = maximum value

$\omega = 2\pi f$ = angular frequency (rad/sec)

$f = \dfrac{1}{T}$ = frequency (Hz)

$T = \dfrac{2\pi}{\omega}$ = period (sec)

θ = phase shift (rad)

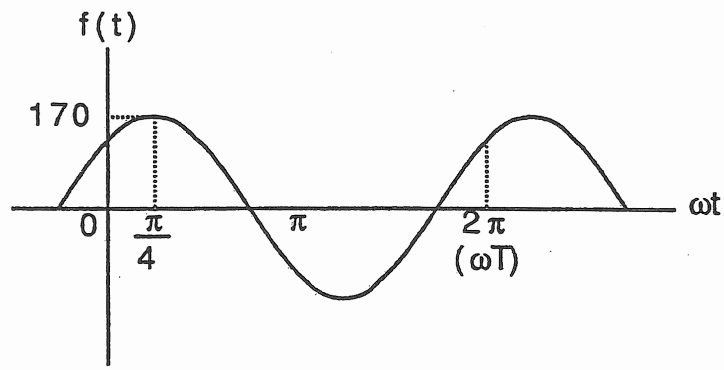
It is common, in power system analysis, to plot Eqn. (1.5) as a function of ωt where the period is 2π radians.

Example 1.1

Given a function,

$$f(t) = 170 \cos(377\, t - \frac{\pi}{4})$$

Sketch this function and find its period, frequency, angular frequency, phase shift and maximum value. Also find its average and rms values.



$$T = \frac{2\pi}{377} = 16.7 \text{ msec}$$

$$f = \frac{1}{T} = 60 \text{ Hz}$$

$$\omega = 2\pi f = 377 \text{ rad/sec}$$

$$F_m = 170$$

$$\theta = \frac{\pi}{4} \text{ rad}$$

$$\langle F \rangle = \frac{1}{\omega T}\int_0^{\omega T} f(\omega t)\, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} 170 \cos(\omega t - \theta)\, d(\omega t)$$

$$= \frac{170}{2\pi}\left[\sin(\omega t - \theta)\right]_0^{2\pi}$$

$$\langle F \rangle = 0$$

→ The average value of any sinusoidal function of time is always zero, regardless of its phase shift.

$$|F| = \sqrt{\frac{1}{\omega T}\int_0^{\omega T} [f(\omega t)]^2\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [170 \cos(\omega t - \theta)]^2\, d(\omega t)}$$

$$= \sqrt{\frac{(170)^2}{2\pi}\left[\frac{\omega t}{2} + \frac{\sin 2(\omega t - \theta)}{4}\right]_0^{2\pi}}$$

$$|F| = \frac{170}{\sqrt{2}} = \frac{F_m}{\sqrt{2}}$$

→ The rms value of any sinusoidal function of time is always $F_m/\sqrt{2}$, regardless of its phase shift.

1-2 PHASOR THEORY AND DIAGRAMS

When analyzing ac circuits, sinusoidal voltages and/or currents must be added, subtracted, multiplied or divided. To do this point by point, or using

trigonometric identities, would be most impractical. Earlier in this century, a brilliant scientist and engineer, Charles Steinmetz, proposed a very efficient method of manipulating sinusoidal functions based on Euler's theorem,

$$e^{j\beta} = \cos\beta + j\sin\beta \tag{1.6}$$

In general, a sinusoidal function can be expressed

$$f(t) = \underbrace{\sqrt{2}}_{\text{instantaneous value}}|A|\cos(\omega t + \phi) \tag{1.7}$$

where $|A|$ = effective value

ϕ = phase shift

From Euler's theorem, Eqn. (1.7) can be expressed,

$$f(t) = \mathcal{R}e\,\{\sqrt{2}\,(|A|\,e^{j\phi})\,e^{j\omega t}\} \tag{1.8}$$

All cosinusoidal functions of time can be expressed in the form of Eqn. (1.8) with the observation that the quantity in parenthesis,

$$A = |A|e^{j\phi} \triangleq |A|\,\underline{/\phi} \tag{1.9}$$

contains all the information needed to completely define a specific sinusoidal function. Equation (1.9) is called the phasor corresponding to the sinusoidal function in Eqn. (1.7). The phasor in Eqn. (1.9) is a complex number and is a directed line element in the complex plane. It has a magnitude and angle in this plane and without exception in electric power engineering, this magnitude is the rms or effective value of the sinusoidal function, rather than its maximum value. From Eqn. (1.8), the phasor rotates with constant magnitude, counterclockwise, at angular velocity, ω, but it is always shown at time = zero, as in Fig. 1.3.

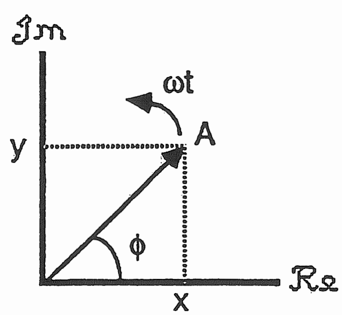


Figure 1.3 Phasor Representation Of A Sinusoidal Function

At any instant of time, the horizontal projection of the phasor in Fig.1.3, multiplied by $\sqrt{2}$, is the instantaneous value of the sinusoidal function in Eqn. (1.7). The quantity to the right of Eqn. (1.9) is defined as the <u>phasor operator</u>, used mostly by power engineers, and it is the phasor in <u>polar</u> form (useful for multiplication or division of phasors),

$$A = |A|e^{j\phi} \triangleq |A|\angle\phi \qquad (1.10)$$

phasor operator

The phasor can also be expressed in <u>rectangular</u> or Cartesian form (useful for addition or subtraction of phasors), which can be written,

$$A = x + j\,y \qquad \text{where } j = \sqrt{-1} \qquad (1.11)$$

$$x = |A| \cos\phi \quad ; \quad y = |A| \sin\phi$$

Phasor manipulation requires changing polar and rectangular forms, which is easily accomplished by observing,

$$|A| = \sqrt{x^2 + y^2} \quad ; \quad \phi = \tan^{-1}\frac{y}{x} \qquad (1.12)$$

<u>Example 1.2</u>

Find the results of the following expressions in polar form and also as a function of time,

(a) $4 + j\,3 + 5\,\angle 53.1°$ (c) $(2 + j\,2)\,(5\sqrt{2}\,\angle 30°)$

(b) $4\sqrt{3}\,\angle 60° + 2\sqrt{3}\,\angle 30°$ (d) $\dfrac{3 + j\,3}{5 + j\,12}$

(a) $4 + j\,3 + 5\,\angle 53.1° = 4 + j\,3 + 5\,(\cos 53.1 + j \sin 53.1°)$

$$= 4 + j\,3 + 3 + j\,4 = 7 + j\,7$$

$$= 9.9\,\angle 45° \quad ; \quad f(t) = \Re e\,\sqrt{2}\,(9.9)\,e^{j0.785}e^{j\omega t} = 14\cos(\omega t + 45°)$$

(b) $4\sqrt{3}\,\angle 60° + 2\sqrt{3}\,\angle 30° = 4\sqrt{3}\,(\cos 60° + j \sin 60°) + 2\sqrt{3}\,(\cos 30° + j \sin 30°)$

$$= 3.46 + j\,6 + 3 + j\,1.732 = 6.46 + j\,7.732$$

$$= 10\,\angle 50.1° \quad ; \quad f(t) = 14.1\cos(\omega t + 50.1°)$$

(c) $(2 + j\,2)\,(5\sqrt{2}\,\angle 30°) = (2\sqrt{2}\,\angle 45°)\,(5\sqrt{2}\,\angle 30°)$

$$= 20\,\angle 75° \quad ; \quad f(t) = 28.3\cos(\omega t + 75°)$$

(d) $$\frac{3+j3}{5+j12} = \frac{4.24\,\angle 45°}{13\,\angle 67.4°} = 0.326\,\angle{-22.4°} \quad ; \quad f(t) = 0.461\cos(\omega t - 22.4°)$$

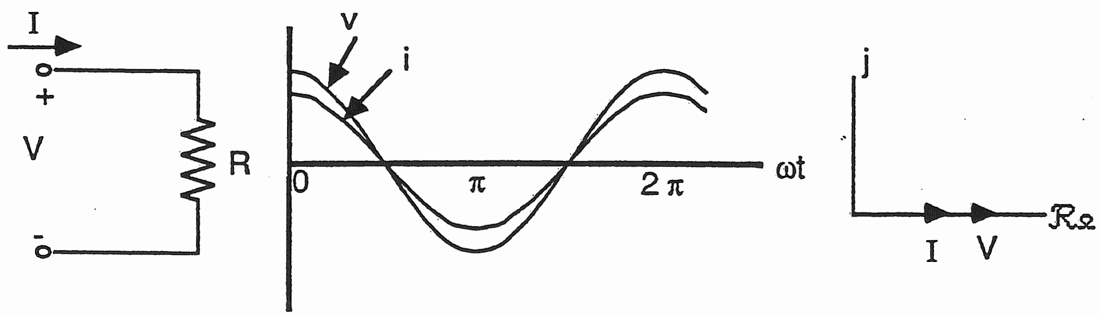
1-3 PHASOR DIAGRAMS OF BASIC CIRCUIT ELEMENTS

Since time is an independent variable in a sinusoidally varying function of time, we can start counting time at any arbitrary instant. If we start counting time from the instant a voltage or current is maximum, and decreasing, we call this function the <u>reference</u> voltage or current, and we find its phasor along the real axis of the complex plane.

$$v = \sqrt{2}\,|V| \cos \omega t$$

$$V = |V| \angle 0^\circ$$

<u>Resistance</u>



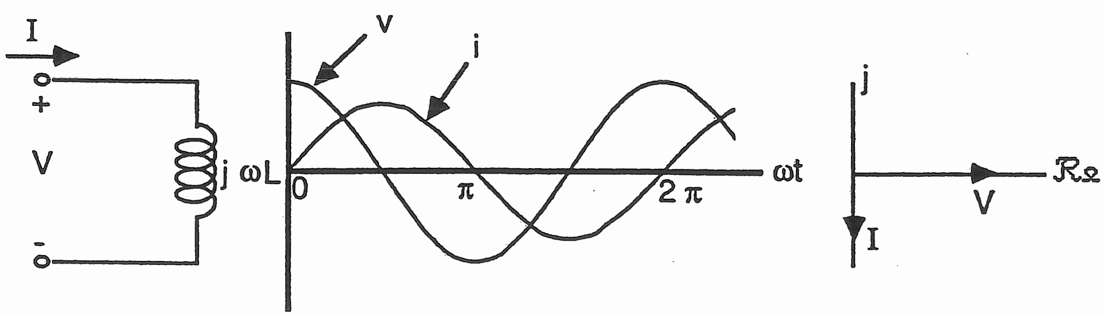
$$I = \frac{V \angle 0^\circ}{R \angle 0^\circ} = \frac{V}{R} \angle 0^\circ \quad \text{(A)}$$

From Eqn. (1.8),

$$i = \sqrt{2}\left(\frac{V}{R}\right) \cos \omega t \qquad \text{(A)}$$

The current through a resistor is in time phase with the voltage across a resistor.

<u>Inductance</u>



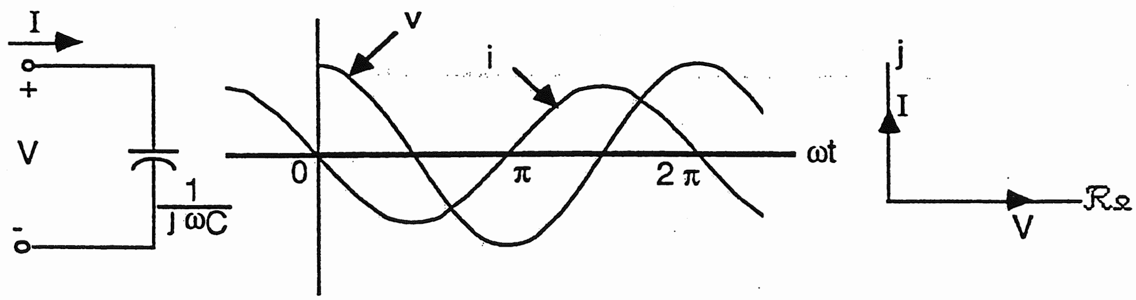
$$I = \frac{V \angle 0^\circ}{j\omega L} = \frac{V \angle 0^\circ}{X_L \angle 90^\circ} = \frac{V}{X_L} \angle -90^\circ \qquad \text{(A)}$$

From Eqn. (1.8)

$$i = \sqrt{2}\,\frac{V}{X_L} \cos(\omega t - 90^\circ) \qquad \text{(A)}$$

The current through an inductance lags the voltage across an inductance by 90°, i.e., the current maximum occurs 90° later in time phase than the voltage maximum.

Capacitance



$$I = \frac{V\angle 0°}{\frac{1}{j\omega C}} = \frac{V\angle 0°}{X_c\angle -90°} = \frac{V}{X_c}\angle 90° \quad \text{(A)}$$

From Eqn. (1.8),

$$i = \sqrt{2}\,\frac{V}{X_c}\cos(\omega t + 90°) \qquad \text{(A)}$$

The current through a capacitor leads the voltage across a capacitor by 90°, i.e, the current maximum occurs 90° earlier in time phase than the voltage maximum.

The ratio of the voltage and current phasors used in the above development is called Ohm's law in ac circuits, from which the concepts of impedance and admittance are defined,

Impedance

$$\frac{V}{I} \triangleq Z = R + j(X_L - X_c) = |Z|\angle\phi \quad (\Omega) \qquad (1.13)$$

where reactance, $X_L = \omega L, \quad X_c = \dfrac{1}{\omega C} \qquad (\Omega)$

and impedance, $|Z| = \sqrt{R^2 + (X_L - X_c)^2}$

$$\phi = \tan^{-1}\left(\frac{X_L - X_c}{R}\right)$$

Admittance

$$\frac{I}{V} \triangleq Y = G + j(B_c - B_L) = |Y|\angle -\phi \quad (\mho) \qquad (1.14)$$

where, $Y = \dfrac{1}{Z}$, G = conductance, B = susceptance (℧)

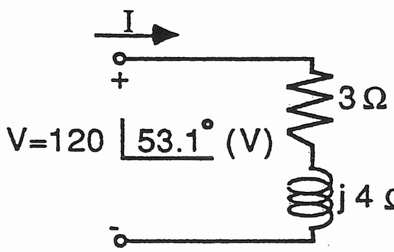
1-4 PHASOR DIAGRAMS OF SERIES AND PARALLEL CIRCUITS

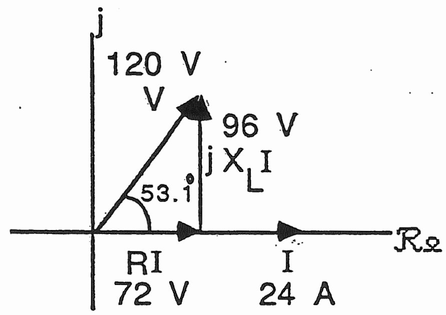
⇒ With the above definitions in mind, impedance is useful in series circuits where current is usually chosen as the reference phasor, whereas admittance is useful in parallel circuits where voltage is usually chosen as the reference phasor.

Example 1.3

Given the following circuits; analyze these circuits and draw their phasor diagrams.

1.



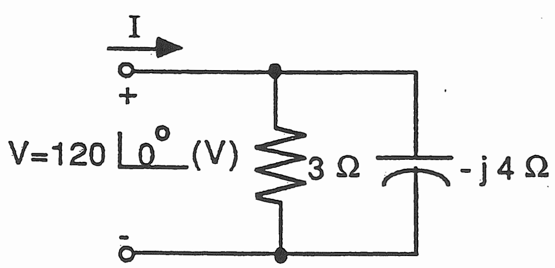


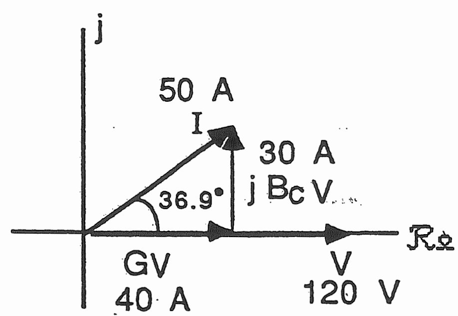
$$V = RI + jX_L I$$

$$I = \frac{V}{Z} = \frac{120\,\underline{/53.1^\circ}}{3+j4} = 24\,\underline{/0^\circ} \qquad \text{(A)}$$

The voltage across the resistance is in phase with its current. The voltage across the inductance leads its current by 90°. The current through the impedance lags the voltage across the impedance by 53.1°, since this is an inductive load.

2.



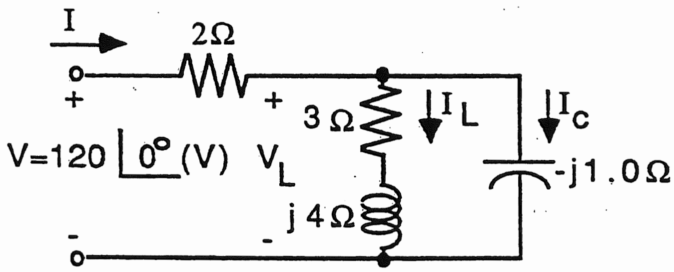


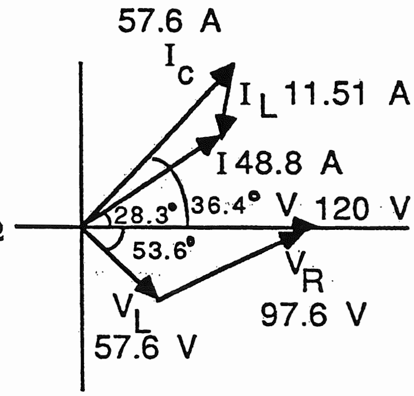
$$I = GV + jB_C V$$

$$= \frac{120\underline{/0^\circ}}{3\underline{/0^\circ}} + \frac{120\underline{/0^\circ}}{4\underline{/-90^\circ}} = 40 + j30 = 50\,\underline{/36.9^\circ} \qquad \text{(A)}$$

The current through the resistance is in phase with its voltage. The current through the capacitance leads its voltage by 90°. The total current through the impedance leads the voltage across the impedance by 36.9°, since this is a capacitive load.

3.





$$Z_i = 2 + \frac{(3+j4)(-j1.0)}{3+j4-j1.0} = 2 + 1.18\,\angle{-81.9°} = 2.46\,\angle{-28.3°} \quad (\Omega)$$

(Z_i is capacitive)

$$I = \frac{V}{Z_i} = \frac{120\,\angle 0°}{2.46\,\angle{-28.3°}} = 48.8\,\angle 28.3° \quad (A)$$

Using current division,

$$I_L = \frac{-j1.0}{3+j4-jl.0} \times 48.8\,\angle 28.3° = 11.51\,\angle{-106.7°} \quad (A)$$

$$I_c = \frac{3+j4}{3+j4-j1.0} \times 48.8\,\angle 28.3° = 57.6\,\angle 36.4° \quad (A)$$

Using voltage division,

$$V_L = \frac{1.18\,\angle{-81.9°}}{2 + 0.1663 - j1.168} \times 120\,\angle 0° = 57.6\,\angle{-53.6°} \quad (V)$$

1-5 POWER IN AC CIRCUITS

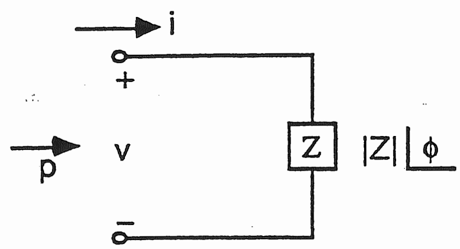


Figure 1.4 AC Circuit

In Fig. 1.4, the voltage is taken as the reference phasor,

$$v = \sqrt{2}\,|V| \cos \omega t \qquad (V) \tag{1.15}$$

$$V = |V| \angle 0^\circ$$

$$I = \frac{V}{Z} = \frac{|V| \angle 0^\circ}{|Z| \angle \phi} = |I| \angle -\phi$$

$$i = \sqrt{2}\,|I| \cos(\omega t - \phi) \qquad (A) \tag{1.16}$$

The phase shift in Eqn. (1.16) is negative since the impedance is inductive as shown in Fig. 1.4; the phase shift would be positive if the impedance was capacitive.

The instantaneous power delivered to the impedance is

$$p = vi = 2\,|V|\,|I| \cos \omega t \cos(\omega t - \phi) \tag{1.17}$$

If the instantaneous voltage and current are both positive or both negative, power flows from the source to the impedance. If the instantaneous voltage or current is negative, power flows from the impedance to the source. Using trigonometric identities, Eqn. (1.17) becomes,

$$vi = |V|\,|I| \cos \phi\,(1 + \cos 2\,\omega t) + |V|\,|I| \sin \phi\,(\sin 2\,\omega t) \tag{1.18}$$

Observe from Eqn. (1.18) that the instantaneous power varies with double frequency. The power that must be supplied continuously to the impedance is the average power of Eqn. (1.18) and is the coefficient of the first term, since the average value of cos 2 ωt and sin 2 ωt is zero.

The average value of the instantaneous power is called real power and is,

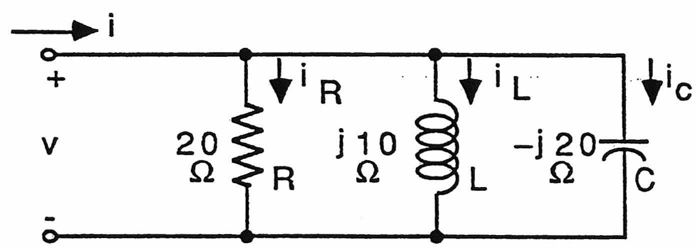
$$P = |V|\,|I| \cos\phi \qquad (W) \qquad (1.19)$$

The magnitude of the second term is called reactive power and is,

$$Q = |V|\,|I| \sin\phi \qquad (VAR) \qquad (1.20)$$

Both real and reactive power have the dimensions, volts times amperes, but to distinguish one from the other, real power is measured in watts, and reactive power in volt-amperes reactive. The angle, ϕ, is always the angle between the rms voltage and current phasors and is determined by the load impedance. Considerable insight into the power equation (1.18) and Eqns. (1.19), (1.20) can be obtained by considering power flow for different load impedances.

Example 1.4

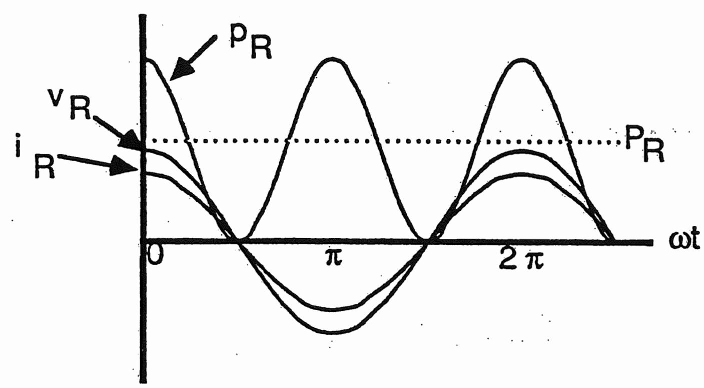


v=141.4 cos ωt (V)

Figure 1.5 RLC Circuit

Calculate the power flow to each branch of Fig. 1.5.

Resistance branch –



$V_R = 100\,\underline{/0^\circ}$ V

$$I_R = \frac{V_R}{R} = \frac{100\,\underline{/0^\circ}}{20\,\underline{/0^\circ}} = 5\,\underline{/0^\circ} \text{ A}$$

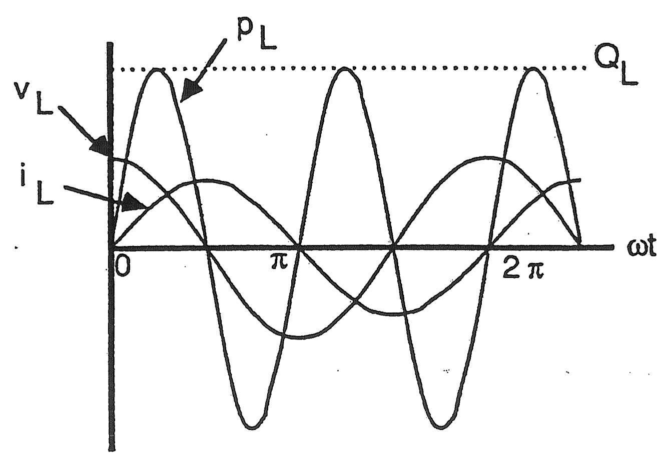
$P_R = V_R I_R \cos \phi_R = (100)(5) \cos 0^\circ = 500$ W

$Q_R = V_R I_R \sin \phi_R = (100)(5) \sin 0^\circ = 0$ VAR

$p_R = 500(1 + \cos 2\,\omega t)$ VA

Observe that the instantaneous power is at double frequency with average value, 500 watts, and since a resistance is dissipative only, it stores no volt-amperes reactive.

Inductance branch –



$V_L = 100\,\underline{/0^\circ}$ V

$$I_L = \frac{V_L}{jX_L} = \frac{100\,\underline{/0^\circ}}{10\,\underline{/90^\circ}} = 10\,\underline{/-90^\circ} \text{ A}$$

$P_L = V_L I_L \cos \phi_L = (100)(10) \cos 90^\circ = 0$ W

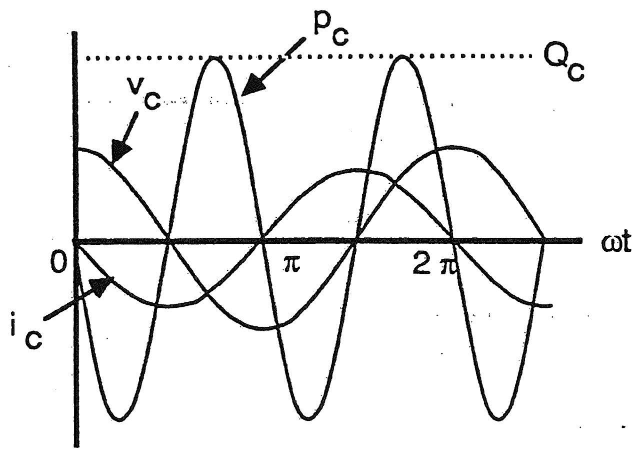
$Q_L = V_L I_L \sin \phi_L = (100)(10) \sin 90^\circ = 1000$ VAR

$p_L = 1000 \sin 2\,\omega t$ VA

Observe that the instantaneous power is at double frequency with magnitude, 1000 VAR, and when the power is positive, energy flows from the source and is stored in the magnetic field of the inductance. When the power is negative, the

magnetic field collapses, and the energy is returned to the source. The average value of the power is zero, with power flow reversing every quarter cycle.

Capacitance branch –



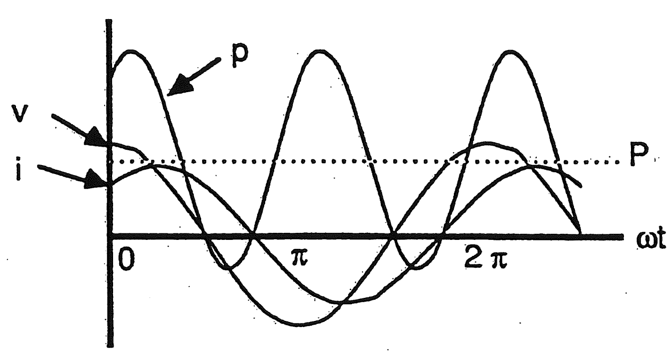
$$V_C = 100\,\underline{/0^\circ}\ \text{V}$$

$$I_C = \frac{V_C}{-jX_C} = \frac{100\,\underline{/0^\circ}}{20\,\underline{/-90^\circ}} = 5\,\underline{/90^\circ}\ \text{A}$$

$$P_C = V_C I_C \cos\phi_C = (100)(5)\cos 90^\circ = 0\ \text{W}$$
$$Q_C = V_C I_C \sin\phi_C = (100)(5)\sin 90^\circ = 500\ \text{VAR}$$
$$p_C = -500 \sin 2\omega t\ \ \text{VA}$$ (negative, since phase shift is positive in Eqn(1.17))

Observe that the instantaneous power is at double frequency with <u>magnitude</u>, 500 VAR; the significance of the minus sign will be discussed in Section 1-7. When power is positive, energy flows from the source and is stored in the electric field of the capacitance. When power is negative, the electric field collapses and the energy is returned to the source. The average power is zero, with power flow reversing each quarter cycle.

Circuit impedance –



$$V = 100 \angle 0^\circ \quad V$$

$$I = I_R + I_L + I_C = 5 \angle 0^\circ + 10 \angle -90^\circ + 5 \angle 90^\circ = 7.07 \angle -45^\circ \quad A$$

$$P = VI \cos\phi = (100)(7.07) \cos 45^\circ = 500 \quad W$$

$$Q = VI \sin\phi = (100)(7.07) \sin 45^\circ = 500 \quad VAR$$

$$p = 500(1 + \cos 2\omega t) + 500 \sin 2\omega t \quad VA$$

Observe that the instantaneous power is at double frequency with average value, 500 W, which is dissipated in the circuit resistance. Observe, also, that the capacitance supplies 500 VAR of the 1000 VAR required by the inductance leaving a net 500 VAR supplied by the source. The power is positive for a relatively long period of time when the source delivers energy to the resistance and inductance. The power is negative for a short period of time when the inductance returns the portion of the energy, not supplied by the capacitance, to the source.

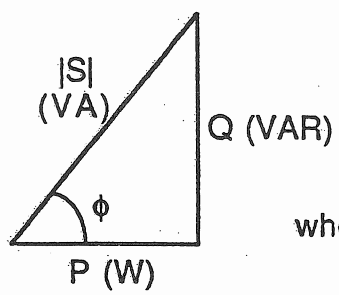
In summary, power flows continuously, with a nonzero average value to a dissipative resistance element, whereas power surges back and forth, with zero average value, to a capacitive or inductive element.

1-6 POWER TRIANGLE

Perhaps the greatest simplification to ac circuit analysis is the power triangle, when the voltage across, or the current through, each circuit element is known. Observe that the real and reactive power equations (1.19) and (1.20), derived from the circuit in Fig. 1.4, define a power triangle,

$$P = VI \cos\phi$$

$$Q = VI \sin\phi$$



where $|S| = VI$ (VA)

Figure 1.6 Power Triangle

The magnitude of the power, S, delivered to the impedance in Fig. 1.4, is called apparent power and its units are volt-amperes to distinguish it from real or reactive power. Some insight into the power triangle can be gained by considering the circuits in Figs. 1.7 and 1.8.

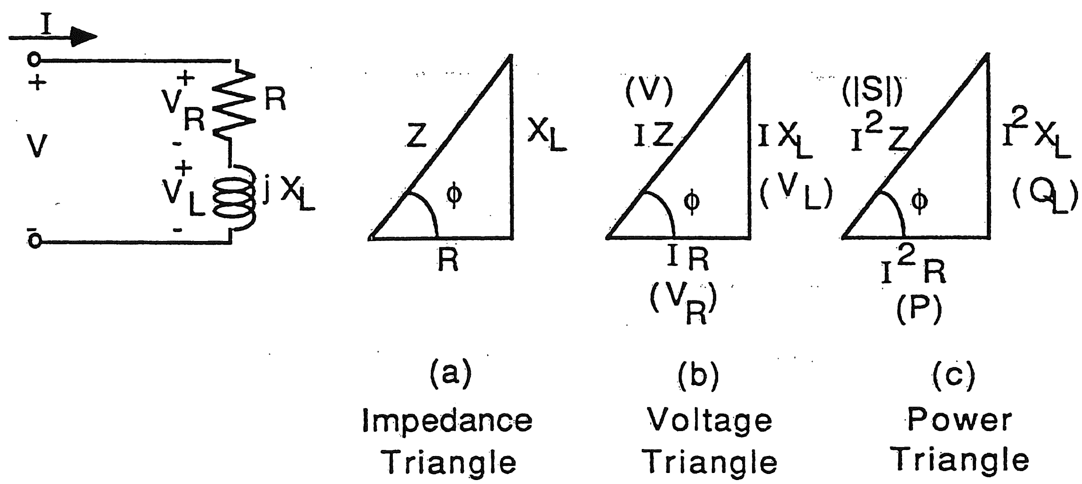


Figure 1.7 Inductive Load

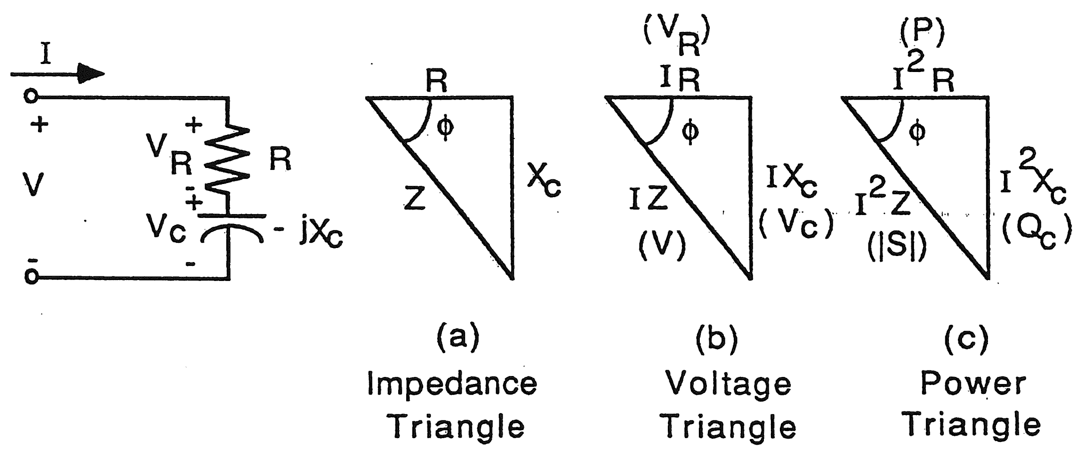


Figure 1.8 Capacitive Load

As can be seen from Figs. 1.7 and 1.8, the impedance, Z, determines the angle, ϕ, between the voltage, V, across, and the current, I, through, this impedance. This angle is called the power factor angle of the load. Observe, also, in these figures, that inductive reactive power, Q_L, is drawn positive and capacitive reactive power, Q_C, is drawn negative. From Figs. 1.7 and 1.8, power can be calculated in several ways,

$$P = VI\cos\phi = I^2 R = \frac{V_R^2}{R} \quad \text{(W)} \tag{1.21}$$

$$Q = VI\sin\phi = I^2 X = \frac{V_x^2}{X} \quad \text{(VAR)} \tag{1.22}$$

$$|S| = VI = I^2 |Z| = \frac{V^2}{|Z|} \quad \text{(VA)} \tag{1.23}$$

where voltage, current, impedance and power factor angle magnitudes only, are used in Eqns. (1.21), 1.22), (1.23). The cos ϕ, in Eqn. (1.21) is called the power factor of the load and since $\cos\phi = \cos(-\phi)$ the power factor must be qualified by current leading or lagging the voltage across the load.

$$pf \triangleq \cos\phi \quad \underset{(+)}{\underline{\text{leading}}} \text{ or } \underset{(+)}{\underline{\text{lagging}}}$$

Carefully note that in Eqns. (1.21) and (1.22), the voltage across the resistance, V_R, or the voltage across the reactance, V_x, is not, in general, the voltage across the impedance, V.

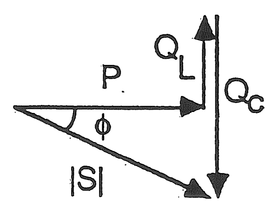
Example 1.5

Calculate and draw the power triangle for the input to the circuit in Example 1.3 - Part 3.

$$P = (48.8)^2 (2) + (11.51)^2 (3) = 5{,}160 \quad W$$

$$Q_L = (11.51)^2 (4) = 530 \quad VAR$$

$$Q_c = (57.6)^2 (1.0) = 3{,}318 \quad VAR$$

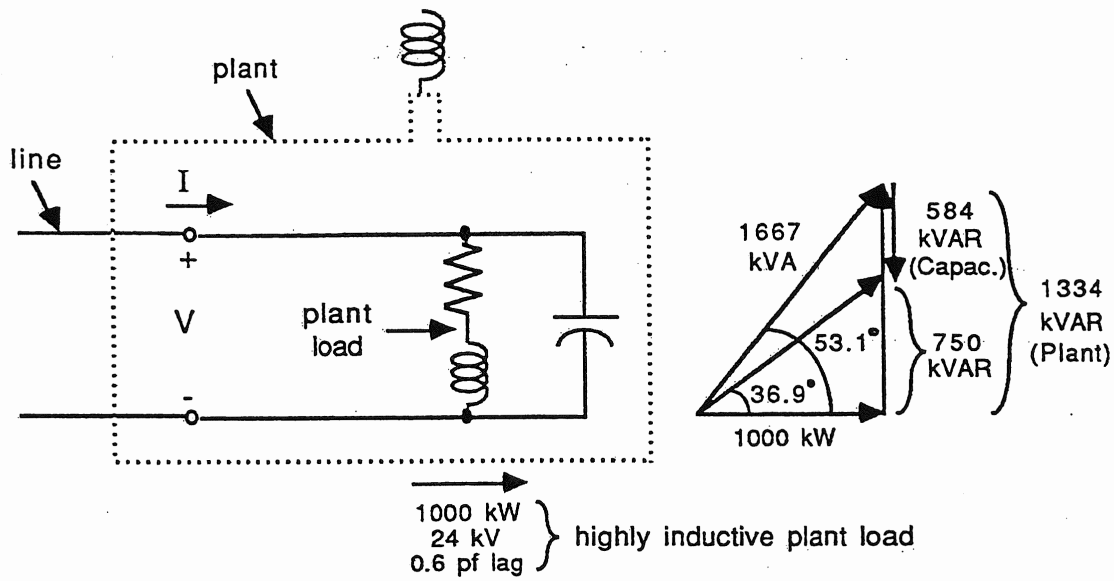


$$\phi = \tan^{-1}\frac{Q_L - Q_c}{P} = -28.3° \quad ; \quad pf = \cos\phi = 0.881 \text{ leading}$$

$$|S| = \frac{P}{\cos\phi} = 5{,}856 \text{ VA} = VI$$

The concept of the power triangle is quite useful in power factor correction of industrial plants in an electric power system. Consider the manufacturing plant in Example 1.6.

Example 1.6



Uncorrected, the load power factor angle is,

$$\phi = \cos^{-1} 0.6 = 53.1°$$

$$|S| = 1000 + j\,1334 = 1667\,\text{kVA} \quad ; \quad I = \frac{1667}{24} = 69.5\,\text{A}$$

With this poor power factor, the line current is excessive, resulting in high transmission line–I^2R losses, the cost of which is not paid by the plant. To avoid penalty charges, the plant power factor is improved to 0.8 lag by adding capacitance in parallel with the load, thus reducing the line current.

Corrected, the plant power factor angle is,

$$\phi' = \cos^{-1} 0.8 = 36.9°$$

$$|S| = 1000 + j750 = 1250\ \text{kVA} \quad ; \quad I = \frac{1250}{24} = 52\,\text{A}$$

The reactive power that must be supplied by the capacitor is,

$$Q_C = 750 - 1334 = -584 \quad \text{kVAR}$$

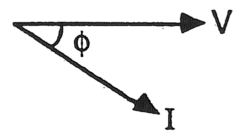
The line current is now reduced, with considerably lower line losses, by resonating the plant at 60 Hz.

1-7 COMPLEX POWER AND DIRECTION OF POWER FLOW

As will become apparent in later chapters, power engineers are concerned with the magnitude and direction of power flow in electric power systems. In Section 1-5, instantaneous power was derived for an <u>inductive</u> load,

$$v = \sqrt{2}\,|V| \cos \omega t \qquad (V)$$

$$i = \sqrt{2}\,|I| \cos (\omega t - \phi) \qquad (A)$$

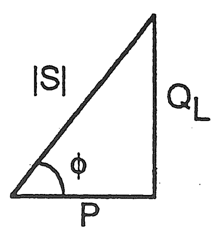


$$p = |V|\,|I| \cos \phi\,(1 + \cos 2\,\omega t) + |V|\,|I| \sin \phi\,(\sin 2\,\omega t) \qquad (VA)$$

where,

$$P = VI \cos \phi \qquad (W)$$

$$Q = VI \sin \phi \qquad (VAR)$$



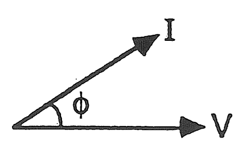
Here, P and Q are positive, where apparent power, S, is a complex variable,

$$S = VI\,\underline{/\phi} = VI \cos \phi + j\,VI \sin \phi = P + j\,Q \qquad (VA) \qquad (1.24)$$

If instantaneous power is derived for a <u>capacitive</u> load,

$$v = \sqrt{2}\,|V| \cos \omega t \qquad (V)$$

$$i = \sqrt{2}\,|I| \cos (\omega t + \phi) \qquad (A)$$

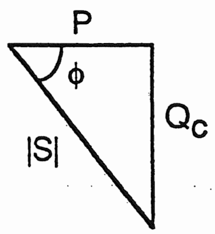


$$p = |V|\,|I| \cos\phi\,(1 + \cos 2\omega t) - |V|\,|I| \sin\phi\,(\sin 2\omega t) \quad \text{(VA)}$$

where,

$$P = VI\cos\phi \quad \text{(W)}$$

$$Q = -VI\sin\phi \quad \text{(VAR)}$$



Here, P is positive and Q is negative, where apparent power, S, is a complex variable,

$$S = VI\angle{-\phi} = VI\cos\phi - j\,VI\sin\phi = P - j\,Q \quad \text{(VA)} \qquad (1.25)$$

In summary, reactive power, Q, is positive for an inductive load and negative for a capacitive load. Therefore, complex power is calculated,

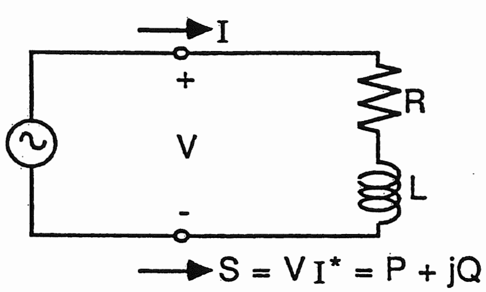
$$S = VI^* \quad \text{(VA)} \qquad = P + j\,Q \quad \text{(inductive load)} \qquad (1.26)$$

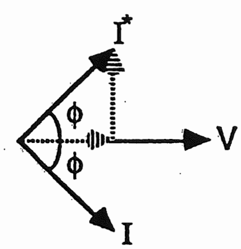
$$= P - j\,Q \quad \text{(capacitive load)}$$

where I is the conjugate of the current. If $S = V^* I$, the signs of the reactive power would be reversed which is incorrect.

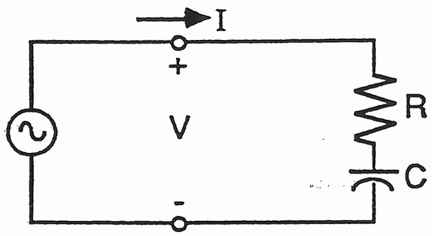
Example 1.7

Determine the complex power delivered to an inductive and capacitive load.

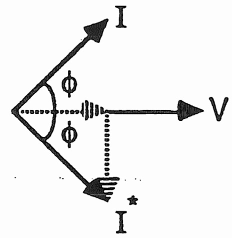




P is positive since the in-phase component of I^* is positive.
Q is positive since the quadrature component of I^* is positive.



$\longrightarrow S = VI^* = P - jQ$



P is positive, since the in-phase component of I^* is positive.
Q is negative, since the quadrature component of I^* is negative.

The positive direction of S in the above circuits is very important, since it determines the directions of its components, P and Q, depending on their signs as a result of the VI^* computation. In the above circuits, for the assumed positive directions of V and I, complex power, S, is positive to the <u>right</u>. If the current is assumed positive to the left, then complex power, S, is positive to the <u>left</u>. With this in mind, let,

$$V = 100\,\underline{/0^\circ} \quad V$$

$$R = 3\ \Omega, \quad X_L = j\,4\ \Omega, \quad X_C = -j\,4\ \Omega$$

<u>Inductive circuit</u> –

$$I = \frac{V}{Z} = \frac{100\,\underline{/0^\circ}}{3+j4} = 20\,\underline{/-53.1^\circ} \quad A$$

$$S = VI^* = (100\,\underline{/0^\circ})(20\,\underline{/53.1^\circ}) = 2000\,\underline{/53.1^\circ} \quad VA$$

$$S = 1200 + j\,1600 \quad VA$$

Since VI^* is the same for the generator or the load, and the signs of P and Q are both positive, real and reactive power both flow in the direction of positive S. It is said, therefore,

1. The generator <u>delivers</u> 1200 watts and the load resistance <u>absorbs</u> 1200 watts.

2. The generator <u>delivers</u> 1600 VAR and the load inductance <u>absorbs</u> 1600 VAR.

Capacitive circuit –

$$I = \frac{V}{Z} = \frac{100\,\underline{/0^\circ}}{3-j4} = 20\,\underline{/53.1^\circ} \quad \text{A}$$

$$S = VI^* = (100\,\underline{/0^\circ})(20\,\underline{/-53.1^\circ}) = 2000\,\underline{/-53.1^\circ} \quad \text{VA}$$

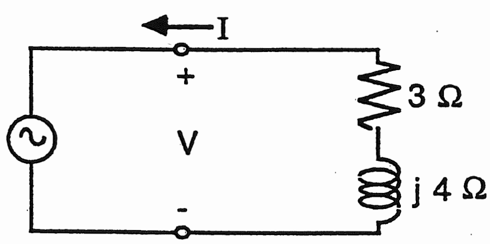
$$S = 1200 - j\,1600 \quad \text{VA}$$

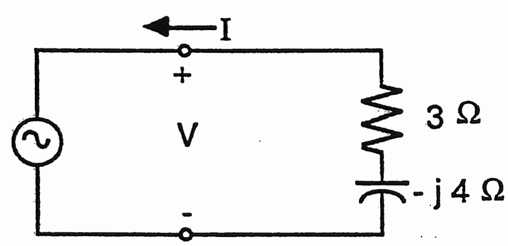
Since VI^* is the same for the generator or the load, P is positive and flows in the direction of positive S, Q is negative and flows in the opposite direction of positive S. It is said, therefore,

1. The generator delivers 1200 watts and the load resistance absorbs 1200 watts.

2. The generator absorbs 1600 VAR and the load capacitance delivers 1600 VAR.

In summary, from the above analysis and regardless of voltage and current convention, resistance always absorbs watts, inductance always absorbs VAR and capacitance always delivers VAR. Complex power, defined $S = VI^*$, then, not only correctly predicts magnitude of power flow but its direction as well. The interested reader can verify the above results using the following voltage and current conventions.

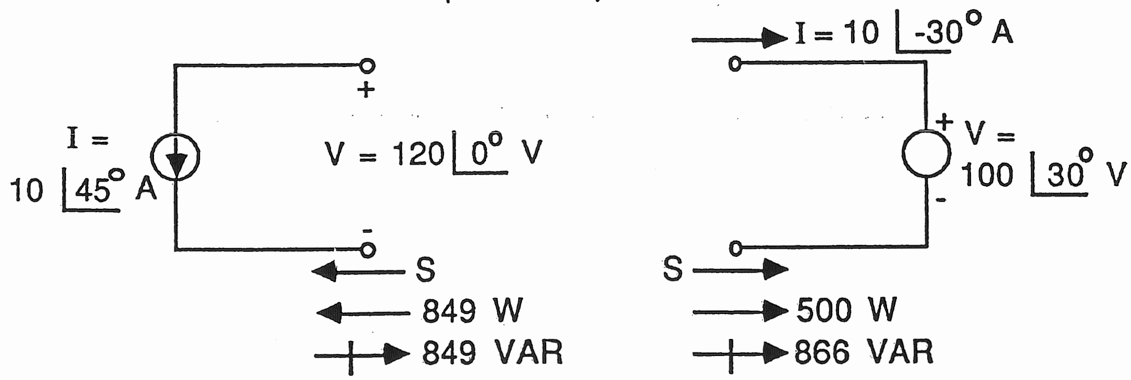




In general, the sign of the in-phase and quadrature components of I^* correctly determines the direction of P and Q, respectively, corresponding to the positive direction of complex power, S. This principle will be used extensively throughout the remainder of the text.

Example 1.8

Calculate the magnitude and direction of complex power for the two sources connected to networks (not shown).



(a) (b)

$$S = VI^* = (120\,\angle 0^\circ)(10\,\angle -45^\circ) \qquad S = VI^* = (100\,\angle 30^\circ)(10\,\angle 30^\circ)$$

$$= 1200\,\angle -45^\circ \text{ VA} \qquad = 1000\,\angle 60^\circ \text{ VA}$$

$$S = 849 - j\,849 \text{ VA} \qquad S = 500 + j\,866 \text{ VA}$$

The important principles of single-phase circuits have been emphasized and will now be extended to three-phase circuits.

1-8 THREE PHASE CIRCUITS

Modern electric power systems are driven by balanced, three-phase, wye connected, generators whose loads can be connected in either wye or delta. Each three-phase generator can be considered as three single-phase generators, delivering balanced line to neutral voltages, 120° apart in phase, as shown in Fig. 1.9.

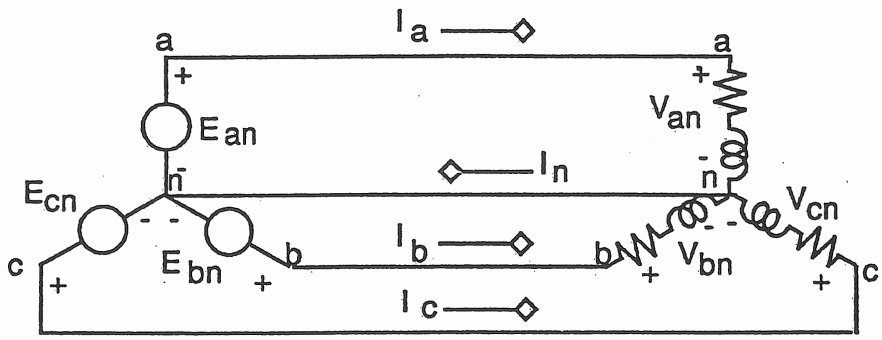
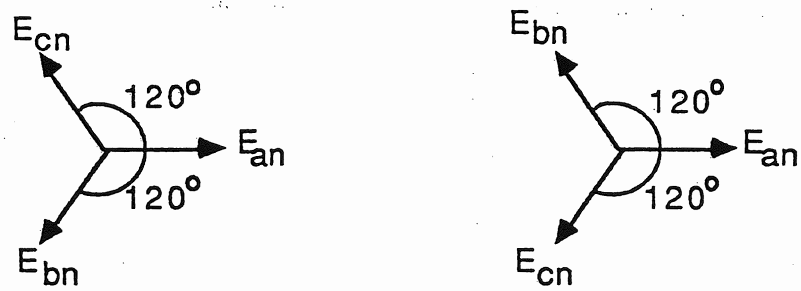


Figure 1.9 Three - Phase Circuit - Wye Load

The generator in Fig. 1.9 delivers a balanced set of line-to-neutral voltages in an abc sequence shown in Fig. 1.10 (a). If the generator is reversed, it delivers a set of voltages in an acb sequence shown in Fig. 1.10 (b).



abc - positive sequence

(a)

acb - negative sequence

(b)

Figure 1.10 Positive and Negative Sequence Voltages

As Fig. 1.10 indicates, for an abc sequence, first the terminal a reaches a positive maximum, then later in time terminal b reaches a positive maximum and still later in time terminal c reaches a positive maximum in voltage. The opposite is true for an acb sequence. Unless otherwise stated, the positive sequence voltage is always used in this text, the negative sequence is defined here since it proves useful in more advanced power system analysis texts. Since the lines connecting the load to the generator in Fig. 1.9 have negligible impedance, Kirchhoff's voltage law around each loop can be written,

$$E_{an} = V_{an} \quad (V)$$
$$E_{bn} = V_{bn} \quad (V) \qquad (1.27)$$
$$E_{cn} = V_{cn} \quad (V)$$

A balanced set of line to neutral voltages is then dropped across the balanced, wye-connected, load impedances.

1-9 BALANCED WYE-CONNECTED LOADS

Using V_{an} as a reference phasor, the load voltages are shown in Fig. 1.11. The memory scheme indicated will prove useful in Sect. 1 -12 - Meas. of Power.

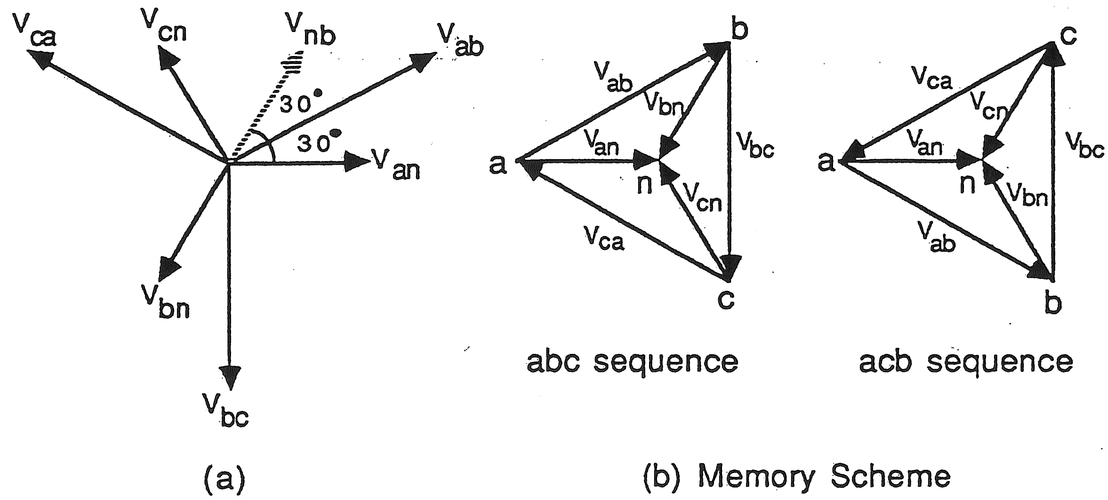


Figure 1.11 Wye - Load, Line and Load Voltages

The line voltages in Fig. 1.10, V_{ab}, V_{bc} and V_{ca}, are found by summing the voltages,

$$\begin{aligned} V_{ab} &= V_{an} + V_{nb} \\ V_{bc} &= V_{bn} + V_{nc} \qquad (1.28) \\ V_{ca} &= V_{cn} + V_{na} \end{aligned}$$

As indicated in Fig. 1.11, the magnitude of the line voltage is,

$$|V_{ab}| = 2\,|V_{an}| \cos 30° = \sqrt{3}\,|V_{an}| \qquad (V) \qquad (1.29)$$

The load currents are balanced and are given as,

$$\begin{aligned} I_a &= I_{an} = \frac{V_{an}}{Z_{an}} = |I_{an}| \angle\theta \qquad (A) \\ I_b &= I_{bn} = \frac{V_{bn}}{Z_{bn}} = |I_{bn}| \angle\theta \qquad (A) \qquad (1.30) \\ I_c &= I_{cn} = \frac{V_{cn}}{Z_{cn}} = |I_{cn}| \angle\theta \qquad (A) \end{aligned}$$

The line currents are the same as the load currents and they lead, lag or are phase with the load voltages by the load power factor angle, θ,

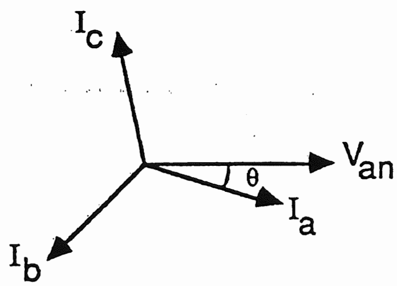


Figure 1.12 Wye - Load, Line and Load Currents

As can be seen from Fig. 1.12, at all instants of time,

$$I_n = I_a + I_b + I_c = 0 \tag{1.31}$$

The neutral line need not be connected but the neutrals of the generator and load are usually grounded for safety and other reasons developed later in this text.

In summary, for a wye-connected load,

$$\begin{aligned} |V_{line}| &= \sqrt{3}\,|V_{load}| \quad (V) \\ |I_{line}| &= |I_{load}| \quad (A) \end{aligned} \tag{1.32}$$

The voltage phasors in Fig. 1.11 can be translated in the complex plane provided their magnitudes and angles remain unchanged. Taking advantage of this principle, the voltage equations (1.28) need not be continually written to find the phase relationship between the line and load voltages; the phasors can be translated to form a closed triangle as indicated in the memory schemes of Fig. 1.11 (b). For an abc sequence, V_{bn} always lags V_{an} by 120°; V_{cn} always leads V_{an} by 120° and is so drawn. The magnitude and angle of the line voltages is then automatically given. For an acb sequence, V_{cn} always lags V_{an} by 120°, V_{bn} always leads V_{an} by 120° and is so drawn. The magnitude and angle of the line voltages is then automatically given. Furthermore the independent variable, time, need not be counted so that V_{an} is the reference phasor; V_{ab} could be the reference phasor, in which case, the entire triangle is rotated, clockwise through 30°. The memory scheme of Fig. 1.11 (b) will prove invaluable in determining phase relationships necessary in three-phase power measurements later in this chapter. For an abc sequence, balanced, inductive load, with θ = 15°, the memory scheme can be drawn,

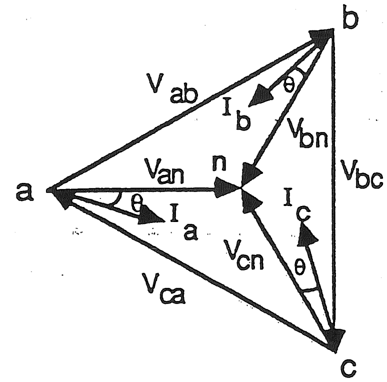
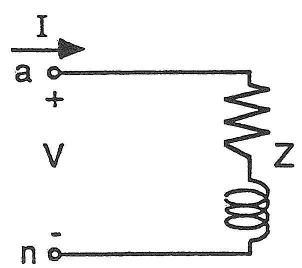


Figure 1.13 Memory Scheme For An Inductive Load

From Fig. 1.13, it can immediately be ascertained that I_a lags V_{ab} by 45°, that V_{bc} lags V_{an} by 90°, that I_c leads V_{an} by 105°, etc. These phase relationships, for a given reference phasor, become very important in power system analysis.

The three-phase load in Fig. 1.9 is a three-port device, each port is always defined line to neutral. This is always true in power system analysis, whether the device is a generator, transmission line, transformer bank or a motor. For a balanced load, exactly the same impedance is seen, line to neutral, whether looking into phase a, b or c. Phase a is usually selected as the per-phase equivalent circuit of a three-phase balanced load; identically the same circuits are obtained for phases b and c. The per-phase equivalent circuit for the load in Fig. 1.9 is,



where -

(*) $I = I_{line}$ (A)

(*) $V_{(load)} = \frac{V_{line}}{\sqrt{3}}$ (V)

Figure 1.14 Wye - Load, Per - Phase Equivalent Circuit

Example 1.9

A balanced, three-phase load consists of impedances, $Z = 10\ \underline{/15^\circ}\ \Omega$, driven by a 230 volt generator. (Voltages and currents, unqualified, are always line quantities in three-phase power analysis). Determine all voltages and currents in a wye connection if V_{an} is the reference phasor.

$$V_{an} = \frac{230}{\sqrt{3}}\underline{/0^\circ} \quad V \qquad ; \qquad v_{an} = \sqrt{2}\,(133)\cos\omega t \quad V$$

$$V_{bn} = 133\,\underline{/-120^\circ} \quad V \qquad ; \qquad v_{bn} = 188\cos(\omega t - 120^\circ) \quad V$$

$$V_{cn} = 133\,\underline{/120^\circ} \quad V \qquad ; \qquad v_{cn} = 188\cos(\omega t + 120) \quad V$$

From Fig. 1.13,

$$V_{ab} = 230\,\underline{/30^\circ} \quad V \qquad ; \qquad v_{ab} = \sqrt{2}\,(230)\cos(\omega t + 30^\circ) \quad V$$

$$V_{bc} = 230\,\underline{/-90^\circ} \quad V \qquad ; \qquad v_{bc} = 325\cos(\omega t - 90^\circ) \quad V$$

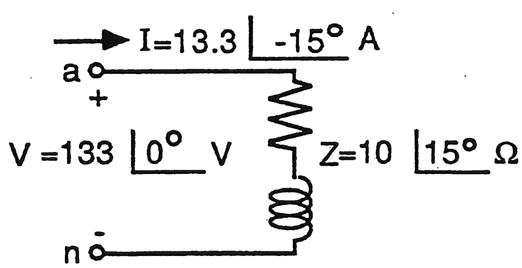
$$V_{ca} = 230\,\underline{/150^\circ} \quad V \qquad ; \qquad v_{ca} = 325\cos(\omega t + 150^\circ) \quad V$$

$$I_a = I_{an} = \frac{V_{an}}{Z} = \frac{133\,\underline{/0^\circ}}{10\,\underline{/15^\circ}} = 13.3\,\underline{/-15^\circ} \quad A \qquad ; i_a = \sqrt{2}\,(13.3)\cos(\omega t - 15^\circ) \quad A$$

$$I_b = I_{bn} = \frac{V_{bn}}{Z} = \frac{133\,\underline{/-120^\circ}}{10\,\underline{/15^\circ}} = 13.3\,\underline{/-135^\circ} \quad A \;\; ; \; i_b = 18.8\cos(\omega t - 135^\circ) \quad A$$

$$I_c = I_{cn} = \frac{V_{cn}}{Z} = \frac{133\,\underline{/120^\circ}}{10\,\underline{/15^\circ}} = 13.3\,\underline{/105^\circ} \quad A \qquad ; i_c = 18.8\cos(\omega t + 105^\circ) \quad A$$

The equivalent circuit is,



1-10 BALANCED DELTA-CONNECTED LOADS

Since loads can be connected in delta as well as wye, the balanced, delta-connected load is shown in Fig. 1.15.

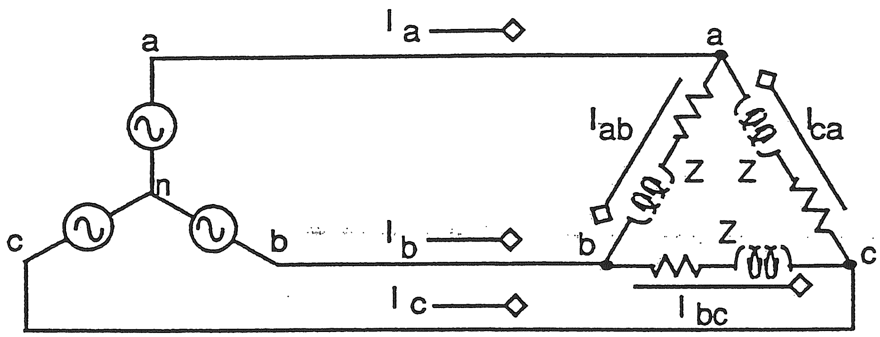


Figure 1.15 Three - Phase Circuit - Delta Load

In Fig. 1.15, the line voltages, V_{ab}, V_{bc} and V_{ca} are the same as the load voltages. The line currents can be found from Kirchhoff's current law at each node,

$$
\begin{aligned}
I_a &= I_{ab} - I_{ca} \qquad \text{(A)} \\
I_b &= I_{bc} - I_{ab} \qquad \text{(A)} \\
I_c &= I_{ca} - I_{bc} \qquad \text{(A)}
\end{aligned} \tag{1.33}
$$

If I_{ab} is the reference phasor,

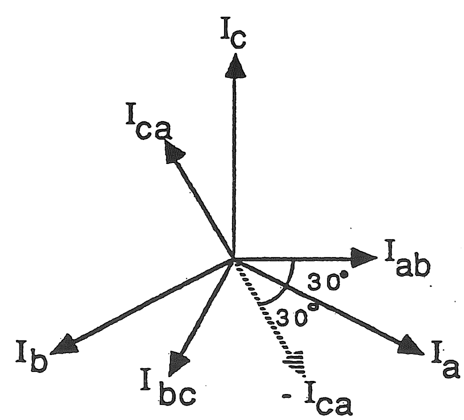


Figure 1.16 Delta Line and Load Currents

As indicated in Fig. 1.16, the magnitude of the line currents, is,

$$|I_a| = 2\,|I_{ab}| \cos 30° = \sqrt{3}\,|I_{ab}| \qquad \text{(A)} \tag{1.34}$$

In summary, for a delta-connected load,

$$
\begin{aligned}
|V_{line}| &= |V_{load}| \qquad \text{(V)} \\
|I_{line}| &= \sqrt{3}\,|I_{load}| \qquad \text{(A)}
\end{aligned} \tag{1.35}
$$

The delta-connected load is also a three-port device, with each port defined line to neutral. Since neutral does not physically exist in a delta-connected load, consider the equivalent wye-connected impedance in Fig. 1.17.

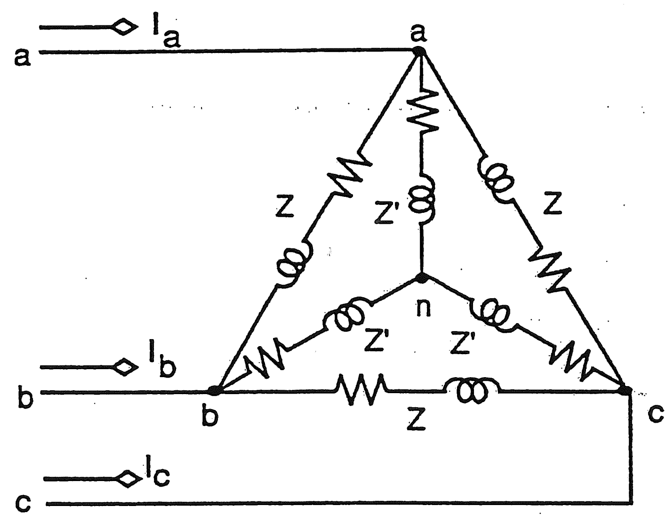


Figure 1.17 Delta - Wye Equivalent Circuits

To establish a neutral point for the delta-connected load, Z, it must be replaced by an equivalent Y-connected load, Z'. This equivalency is established if the line voltages and the resulting line currents are the same for either the delta or the equivalent wye-conected circuits.

Delta-connected circuit alone –

$$I_{ab} = \frac{V_{ab}}{Z} \quad ; \quad Z = \frac{V_{ab}}{I_a/\sqrt{3}} \quad (\Omega) \qquad (1.36)$$

Wye-connected circuit alone -

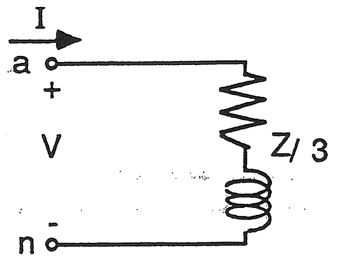
$$I_a = \frac{V_{ab}/\sqrt{3}}{Z'} \quad ; \quad Z' = \frac{V_{ab}/\sqrt{3}}{I_a} \quad (\Omega) \qquad (1.37)$$

If Eqns. (1.36) and (1.37) are divided,

$$Z' = \frac{Z}{3} \quad (\Omega) \qquad (1.38)$$

(*)Equation (1.38) states that a balanced, wye-connected impedance, which is one-third of a delta-connected impedance, will draw exactly the same line currents and consume exactly the same power as the delta-connected impedance for the same, balanced, line voltages.

The per-phase equivalent circuit can now be defined for the delta load,



where -

$$I = I_{line} \quad (A)$$

$$V = \frac{V_{line}}{\sqrt{3}} \quad (V)$$

Figure 1.18 Delta - Load, Per - Phase Equivalent Circuit

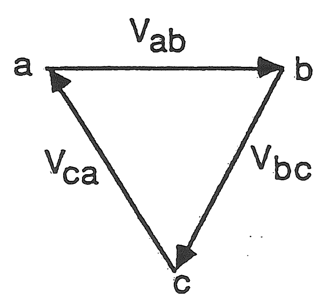
The interested reader can verify the fact that a delta-connected admittance can be replaced by an equivalent, wye-connected admittance if,

$$Y' = 3\,Y \qquad (\mho) \qquad (1.39)$$

As will be seen subsequently, in system analysis, when equivalent, per-phase circuits are discussed, neutral must be established throughout the system by replacing all delta-connected components by equivalent wye-connected components.

Example 1.10

A balanced, three-phase load consists of impedances, $Z = 10 \angle 15° \ \Omega$, driven by a 230 volt generator. (Voltages and currents, unqualified, are always line quantities in three-phase power analysis). Determine all voltages and currents in a delta connection if V_{ab} is the reference phasor.



$$V_{ab} = 230 \angle 0° \qquad V$$

$$V_{bc} = 230 \angle -120° \qquad V$$

$$V_{ca} = 230 \angle 120° \qquad V$$

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{230\angle 0^\circ}{10\angle 15^\circ} = 23\angle{-15^\circ} \quad \text{A}$$

$$I_{bc} = \frac{V_{bc}}{Z} = \frac{230\angle{-120^\circ}}{10\angle 15^\circ} = 23\angle{-135^\circ} \quad \text{A}$$

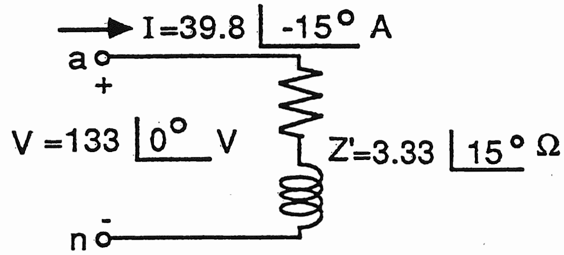
$$I_{ca} = \frac{V_{ca}}{Z} = \frac{230\angle 120^\circ}{10\angle 15^\circ} = 23\angle 105^\circ \quad \text{A}$$

$$I_a = I_{ab} - I_{ca} = 39.8\angle{-45^\circ} \quad \text{A}$$

$$I_b = I_{bc} - I_{ab} = 39.8\angle{-165^\circ} \quad \text{A}$$

$$I_c = I_{ca} - I_{bc} = 39.8\angle 75^\circ \quad \text{A}$$

The equivalent circuit is,



1-11 POWER IN BALANCED THREE-PHASE CIRCUITS

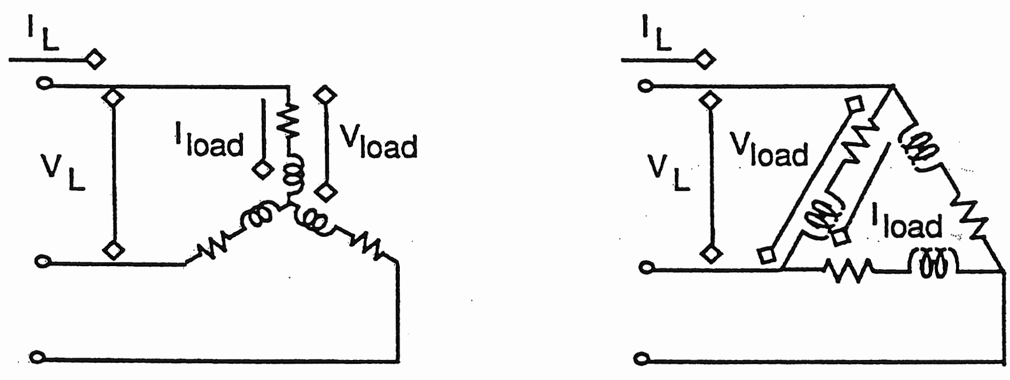


Figure 1.19 Total Power in Wye and Delta Loads

The total power delivered to either the wye or delta-connected loads in Fig. 1.19 is the sum of the powers delivered to each load impedance,

basic	wye	delta	
$\lvert S\rvert = 3\, V_{load}\, I_{load}$	$= 3 \frac{V_L}{\sqrt{3}} I_L$	$= 3\, V_L \frac{I_L}{\sqrt{3}}$	(VA)
$P = 3\, V_{load}\, I_{load} \cos\theta$	$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\theta$	$= 3\, V_L \frac{I_L}{\sqrt{3}} \cos\theta$	(W)
$Q = 3\, V_{load}\, I_{load} \sin\theta$	$= 3 \frac{V_L}{\sqrt{3}} I_L \sin\theta$	$= 3\, V_L \frac{I_L}{\sqrt{3}} \sin\theta$	(VAR)

where θ is the angle between $\underline{V_{load}}$ and $\underline{I_{load}}$.

In terms of line quantities, the total power delivered to either a wye or delta-connected load has the same expressions,

$$\lvert S\rvert = \sqrt{3}\, V_L\, I_L \qquad \text{(VA)} \tag{1.40}$$

$$P = \sqrt{3}\, V_L\, I_L \cos\theta \qquad \text{(W)} \tag{1.41}$$

$$Q = \sqrt{3}\, V_L\, I_L \sin\theta \qquad \text{(VAR)} \tag{1.42}$$

where θ is the angle between V_{load} and I_{load}, and $\cos\theta$ is the power factor of the load, for I_{load} leading, lagging or in phase with V_{load}.

The power equations (1.40), (1.41), (1.42) are extremely important in power system analysis, since the ratings of three-phase components are always given in terms of line voltage and current and total power. Three-phase voltage, current and power, unqualified, are always line and total power.

Observe in Eqns. (1.40) and (1.41), the magnitude of the line current can be calculated for either a wye or delta connection,

$$I_L = \frac{\lvert S\rvert \text{ (VA)}}{\sqrt{3}\, V_L} = \frac{P \text{ (W)}}{\sqrt{3}\, V_L \cos\theta} \qquad \text{(A)} \tag{1.43}$$

It must be realized that three-phase power, in terms of line and total quantities, is distinguished from single-phase power by the $\sqrt{3}$. Furthermore, the line current is determined by the numerator in Eqn. (1.43); if the numerator is given in VA, the power factor is not used; if the numerator is given in W, the power factor must be used.

Example 1.11

(a) A three-phase, Y-connected, generator is rated,100 MVA, 13.8 kV. Calculate rated current –

$$I_L \text{ (rated)} = \frac{(100{,}000)}{\sqrt{3}\,(13.8)} = 4{,}180 \quad \text{A}$$

(b) A three-phase,Δ-connected, load is rated 100 MW, 0.8 pf lagging,13.8 kV. Calculate rated current and load impedance –

$$I_L \text{ (rated)} = \frac{(100{,}000)}{\sqrt{3}\,(13.8)(0.8)} = 5{,}230 \quad \text{A}$$

$$Z' = \frac{(13{,}800)/\sqrt{3}\ \underline{/0^\circ}}{(5{,}230)\ \underline{/-36.9^\circ}} = 1.52\ \underline{/36.9^\circ} \quad \Omega$$

$$Z = 3\,Z' = 4.56\ \underline{/36.9^\circ} \quad \Omega$$

Calculate the power delivered to the load under rated conditions –

$$|S| = \sqrt{3}\,V_L\,I_L = \sqrt{3}\,(13{,}800)(5{,}230) = 125 \quad \text{MVA}$$

$$P = \sqrt{3}\,V_L\,I_L \cos\theta = \sqrt{3}\,(13{,}800)(5{,}230)\cos 36.9^\circ = 100 \quad \text{MW}$$

$$Q = \sqrt{3}\,V_L\,I_L \sin\theta = \sqrt{3}\,(13{,}800)(5{,}230)\sin 36.9^\circ = 75 \quad \text{MVAR}$$

1-12 MEASUREMENT OF POWER IN THREE-PHASE CIRCUITS

1. Single-phase wattmeter

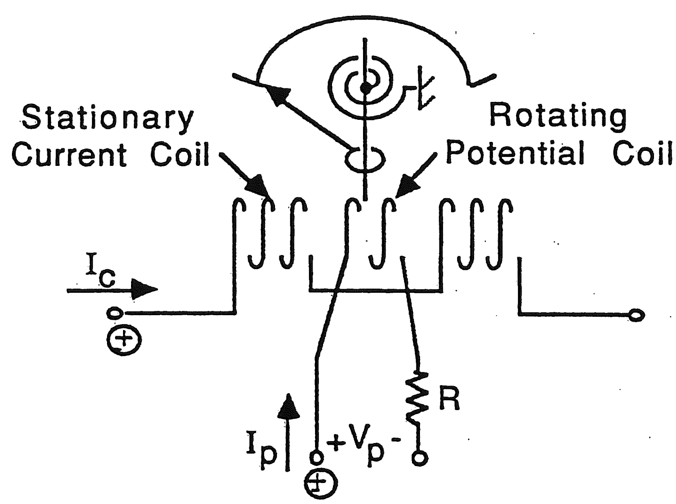


Figure 1.20 Single - Phase Wattmeter

The single-phase wattmeter consists of a current coil and a potential coil in series with a high resistance, R. The instantaneous power delivered to a load is the product of the instantaneous current and voltage. The current through the potential coil is approximately equal to the voltage across the potential-coil circuit divided by R. If positive currents flow into the ± terminals of each coil, the interaction between the magnetic fields of these currents will drive the potential coil, mounted on a spring-loaded shaft, up-scale. Because of the inertia of the potential coil and shaft, the deflection is constant for given coil currents at the average value of the power being measured. The wattmeter is calibrated so that it will always read the product of the rms voltage across the potential coil circuit times the rms current through the current coil times the cosine of the angle between them.

$$P = V_p I_c \cos {}_p\theta_c \qquad \text{(W)}$$

The ± terminals of the coils in Fig. 1.20 are very important in the circuit connection of a wattmeter. The correct connection is shown in Fig. 1.21.

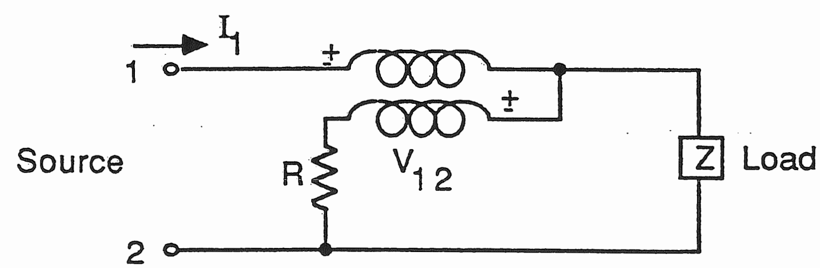


Figure 1.21 Wattmeter Circuit Connection

As connected in Fig. 1.21, the wattmeter will read,

$$P = V_{12} I_1 \cos {}_v\theta_I \qquad \text{(W)}$$

$$(P \neq V_{21} I_1 \cos {}_v\theta_I)$$

The ± terminal of the potential circuit must always be connected to the current-coil line; if the potential circuit is reversed, because of high R, the potential of the potential coil is at full input voltage with respect to line 1 and the difference in voltage between the two coils could break them down thus destroying the wattmeter. As shown in Fig. 1.21, each coil is at the same potential thus minimizing the above risk.

In summary, a wattmeter will always read the product of the voltage across it times the current through it, times the cosine of the angle between the voltage and current.

2. Three-wattmeter measurement of power.

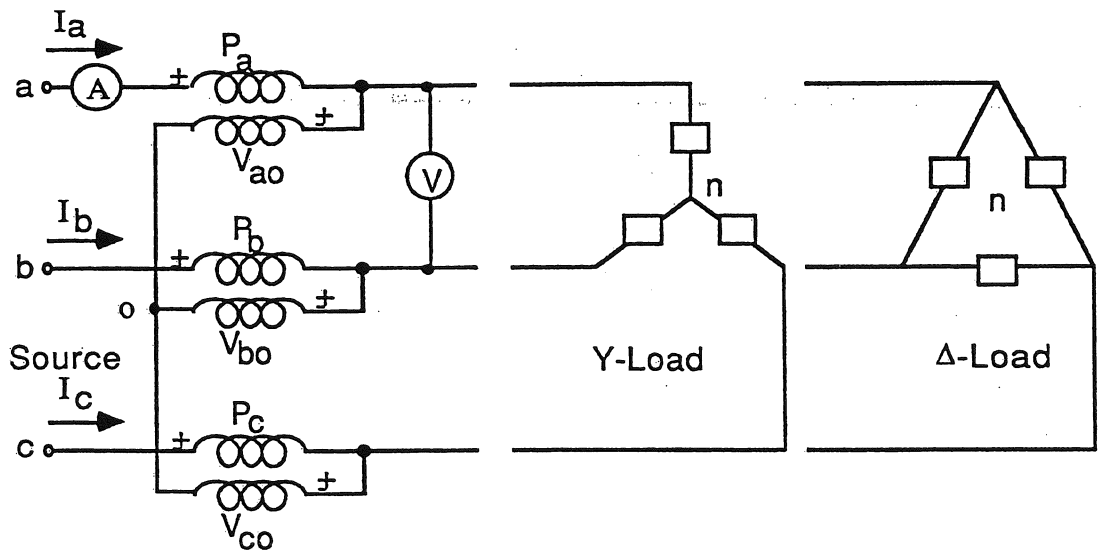


Figure 1.22 Three Wattmeter Connection

The instantaneous power delivered to either wye or delta load is,

$$p = v_{an}\, i_a + v_{bn}\, i_b + v_{cn}\, i_c \qquad \text{(VA)} \qquad (1.44)$$

The power read by the wattmeter is,

$$P = V_{ao} I_a \cos\theta_a + V_{bo}\, I_b \cos\theta_b + V_{co}\, I_c \cos\theta_c \qquad \text{(W)} \qquad (1.45)$$

$$P = \quad P_a \quad + \quad P_b \quad + \quad P_c$$

The average value of Eqn. (1.44) is indeed the same as Eqn. (1.45) since, for a balanced load, the neutral of the wattmeters is at the same potential as the neutral of the loads. If the neutral of the wattmeters is <u>not</u> connected to the neutral of the loads, it can be shown that the sum of the wattmeter readings is the correct total power for unbalanced as well as balanced loads. For balanced load,

$$\cos\theta = \frac{P_a + P_b + P_c}{\sqrt{3}\, VI}$$

3. Two-wattmeter measurement of power

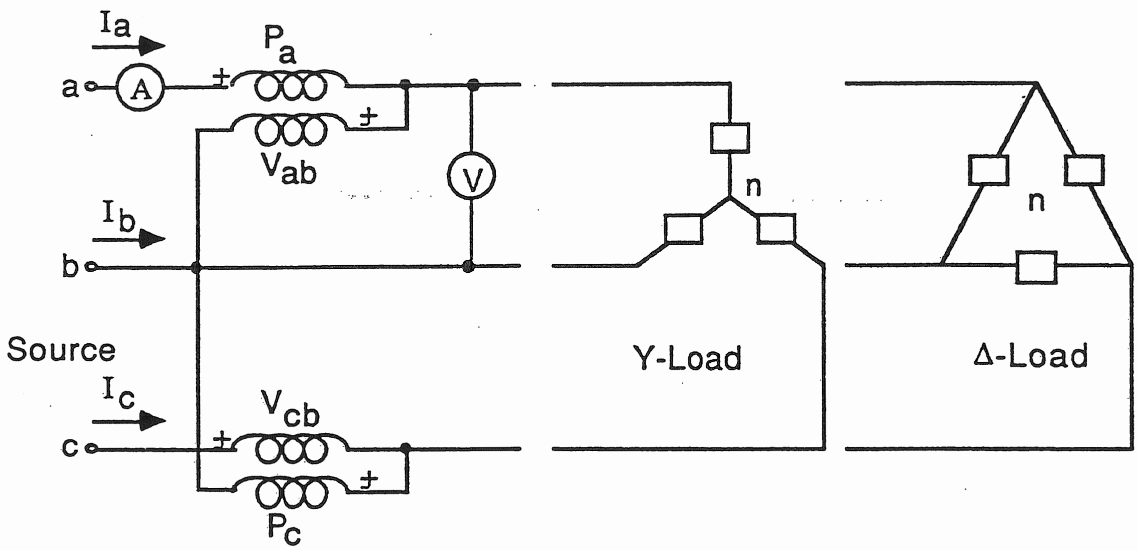


Figure 1.23 Two Wattmeter Connection

The instantaneous power delivered to either wye or delta load is,

$$p = v_{an}\, i_a + v_{bn}\, i_b + v_{cn}\, i_c \qquad \text{(VA)} \tag{1.46}$$

where,

$$v_{an} = v_{ab} + v_{bn} \tag{1.47}$$

$$v_{cn} = v_{cb} + v_{bn}$$

Substituting Eqn. (1.47) in Eqn. (1.46),

$$p = (v_{ab} + v_{bn})\, i_a + v_{bn}\, i_b + (v_{cb} + v_{bn})\, i_c$$

$$= v_{ab}\, i_a + v_{cb}\, i_c + v_{bn}(\overset{0}{i_a + i_b + i_c})$$

$$p = v_{ab}\, i_a + v_{cb}\, i_c \tag{1.48}$$

Equation (1.48) is true because, according to Kirchhoff's current law, the sum of the line currents, in Fig. 1.23, balanced or unbalanced, is zero.

The power read by the wattmeter is,

$$P = V_{ab}\; I_a \cos\theta_a + V_{cb} I_c \cos\theta_c \qquad \text{(W)} \tag{1.49}$$

$$P = \quad P_a \quad + \quad P_c$$

which is the average power of Eqn. (1.48).

Care must be taken, however, in interpreting the readings of each wattmeter in Eqn. (1.49), as shown in Fig. 1.24.

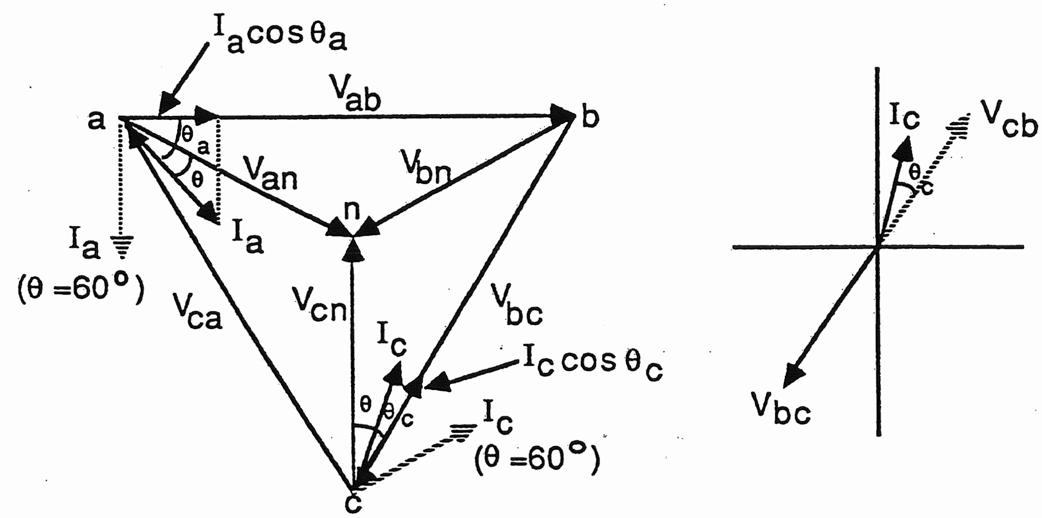


Figure 1.24 Inductive-Load Voltage and Current Phasors

For a balanced inductive load, I_a lags V_{an}, and I_c lags V_{cn} by the power factor angle, θ, of the load, as indicated in Fig. 1.24. The wattmeter readings, however, from Eqn. (1.49), are proportional to the projection of I_a on V_{ab} through an angle θ_a, and to the projection of I_c on V_{cb} through an angle θ_c, as indicated in Fig. 1.24. For an <u>inductive</u> load, Eqn. (1.49) can be rewritten.

$$P = V_{ab} I_a \cos(30° + \theta) + V_{cb} I_c \cos(30° - \theta) \quad \text{(W)} \qquad (1.50)$$

$$= \quad P_a \quad + \quad P_c$$

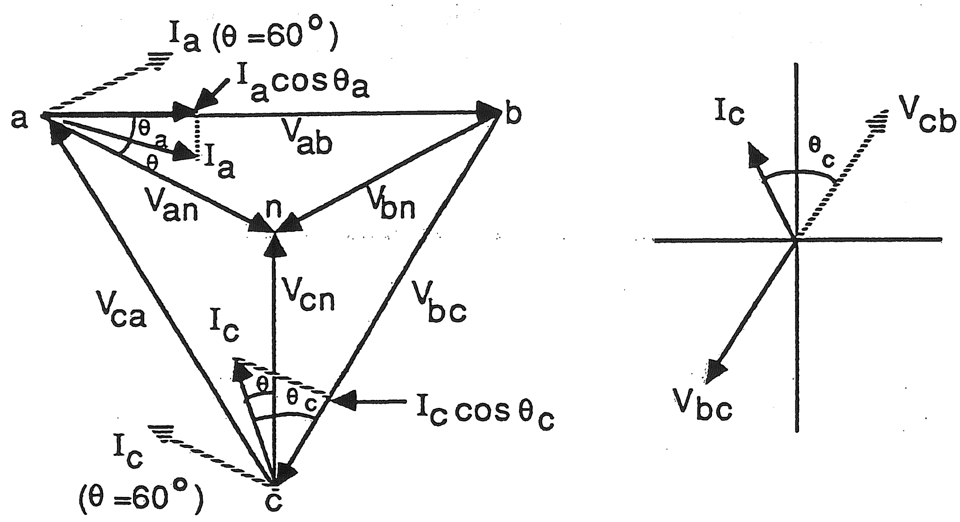


Figure 1.25 Capacitive-Load Voltage and Current Phasors

For a balanced, capacitive load, I_a leads V_{an}, and I_c leads V_{cn} by the power factor angle, θ, of the load as indicated in Fig. 1.25. For a capacitive load, Eqn. (1.49) can be rewritten,

$$P = V_{ab}\, I_a \cos\,(30° - \theta) + V_{cb} I_c \cos\,(30° + \theta) \quad (W) \qquad (1.51)$$

When the load power factor equals 0.5, $\theta = 60°$; the projection of I_a on V_{ab}, in Fig. 1.24, is zero; therefore $P_a = 0$ for an inductive load. For a capacitive load, when the load power factor is 0.5, $\theta = 60°$, the projection of I_c on V_{bc} in Fig. 1.25 is zero; therefore $P_c = 0$ for a capacitive load. Thus, when the power factor of a balanced three-phase load is,

1. pf = 1.0, ($\theta = 0°$), $P_a = P_c$ and $P = P_a + P_c$ (W)
2. pf = 0.5 lagging, ($\theta = 60°$), $P_a = 0$ and $P = P_c$ (W)
3. pf = 0.5 leading, ($\theta = 60°$), $P_c = 0$ and $P = P_a$ (W)
4. pf < 0.5 lagging, ($\theta > 60°$), P_a = neg. and $P = -P_a + P_c$ (W)
5. pf < 0.5 leading, ($\theta > 60°$), P_c = neg. and $P = P_a - P_c$ (W)

Equations (1.50) and (1.51) confirm the above statements and one or the other wattmeter can read negative if the load power factor is less than 0.5. When a wattmeter reads negative, the current coil (not the potential circuit) must be reversed, in which case, the total power for the two-wattmeter method is the algebraic sum of the wattmeter reading for any load power factor.

It can be shown that the algebraic sum of the wattmeter readings is the correct total power for unbalanced as well as balanced loads. For balanced loads,

$$\cos\theta = \frac{P_a + P_c}{\sqrt{3}\,VI}$$

1-13 SUMMARY

Electric power systems are driven by sinusoidal voltages with resulting sinusoidal currents. Power system analysis is greatly simplified by representing instantaneously varying, sinusoidal quantities as,

$$f(t) = \text{Re}\,\{\sqrt{2}\,(Ae^{j\phi})\,e^{j\omega t}\}$$

where the phasor corresponding to this quantity is,

$$F = Ae^{j\phi}$$

The phasor is a directed line element in the complex plane and can be expressed in polar or rectangular form. Sinusoidally varying quantities can then be added, subtracted, multiplied or divided, very efficiently, by vectorially manipulating their phasors. Phasor diagrams, then, become very important in the definition of impedance and admittance and their effect in power circuits.

Power flow is basic to power system analysis and for single-phase circuits, apparent, real and reactive power is,

$$|S| = VI \qquad \text{(VA)}$$

$$P = VI\cos\theta \qquad \text{(W)}$$

$$Q = VI\sin\theta \qquad \text{(VAR)}$$

whereas for three-phase circuits,

$$|S| = \sqrt{3}\,V_L I_L \qquad \text{(VA)}$$

$$P = \sqrt{3}\,V_L I_L\cos\theta \qquad \text{(W)}$$

$$Q = \sqrt{3}\,V_L I_L\sin\theta \qquad \text{(VAR)}$$

Care must be taken to remember that three-phase power equations are characterized by the $\sqrt{3}$ - factor and that the power factor angle, θ, is always the angle between the load voltage and current.

Complex power is equally important since power magnitude and direction are given by,

$$S = VI^* \text{ (VA)}$$

and the corresponding power triangle proves invaluable in many power system analyses.

Three-phase circuits are characterized by wye or delta-connected loads where, for a,

wye load		delta load
$\lvert V_L \rvert = \sqrt{3}\,\lvert V_{load} \rvert$	;	$\lvert V_L \rvert = \lvert V_{load} \rvert$
$\lvert I_L \rvert = \lvert I_{load} \rvert$	;	$\lvert I_L \rvert = \sqrt{3}\,\lvert I_{load} \rvert$

Balanced, three-phase components, including generators, transmission lines, transformer banks, motors and passive loads, all have three input ports. Each port is defined, line to neutral, and neutral must be defined throughout the power system, in which case all delta connections are replaced by their wye equivalents. The per-phase equvalent circuit, then, is used extensively in power system analysis.

Power flow can be measured in three-phase circuits, using the two or three-wattmeter method. Care must be exercised in observing the coil polarities when connecting a wattmeter in a circuit. Depending on the load power factor, one or the other wattmeter can read negative in the two-wattmeter method as indicated by its equation,

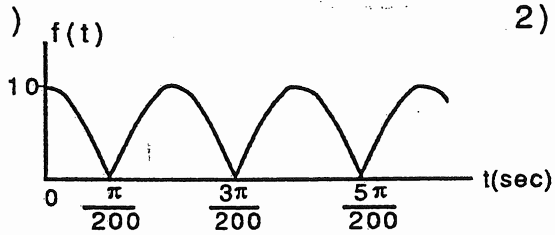
$$P = V_{ab} I_a \cos(30° \pm \theta) + V_{cb} I_c \cos(30° \mp \theta) \quad \text{(W)}$$

PROBLEMS

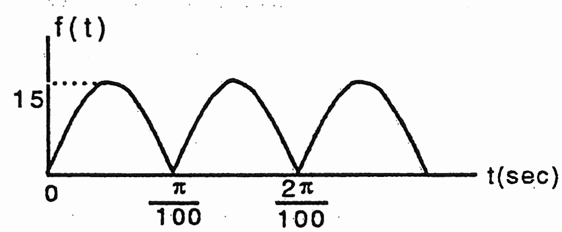
1.1 For each of the following sinusoidal or triangular waveforms, find,

(a) the period, frequency and angular frequency of the waveform.
(b) the average value of the waveform.
(c) the rms value of the waveform.

1)



2)



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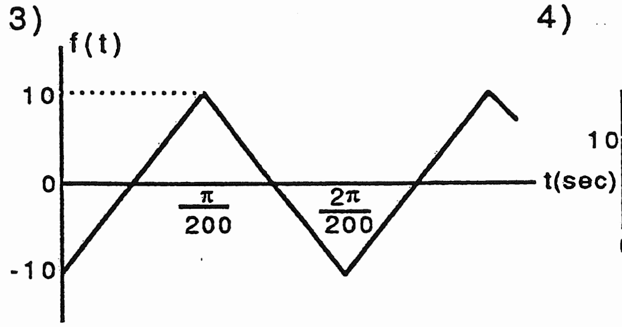

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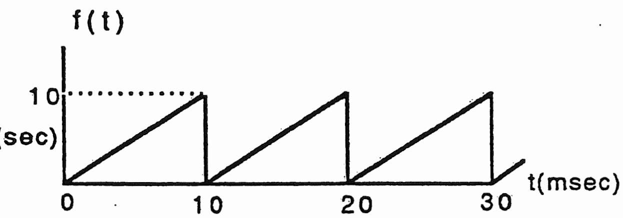


1.2 A current i(t) is given, (t in sec.),

$$i(t) = 2t - \cos t \quad \text{(A)}$$

and is repetitive with the period of the cosine, [$f(t) = f(t + nT)$].

(a) Find the average and rms values for i(t).

(b) What is the repetition frequency, Hz, of f(t)?

(c) If the current, i(t), flows through a resistor, 10 Ω, find the power dissipated in the resistor.

1.3 Convert the following expressions to rectangular form,

(a) $10\,\underline{/60^\circ}$

(b) $12\,e^{j\pi/6}$

(c) $\dfrac{4\sqrt{2}\,\underline{/45^\circ}}{2+j2}$

(d) $20\,\underline{/30^\circ} - 5\,\underline{/105^\circ}$

(e) $(8\,\underline{/45^\circ})(6 + j\,6)$

(f) $2 + j\,5 + 7\,\underline{/45^\circ}$

1.4 Given the voltages,

$v_1(t) = 15 \sin(\omega t + 60°)$

$v_2(t) = 20 \cos \omega t$

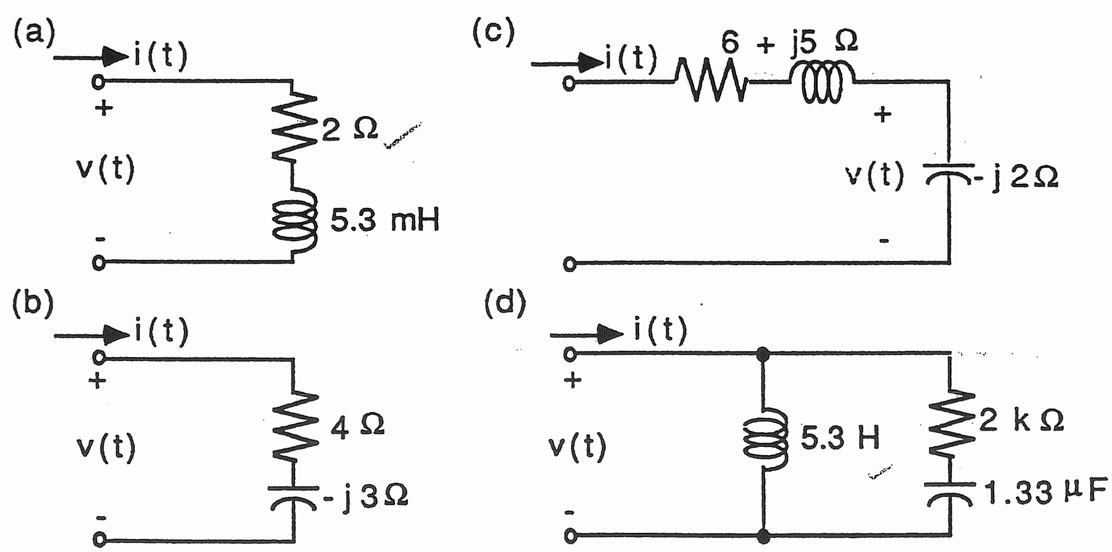
$v_3(t) = 30 \sin(\omega t + \frac{\pi}{6})$

(a) Find the phasors V_1, V_2, V_3

(b) Find $V_4 = V_1 + V_2 + V_3$

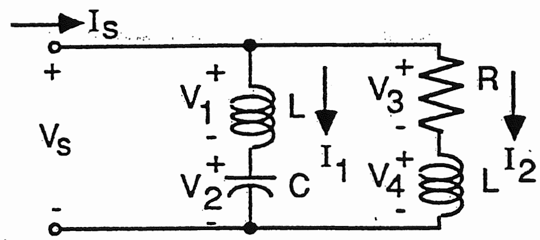
(c) Write $v_4(t)$.

1.5 If v(t) = 170 cos(377t + 30°), find the current, i(t), for each of the following circuits,



1.6 Draw and numerically label a phasor diagram showing <u>all</u> the voltages and currents for each of the circuits in Problem 1.5.

1.7 For the circuit shown,



$$V_S = 120\,\angle 0^\circ\ \text{V}$$
$$\omega = 100\ \text{rad/sec}$$
$$L = 0.1\ \text{H}$$
$$C = 2\ \text{mF}$$
$$R = 10\ \Omega$$

(a) Find the currents and voltages.

(b) Draw and label the phasor diagram corresponding to (a).

1.8 Given the voltage across and the current through an impedance,

$$v(t) = V_{max} \sin \omega t$$

$$i(t) = I_{max} \sin (\omega t + \phi)$$

(a) Plot v, i, and p for a positive value of ϕ, for a negative value of ϕ, and for $\phi = 0$, where p is the instantaneous value.

(b) Show that the average power is,

$$P = \frac{V_{max} I_{max}}{2} \cos \phi \quad \text{(W)}$$

1.9 Find the real and reactive power delivered or absorbed by the source in Problem 1.7. Draw the input power triangle.

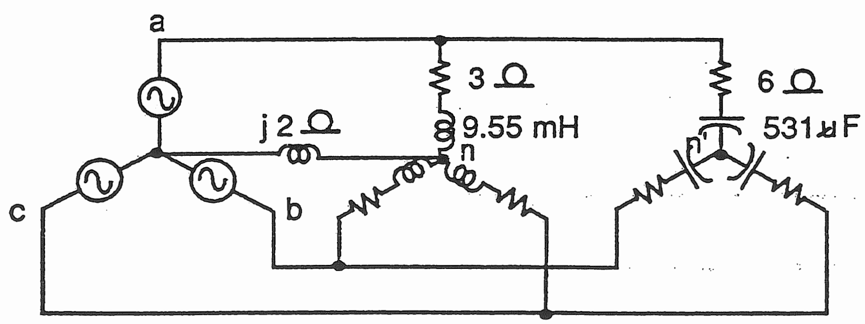
1.10. A single-phase motor draws a current of 20A at 0.8 pf lagging when operated from a 220 V, 60 Hz source. Find the value of the capacitance across the motor, that will raise the power factor to 0.9 lagging.

1.11 A balanced, three-phase, Y-connected load is connected to an abc sequence generator with $V_{an} = 100 \angle 0°$ V. Find the load and line voltages and draw and label the voltage-phasor diagram.

1.12 A balanced, three-phase, delta-connected load is connected to an acb sequence generator with $I_a = 10 \angle 60°$ A. Find the load and line currents and draw and label the current-phasor diagram.

1.13 In Figure 1.9, the 208 volt generator drives a balanced 20 + j 0 Ω load impedance. Find the line currents, the neutral current and the total power delivered to the load.

1.14



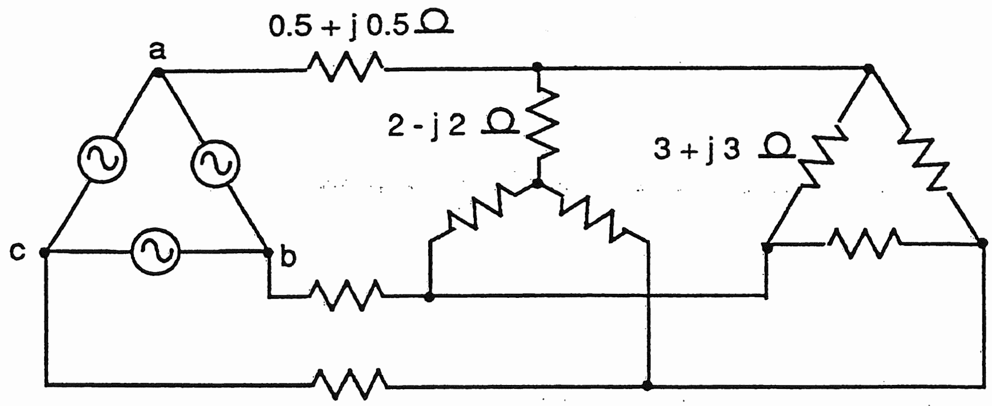
Generator : 3ϕ, 440 V, 50 Hz
Loads : Balanced

(a) Draw the per-phase equivalent circuit.

(b) Find the currents of each load and the source.

(c) Find the total real and reactive power absorbed or delivered by each load and the source.

1.15



A balanced system is shown with,

$$v_{ab} = 294.2 \cos(\omega t + 30°) \quad V$$

(a) Draw a per-phase equivalent circuit.
(b) Find the load voltages and currents of the Y-connected load.
(c) Find the total real and reactive power and power factor of the Y-connected load.
(d) Find the load voltages and currents of the Δ-connected load.
(e) Find the total real and reactive power and power factor of the Δ-connected load.
(f) Find the source line currents.
(g) Find the total real and reactive power and power factor of the source.
(h) Sketch and numerically label the input power triangle to this system.

1.16 A three-phase induction motor draws an input current of 15A at 0.8 pf lagging, when operated from a 240 V source. If the efficiency of the motor is 95%, find,

(a) the total real and reactive power supplied to the motor.
(b) the output power of the motor.
(c) the balanced, parallel, delta-connected value of the impedance that allows the motor to draw the same current at unity pf.

1.17 A balanced Y-connected load of 3 + j 4 Ω is connected to a balanced 208-volt source.

(a) Using the three-wattmeter method, calculate the wattmeter readings, P_a, P_b, P_c.
(b) Using the two-wattmeter method, calculate the wattmeter readings, P_a, P_c.
(c) Draw and label the voltage-current phasor diagrams corresponding to each method.

1.18 Do Problem 4.17 for a balanced, Δ-connected load of 3 - j 9 Ω.

CHAPTER 2

MAGNETIC CIRCUITS

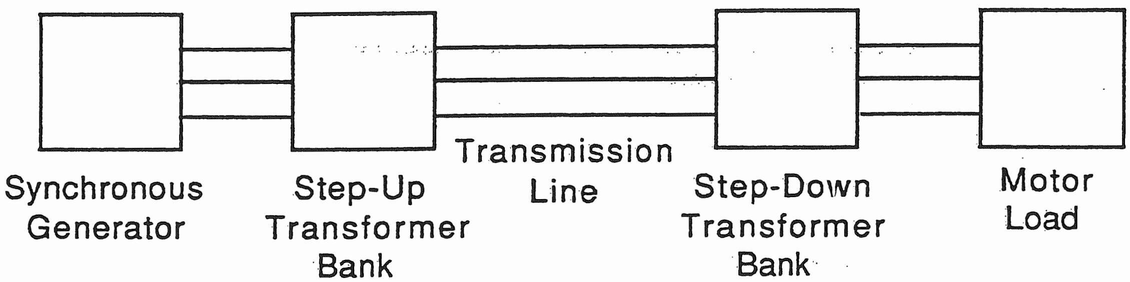


Figure 2.1 Elementary Power System

As shown in the elementary power system of Fig. 2.1, electric power systems consist primarily of components that utilize magnetic fields. The generators, transformers, transmission lines and inductive loads that comprise a power system all convert, transfer or absorb power by means of a magnetic field. Large blocks of power are transferred by these systems which is made possible by the use of ferrous materials that permit high magnetic densities. A simplified analysis of magnetic circuit is required, therefore, to establish the characteristics and performance of the power system components that use these ferrous materials.

2-1 MAXWELL'S "MMF" LAW

A basis for the analysis of all magnetic circuits is Maxwell's mmf law and corollary in integral form.

$$\int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right)\cdot d\mathbf{a} = \oint_C \mathbf{H}\cdot d\mathbf{l} \qquad \text{(A)} \qquad (2.1)$$

where J = current density, A/m^2
D = electric flux density, C/m^2
H = magnetic intensity, A/m

$$\oint_S \mathbf{B}\cdot d\mathbf{a} = 0 \qquad \text{(Wb)} \qquad (2.2)$$

where B = flux density, Wb/m^2 or Teslas

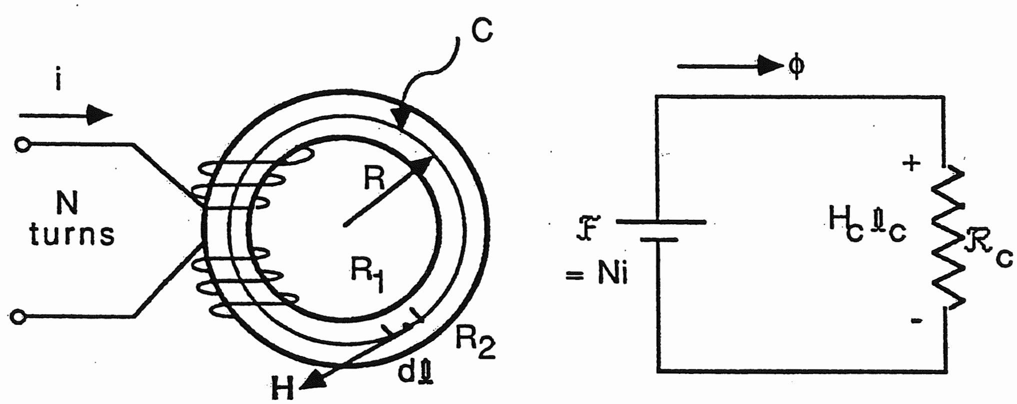
In Eqn. (2.1) the time variation of the electric field is so small at power system frequencies that it is negligible and Maxwell's mmf law becomes Ampere's Law.

$$\int_S \mathbf{J} \cdot d\mathbf{a} = \oint_C \mathbf{H} \cdot d\mathbf{l} \qquad \text{(A)} \qquad (2.3)$$

Maxwell's mmf law in Eqn. (2.3) is a very general law in that any closed contour, C, can be chosen in space that bounds any surface, S. If the chosen contour encloses a current distribution over S, (lefthand integral) then a magnetic field, H, is produced (righthand integral). This law, in integral form, is usually practical only for symmetrical magnetic devices; such a device is a toroid.

2-2 TOROID

The basic magnetic circuit, from which more-complicated circuits can be analyzed, is the toroidal coil. The toroidal coil of Fig. 2.2 has a doughnut-shaped core of circular cross section with a finely wound coil of wire of N turns.



(a) Toroid Magnetic Circuit

(b) Analog Magnetic Circuit

Figure 2.2

The toroid in Fig. 2.2(a) is placed in a polar coordinate system concentric with the origin. Since the toroid possesses circular symmetry, a circular contour, C, with radius, R, will be chosen concentric with the origin, halfway between the toroid radii, R_1 and R_2. This contour is known as the mean path for

the flow of flux. The current, i, flowing through N-turns creates a magnetic field within the toroid and the direction of this field is predicted with the right-hand rule. The coil is grasped with the fingers in the direction of current flow and the thumb indicates the direction of flux flow. The magnetic intensity, H, A-t/m is assumed constant, because of symmetry, and tangential at all points along the contour in the direction predicted by the right-hand rule. The current density, J, A/m^2, in the coil pierces the surface, S, which is bounded by the contour and is perpendicular to this surface. The dot products of both sides of Eqn. (2.3) are summed and this equation becomes,

$$N\,i = H\,2\pi R \qquad \text{(A-t)} \qquad (2.4)$$

Equation (2.4) reveals the existence of a magnetic field whenever the contour of radius R encloses a net current N i. The magnetic intensity of this field is,

$$H = \frac{N\,i}{2\pi R} \qquad \text{A-t/m}$$

and is plotted versus the contour radius in Fig. 2.3.

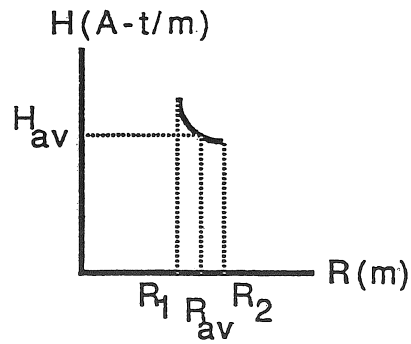


Figure 2.3 Toroid Magnetic Field

For contour radii smaller than R_1 and larger than R_2, no net current is enclosed and therefore no field exists outside the toroid. For a given current, i, the field within the toroid varies inversely with radius R. If the cross section radius of the toroid is small compared to the toroid dimensions, the magnetic intensity variation is small and can be considered constant at all points within the toroid volume at its average value along the mean path, ℓ_c, in the core.

$$H_{av} = \frac{N\,i}{2\pi R_{av}} = \frac{N\,i}{\ell_c} \qquad \text{(A-t/m)} \qquad (2.5)$$

With this assumption, Maxwell's MMF law in Eqn (2.5) is single-dimensionalized for mathematical tractability in the analysis of more complex magnetic circuits. The average magnetic intensity within the core is related to the flux density by the permeability, μ, of the core material.

$$B_{av} = \mu H_{av} \qquad (T) \qquad (2.6)$$

Equation (2.6) is nonlinear for ferrous materials such as cast iron, steel and steel alloys; it is linear for all nonferrous materials such as air, wood, porcelain, copper, etc. The magnetization curves of Eqn. (2.6) are plotted in Fig. 2.4.

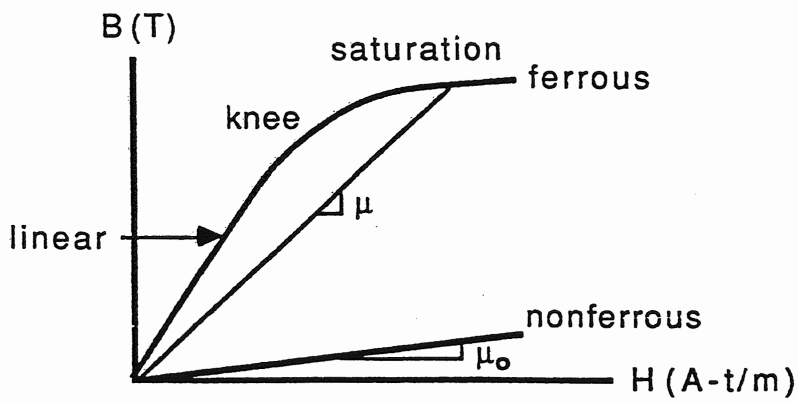


Figure 2.4 Ferrous and Nonferrous Magnetization Curves

As Fig. 2.4 indicates, the permeability of ferrous materials to flux flow is variable with flux density. The knee of the curve separates the linear and saturation regions and its flux density is typically 1.0 T. Permeability is greatest in the linear region and is usually 2000-6000 times the permeability of nonferrous materials. The permeability of nonferrous materials is constant with flux density and has the value of free space, $\mu_o = 4\pi \times 10^{-7}$ T/A-t/m or H/m. Since ferrous materials are highly permeable they are used in most power system components. Keeping in mind that the permeability of ferrous materials is a variable in Eqn. (2.6), Equation (2.5) can be written,

$$B_{av} = \frac{\mu}{l_c} N\, i \qquad (T) \qquad (2.7)$$

The flux density in Eqn. (2.7) is considered constant within the toroid at its average value and is integrated over the toroid cross section to obtain the magnetic flux.

$$\phi = \int_{S_1} B \cdot da = B_{av} A_c \qquad \text{(Wb)} \qquad (2.8)$$

where area, S_1, is the cross section of the core through which the flux flows.

Equation (2.7) then becomes,

$$\phi = \frac{\mu A_c}{\ell_c} N\, i \qquad \text{(Wb)} \qquad (2.9)$$

The current flowing through N-turns in Eqn. (2.9) is defined as the magneto-motive force, $\mathcal{F}$, of the toroid.

$$\mathcal{F} = N\, i = \frac{\ell_c}{\mu A_c} \phi = \mathcal{R}_c\, \phi \qquad \text{(A-t)} \qquad (2.10)$$

Equation (2.10) can be interpreted as a magnetomotive force in a magnetic circuit that causes flux to flow through a reluctance which is directly proportional to the mean length of path and inversely proportional to the permeability of the core material and cross-section area.

2-3 ANALOG MAGNETIC CIRCUIT

Equation (2.10) is analogous to an electric circuit whose electromotive force is,

$$e = R\, i$$

The analog of the magnetic circuit for the toroid is shown in Fig. 2.2(b). Here the magnetomotive force rise of N i, A-t causes a flux, ϕ, to flow around a closed mean path , ℓ_c, whose reluctance is $\mathcal{R}_c = \ell_c / \mu A_c$. The magnetic drop across the reluctance is, according to Maxwell's mmf law, equal to $H_c \ell_c$, A-t.

Example 2.1

Consider a toroid with a coil of 500 turns that carries a current of 160 mA. The mean length of path is 0.4 m and the cross-section area is 4 x 10^{-4} m^2. What is the flux if the core is made of silicon steel? of porcelain ?

Silicon Steel	Porcelain
$\mathcal{F} = N\,i = (500)(0.16) = 80$ A-t	$\mathcal{F} = N\,i = (500)(0.16) = 80$ A-t
$H = N\,i/\ell_c = 80/0.4 = 200$ A-t /m	$H = N\,i/\ell_c = 80/0.4 = 200$ A-t /m
$B = 1.0$ T (from Fig. 2.5)	$B = \mu_o H = (4\pi \times 10^{-7})(200) = 2.5 \times 10^{-4}$ T
$\phi = BA = (1.0)(4 \times 10^{-4}) = 400\ \mu$Wb	$\phi = BA = (2.5 \times 10^{-4})(4 \times 10^{-4}) = 0.1\ \mu$Wb

Example 2.1 is important because it illustrates a general method of analyzing magnetic circuits. When the mmf driving the magnetic circuit is given, the average magnetic intensity is found using the mean-path length in the core. The flux density can then be found, graphically if the core is ferrous, and analytically if the core is nonferrous. Finally, the flux is calculated using the cross-section area.

The calculation order F, H, B, ϕ is useful when the mmf is given. The opposite order is useful when a desired flux is given. Several observations can be made concerning Example 2.1.

When the core is ferrous, a current of 160 mA flowing through 500 turns establishes a magnetic field at the knee of the magnetization curve; currents higher than this will drive the core into saturation and currents lower than this will permit linear operation.The permeability of the ferrous core at a flux density of 1.0 T is calculated as, $\mu = B/H = 0.005$ H/m. The relative permeability is,

$$\mu_r = \mu/\mu_o = 4000$$

(*) i.e., the ferrous core is 4000 times more permeable than the nonferrous core. Reluctance, in general, is never calculated since it varies, for ferrous cores, with the driving mmf.

Maxwell's mmf law can now be usefully stated as the sum of the magnetic rises equals the sum of the magnetic drops around any closed magnetic path,

$$\int_S \mathbf{J}\cdot d\mathbf{a} = \oint_C \mathbf{H}\cdot d\boldsymbol{\ell} \qquad \text{(A)} \qquad (2.11)$$

$$\Sigma N i = \Sigma H \ell = \Sigma \mathcal{R} \phi$$

Equation (2.11) is analogous to Kirchhoff's voltage law.

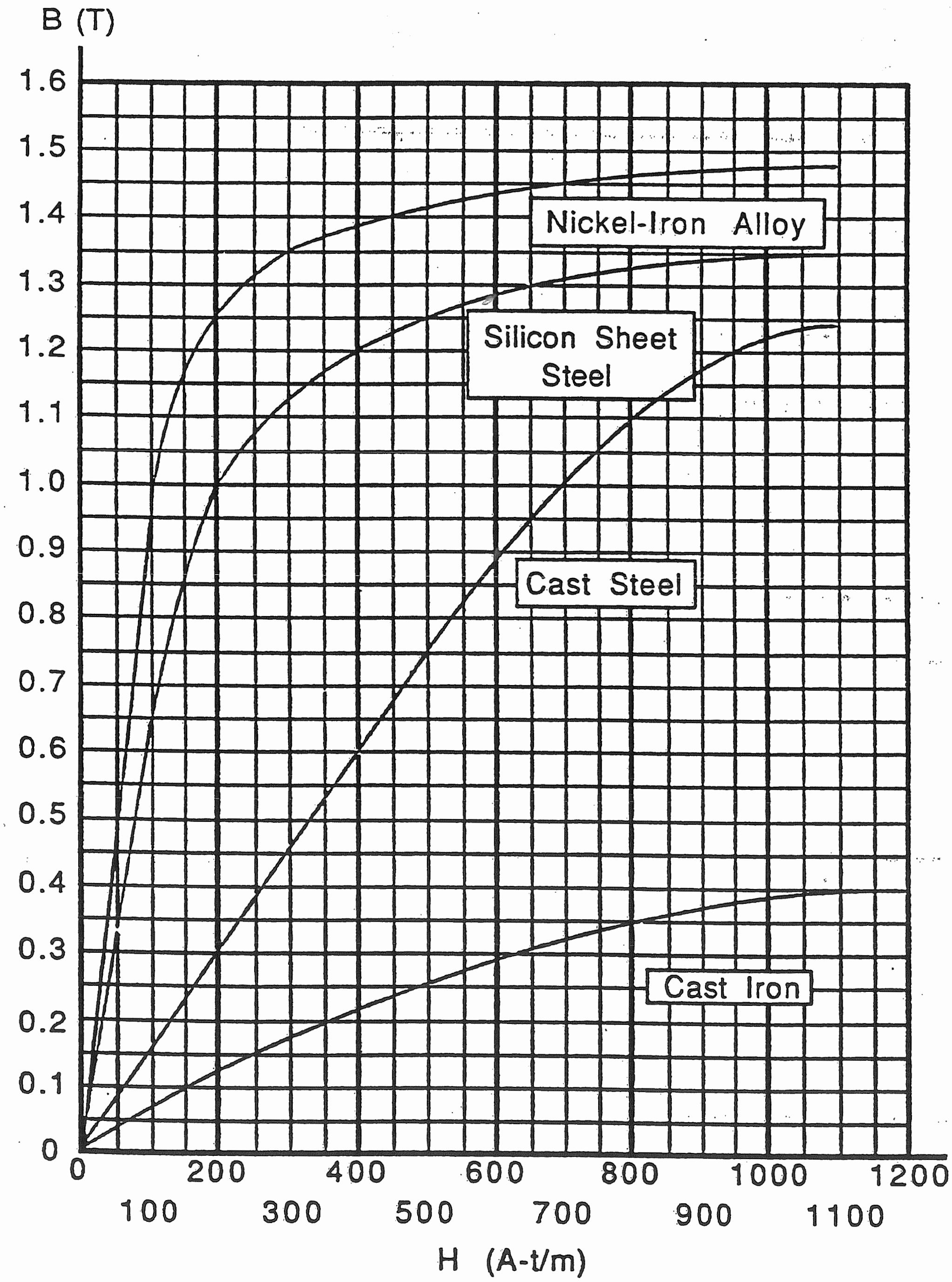


Figure 2.5 Magnetization Curves for Typical Magnetic Materials

Maxwell's corollary law is,

$$\oint_S \mathbf{B} \cdot d\mathbf{a} = 0 \qquad \text{(Wb)} \qquad (2.12)$$

$$\Sigma\ \phi = 0$$

and this law can be usefully stated as the sum of the magnetic fluxes that enter a magnetic node must be equal to the sum of the magnetic fluxes that leave this node. Equation (2.12) is analogous to Kirchhoff's nodal law. In fact, Eqns. (2.11) and (2.12) permit the analysis of magnetic circuits using all of the familiar electric-circuit, analytic techniques. The analogous magnetic and electric quantities are listed in Table 2.1.

Magnetic		Electric	
Magnetomotive Force	$\mathcal{F}$	Electromotive Force	e
Magnetic Intensity	**H**	Electric Intensity	**E**
Flux Density	**B**	Current Density	**J**
Flux	ϕ	Current	I
Reluctance	$\mathcal{R}$	Resistance	R
Permeability	μ	Conductivity	σ

Table 2.1 Analogous Magnetic and Electric Quantities

2-4 A GENERAL ANALYTIC PROCEDURE

The application of Maxwell's mmf law to the toroid will now be extended to more complex magnetic circuits. Equations (2.11) and (2.12) will be the basis for analyzing series and parallel magnetic circuits. Before these equations are used, however, a definite procedure is recommended.

1. Observe the magnetic device and identify all the rises in mmf (coils). Determine the directions of these mmfs using the right-hand rule.

2. Determine the distribution of flux flow and identify all magnetic nodes, i.e., points where the flux divides.

3. Using good engineering judgment, determine mean paths for flux flow throughout the device.

4. Draw the analog magnetic circuit based on the above judgments. This circuit is also useful for applying known concepts of electric-circuit analysis.

5. When drawing the analog magnetic circuit, reluctances must be chosen for each mean length of path. Realizing that $\mathcal{R} = \ell/\mu A$, a different reluctance is chosen for each mean length of path whose permeability (material) and cross-section are reasonably constant. Ferrous paths are represented by variable reluctances and nonferrous paths by constant reluctances.

6. The mmf dropped across each reluctance or the flux through each reluctance is determined by the F, H, B, ϕ analysis used in Example 2.1 combined with Eqns. (2.11) and (2.12).

2-5 SERIES MAGNETIC CIRCUITS

Magnetic circuits and their analysis are very important in visualizing some of the components of power systems-especially energy-conversion components such as motors and generators. Energy cannot be converted unless an air gap exists in the magnetic circuit. The air gap is very important because it is here that a very strong magnetic field should be present. The conditions for obtaining this strong field are emphasized in Example 2.2 and the characteristics of an air gap are illustrated in Fig. 2.6.

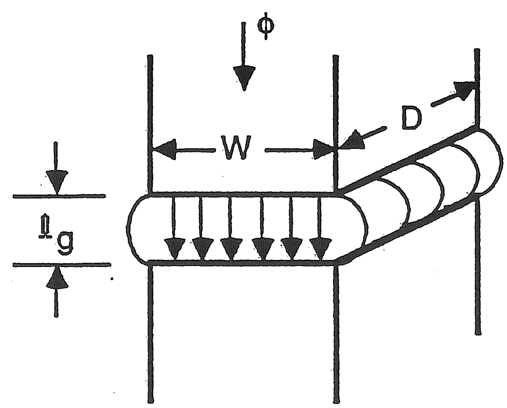


Figure 2.6 Air-Gap Fringing

The difference in magnetic potential across the air gap in Fig.2.6 is usually very large resulting in flux fringing. Because of air gap fringing, the effective area of the gap through which the flux flows is larger than that of the core. One way to account for this would be to calculate the effective gap area by adding ℓ_g to each dimension of the core. This is not done for small gaps,

$\ell_g << \ell_c$, since the overall accuracy of the graphical analysis does not warrant this correction. Fringing for small gaps is usually neglected. A series magnetic circuit with an air gap will be considered in Example 2.2. The magnetization curves for typical magnetic materials are given in Fig. 2.5.

Example 2.2

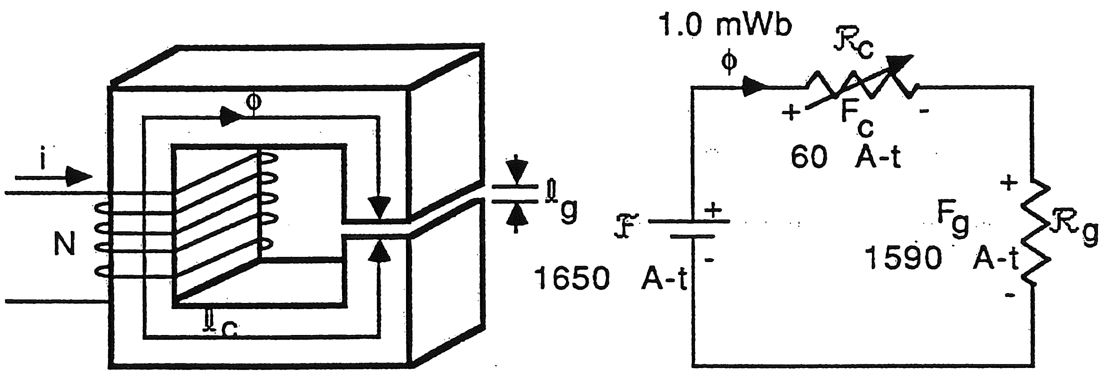


Figure 2.7 Series Magnetic Circuit

A silicon-steel core has a uniform cross-section area of 10 cm^2, a mean path-length of 30 cm, and a gap length of 2 mm. Find the current in the 1000-turn coil that will result in an air-gap flux of 1.0 mWb. Neglect air-gap fringing.

Silicon Steel	Gap
$\phi = 0.001$ Wb	$\phi = 0.001$ Wb
$B_c = \phi/A_c = 0.001/10\times10^{-4} = 1.0$ T	$B_g = \phi/A_g = 0.001/10\times10^{-4} = 1.0$T
$H_c = 200$ A-t/m (From Fig. 2.5)	$H_g = B_g/\mu_o = 1.0/4\pi \times 10^{-7} = 796{,}000$ A-t/m

$F_c = H_c\ell_c = (200)(30 \times 10^{-2}) = 60$ A-t $F_g = H_g\ell_g = (796{,}000)(2 \times 10^{-3}) = 1590$ A-t

$$\mathcal{F} = F_c + F_g = 60 + 1590 = 1650 \text{ A-t}$$
$$i = \mathcal{F}/N = 1650/1000 = 1.65 \text{ A}$$

Aside from the procedure for analysis in Example 2.2, an important observation in a well-designed core with an air gap is that the mmf dropped across the iron is negligible compared with the mmf dropped across the gap. The ferrous path simply acts as a channel of low reluctance so that the full driving mmf is dropped across the gap with a resultant high gap-flux density.

2-6 PARALLEL MAGNETIC CIRCUITS

Most magnetic circuits encountered in power systems are complex, resulting in parallel or series-parallel magnetic circuits. It is here that drawing the analog magnetic circuit becomes most important in applying electric-circuit principles to the solution of magnetic circuit problems. A typical parallel magnetic circuit analysis is given in Example 2.3.

Example 2.3

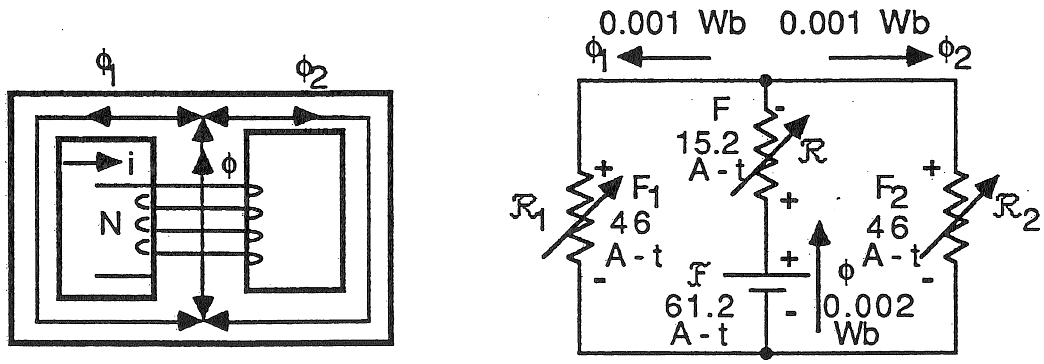


Figure 2.8 Parallel Magnetic Circuit

The silicon-steel core shown in Fig. 2.8 has a rectangular cross section and the cross-section area of legs 1 and 2 is uniform at 10 cm^2; the cross-section area of the center leg is 20 cm^2. The mean length of path of legs 1 and 2 is each 23 cm. The mean length of path of the center leg is 7.6 cm. What current is required in the 100-turn coil to establish a flux in the center leg of 2.0 mWb?

<u>center leg</u>	<u>since $\mathcal{R}_1 = \mathcal{R}_2 = \ell/\mu A$, consider leg 1 or 2</u>
$\phi = 0.002$ Wb	$\phi_1 = \phi_2 = \phi/2 = 0.002/2 = 0.001$ Wb
$B = \phi/A = 0.002/20 \times 10^{-4} = 1.0$ T	$B_1 = \phi_1/A_1 = 0.001/10 \times 10^{-4} = 1.0$ T
$H = 200$ A-t/m (From Fig. 2.5)	$H = 200$ A-t/m (From Fig. 2.5)
$F = H\ell = (200)(7.6 \times 10^{-2}) = 15.2$ A-t	$F_1 = F_2 = H_1\ell_1 = (200)(23 \times 10^{-2}) = 46$ A-t

$$\mathcal{F} = F + F_1 = 15.2 + 46 = 61.2 \text{ A-t}$$

$$i = \mathcal{F}/N = 61.2/100 = 0.612 \text{ A}$$

To this point, magnetic circuits have been driven with dc or time-invariant currents. They could also be driven by time-varying currents, in which case, Maxwell's mmf law still holds for each instant of time, and the foregoing analyses are valid at each instant. The effect, however, on the magnetic core and the time-varying electric circuit driving the core will now be examined.

2-7 MAXWELL'S "EMF" LAW

$$\oint_C \mathbf{E}' \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a} \tag{2.13}$$

where E' = E + u x B = electric intensity in a frame moving at u, m/s.
E = electric intensity in a fixed frame, V/m
u = velocity of contour, m/sec
B = flux density, Wb/m^2 or Teslas

Maxwell's emf law in Eqn. (2.13) is a very general law in that any closed contour, C, can be chosen in space that bounds any surface, S. If the chosen contour is stationary and encloses a time-varying flux density distribution over S, or if a moving or time-varying contour encloses a constant or time-varying flux density distribution over S, an electric field will be induced along the contour. Maxwell's emf law is also called Faraday's law.

For the purposes of this section, the chosen contour will be stationary, (u=o), enclosing a time-varying, uniform flux density distribution. Equation (2.13) then becomes,

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a} \quad \text{(V)} \tag{2.14}$$

now, $$e \triangleq \int_C \mathbf{E} \cdot d\mathbf{l} \quad \text{(V)} \qquad \text{and} \qquad \phi = \int_S \mathbf{B} \cdot d\mathbf{a} \quad \text{(Wb)}$$

then, $$e = -\frac{d\phi}{dt} \quad \text{(V)} \tag{2.15}$$

where, e is the total emf induced across the coil.

Consider now, a magnetic circuit driven by a time-variant source.

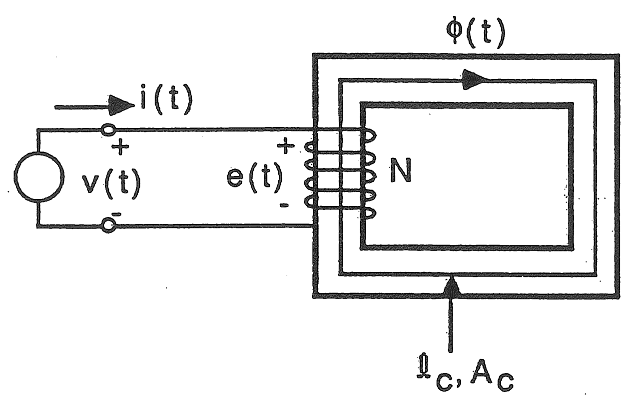


Figure 2.9 Timevarying Magnetic Field

The time-variant source in Fig. 2.9 produces a time-varying field in the core of the magnetic circuit; Maxwell's mmf law still holds at every instant of time but his emf law now becomes very important. The N-turn coil in Fig. (2.9), (refigured for clarity), will be chosen as the contour, C, in Eqn. (2.14). This refigured contour bounds a surface,S, as shown in Fig. (2.10).

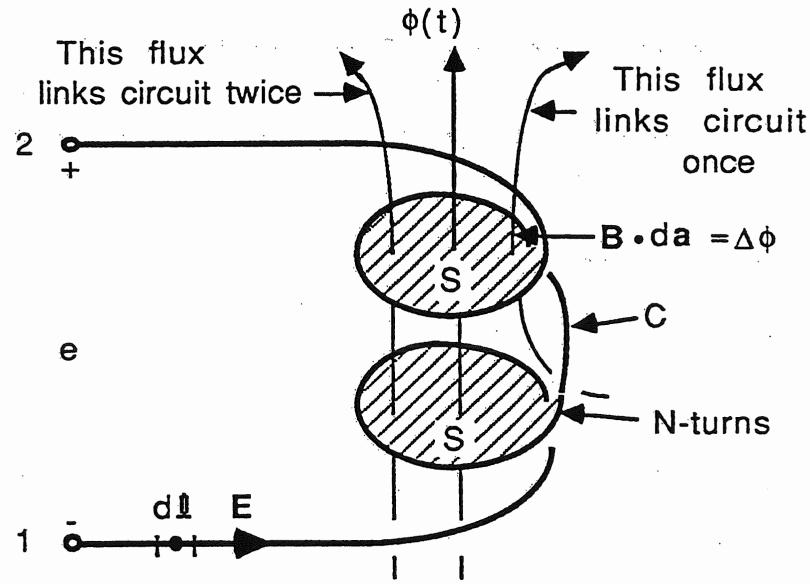


Figure 2.10 Flux Linkages of Figure 2.9

The electric circuit in Fig. 2.10 is shown at the instant of time when the flux, ϕ, is <u>decreasing.</u> An electric field, **E,** will be induced, as indicated, according to,

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{a}$$

In the left-hand integral, $\mathbf{E}\cdot d\mathbf{l}$ is the incremental emf induced along the contour, and the electric field, when line integrated from 1 to 2, is the summation of these incremental emfs or the total induced emf, e, across the coil terminals.

In the right-hand integral, $\mathbf{B}\cdot d\mathbf{a}$ is defined as the <u>incremental flux linkage</u>

of the coil, i.e., each increment of flux, $\mathbf{B}\cdot d\mathbf{a}$, that pierces the surface, S, constitutes a flux linkage. The normal component of the magnetic field, when integrated over the entire surface, S, is the summation of these incremental flux linkages or is the total flux linkage of the coil,

$$\lambda \triangleq N\phi \qquad \text{(Wb-t)}$$

From the above integral equation, in terms of flux linkage, Faraday's law can be written,

$$e = -\frac{d(N\phi)}{dt} = -\frac{d\lambda}{dt} \qquad (V) \qquad (2.16)$$

where λ = total flux linkages of the coil (Wb-t).

If all of the flux links all of the turns, then,

$$e = -N\frac{d\phi}{dt} = -\frac{d\lambda}{dt} \qquad (V)$$

In the form of Eqn. (2.16), Faraday's law states that the emf induced in a stationary open (no current flow) or closed (current flow) circuit is equal to the negative time-rate of change of the magnetic flux linking the circuit. In this form, the direction of the emf is such that if the coil were short-circuited, at any instant of time, the induced emf would drive a current in such a direction as to oppose the change in flux linkage - this is known as Lenz's law. For the remainder of this text, the induced emf will be defined as,

$$|e| \triangleq \frac{d(N\phi)}{dt} = \frac{d\lambda}{dt} \qquad (2.17)$$

and Lenz's law will determine its direction.

2-8 INDUCTANCE OF SINGLE-COIL MAGNETIC CIRCUITS

In the analysis of time-varying power systems, it is desirable to model the components of this system-motors, generators, transformers and transmission lines- with electric-circuit elements. The electric-circuit element that represents the magnetic field of each of these components is inductance. The inductance of a coil is defined as the ratio of the flux linkages of the coil to the current that produced these flux linkages,

$$L \triangleq \frac{N\phi}{i} = \frac{\lambda}{i} \qquad \text{(Wb-t/A or H)} \qquad (2.18)$$

or,

$$\lambda = Li \qquad \text{(Wb-t)}$$

The inductance in Eqn. (2.18), as will be seen, is a physical property of a N-turn coil wound on any magnetic circuit. An important assumption is made at this point. The definition for inductance is valid only when ferrous magnetic circuits are operated in the linear region of their B-H curves or the magnetic circuits have air gaps assuring linearity.

Consider the magnetic circuit of Fig. 2.11

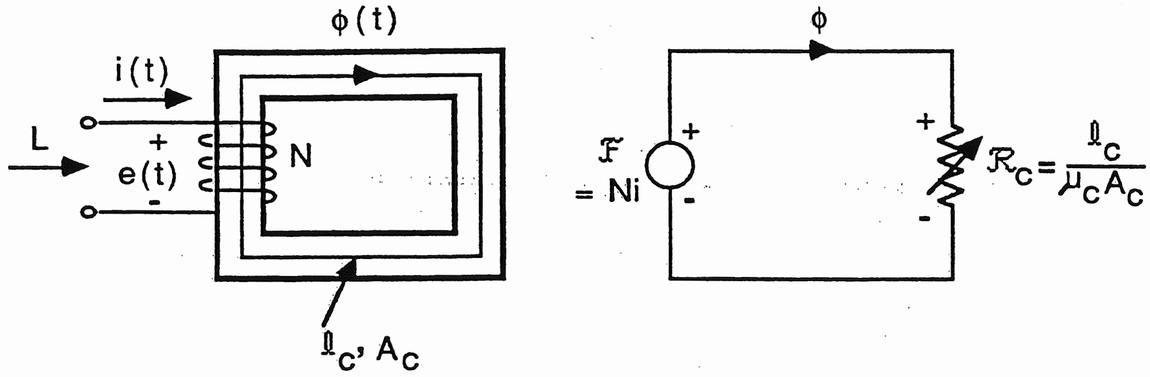


Figure 2.11 A Timevarying Magnetic Circuit

The inductance looking into the coil terminals of Fig. 2.11 can be found by using its analog magnetic circuit to calculate the flux linkages per unit current that link this coil.

$$\phi = Ni/\mathcal{R}_c = \frac{\mu_c A_c N}{\ell_c} i \qquad \text{(Wb)} \qquad (2.19)$$

$$L = \frac{N\phi}{i} = \frac{\lambda}{i} = \frac{\mu_c A_c N^2}{\ell_c} \qquad \text{(H)} \qquad (2.20)$$

Equation (2.19), multiplied by N, is plotted in Fig. 2.12.

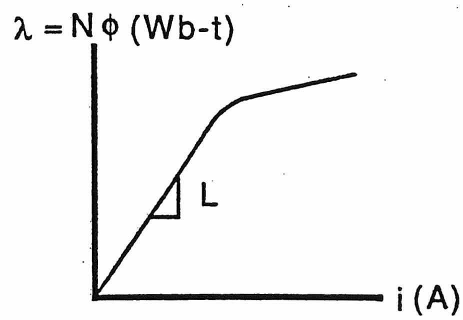


Figure 2.12 Magnetization Curve of Figure 2.11

Inductance, as indicated in Fig. 2.12, is the slope of the λ-i curve and is constant in the linear region of this curve. Inductance, from Eqn. 2.20, is a function only of the geometry of the coil and the magnetic circuit, and can be calculated for any magnetic circuit in a manner similar to the foregoing analysis.

Faraday's emf induced in the coil can now be found in terms of the coil inductance. Since $\lambda = L\,i$,

$$e = \frac{d(N\phi)}{dt} = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} \qquad \text{(V)}$$

and if the inductance is constant,

$$e = L\frac{di}{dt} \qquad \text{(V)} \qquad (2.21)$$

The energy stored in the magnetic field of the circuit in Fig. 2.11 can now be found by calculating the instantaneous power delivered to the coil and realizing that this energy <u>must be stored</u> in the field.

$$p = e\,i = \frac{d\lambda}{dt}i = \frac{dW_f}{dt} \qquad \text{(W)}$$

or,

$$dW_f = i\,d\lambda$$

then,

$$W_f = \int_0^{\lambda} i\,d\lambda \qquad \text{(J)} \qquad (2.22)$$

Graphically, this integral is shown in Fig. 2.13.

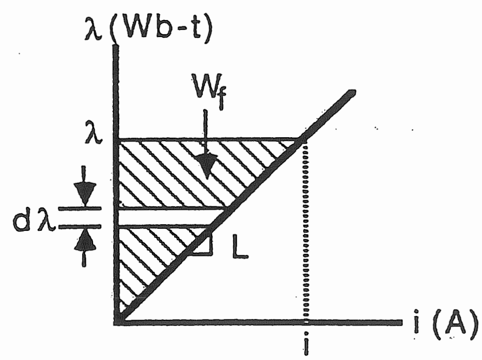


Figure 2.13 Energy Stored in the Magnetic Field

It is important to observe from Fig. 2.13, the energy stored is the area to the <u>left</u> of the magnetization curve, i.e., for a linear λ-i relationship,

$$W_f = \frac{1}{2}\lambda i = \frac{1}{2}L i^2 \qquad \text{(J)} \qquad (2.23)$$

Example 2.4

(a) Consider the silicon-steel toroid of Example 2.1 with a coil of 500 turns, carrying a current of 160 mA with a mean length of path of 0.4 m and a cross-section area of $4x10^{-4}$ m^2. What is the coil inductance, (H)?

$$L = \frac{\mu_c A_c N^2}{l_c} = \frac{(1/200)(4x10^{-4})(500)^2}{0.4} = 1.25 \text{ H}$$

or,

$$= \frac{N\phi}{i} = \frac{(500)(400x10^{-6})}{0.16} = 1.25 \text{ H}$$

b) Calculate the energy stored in the magnetic field.

$$W_f = \frac{1}{2}\lambda i = (\frac{1}{2})(500)(400 \times 10^{-6})(0.16) = 0.016 \text{ Joule}$$

or,

$$= \frac{1}{2} L i^2 = (\frac{1}{2})(1.25)(0.16)^2 = 0.016 \text{ Joule}$$

(c) If the coil current is varying with time, $i = 0.16 \sin 377\, t$ (A);

What is the Faraday emf generated across the coil terminals?

$$e = L\frac{di}{dt} = (1.25)(0.16)(377) \cos 377\, t$$

$$= 75.4 \cos 377\, t \qquad \text{(V)}$$

When ferromagnetic cores are cyclically excited, the magnetic state of the cores involves a storage and release of energy which is not completely reversible, i.e., two important losses are incurred. The first loss is called the hysteresis loss and is associated with the fact that the magnetization curve for cyclic excitation is not single valued but is a double valued, nonlinear function of the mmf driving the core as shown in Fig. 2.14.

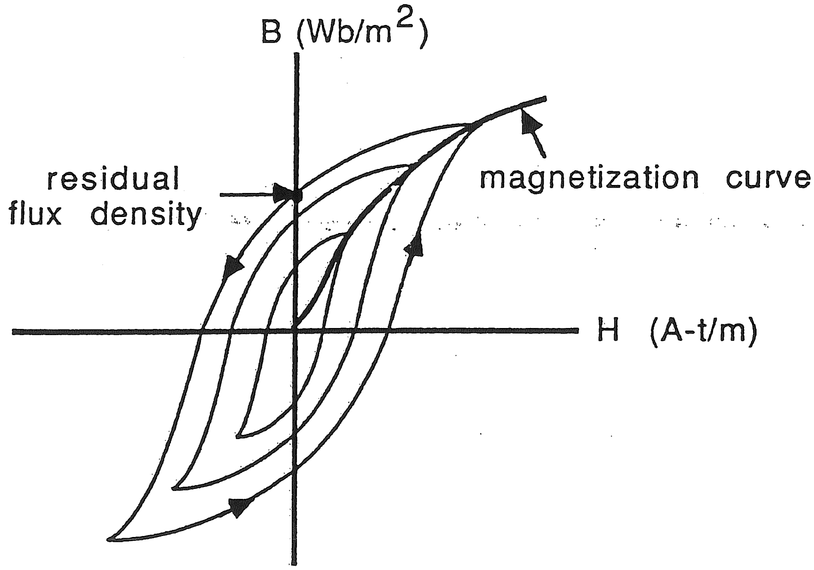


Figure 2.14 Ferromagnetic Hysteresis Loops and Magnetization Curve

Several hysteresis loops are shown in Fig. 2.14 corresponding to different maximum values of the cyclic mmfs driving the core. For a given hysteresis loop, the magnetic state of the core changes counterclockwise around the loop as the arrows indicate. The magnetization curve is the locus of the tips of the hysteresis loops and is often used to represent the magnetic state of the core when hysteresis is neglected or when the loops are very narrow for highly-permeable, core materials.

The phenomenon of hysteresis is peculiar only to core materials consisting of iron and its alloys-principally nickel, cobalt and silicon which contain many magnetic dipoles (micro, north-south poles) whose axes are easily rotated when a cyclic magnetic field is applied. As the mmf, in Fig. 2.14, approaches a positive maximum, more and more of the magnetic dipoles line up in one direction, thus magnetizing the core material as is evidenced by increased flux density. As the mmf decreases to zero, some, but not all of the dipoles are rotated thus demagnetizing the iron, resulting in a decreased flux density called residual. As the mmf reverses to a negative maximum, magnetization of the iron reverses and the cycle is repeated. This process of rotating the magnetic dipoles in a cyclic magnetic field involves an energy exchange as seen in Fig. 2.15.

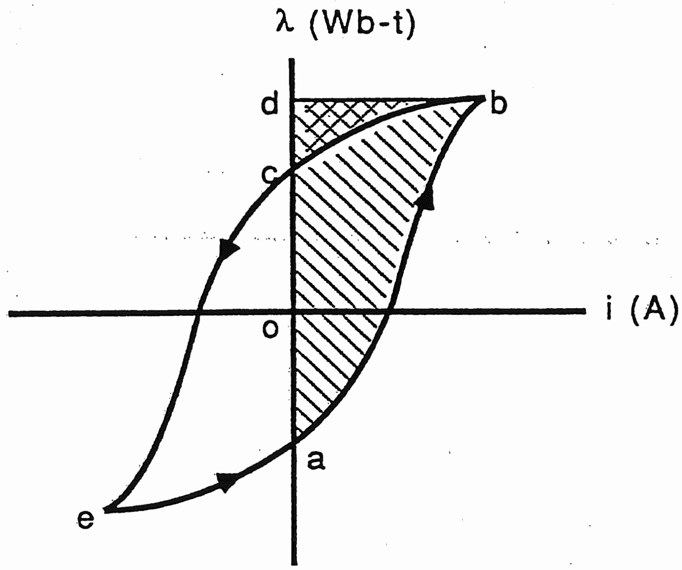


Figure 2.15 Ferromagnetic Hysteresis Loss

For the core given in Fig. 2.11, which is singly excited, with a coil of N-turns, mean path length, ℓ_c, and cross-section, A_c, the flux linkages vs the cyclic, driving mmf is plotted in Fig. 2.15. When the mmf is positive, the area, abd, is the energy taken from the source and stored in the magnetic field. As the mmf decreases to zero, the area, cbd, is the energy returned to the source. The difference between these two areas, abc, is the energy expended as heat in the core during the positive mmf half-cycle. By the same token, during the negative mmf half-cycle, the energy expended in heat is area cea. The hysteresis loss, then, is determined by the area bounded by the hysteresis loop and the frequency of the cyclic, driving mmf.

The second loss is the eddy-current loss which occurs when a time varying flux exists in ferromagnetic cores. Since iron is a conductive material, short-circuit currents flow around closed paths in the cross-section of the core because of Faraday emfs generated due to time-changing flux linkages that link these closed paths. The resulting heat losses are appreciable unless the iron core is constructed of thin laminations that break up these closed paths and thus considerably reduce these heat losses. The eddy-current loss, then, is determined by the thickness of the laminations and the magnitude and frequency of the magnetic field. The hysteresis and eddy-current losses are called the core or field losses and become important in determining the efficiency of electromagnetic devices. These losses will be considered in detail in the chapter on transformers.

2-9 INDUCTANCE OF MULTI-COIL MAGNETIC CIRCUITS

Very often power system components have more than one coil wound on their magnetic circuits. The analysis for calculating the inductance for these circuits is similar to the analysis of Section 2-8. It is again emphasized that inductance is a linear concept, i.e., inductance is meaningless unless operation is constrained to the linear portion (below the knee) of the magnetization curve of a ferrous core. Consider the two-coil magnetic circuit in Fig.2.16, where the currents are chosen so that the mmfs are <u>subtractive</u>,

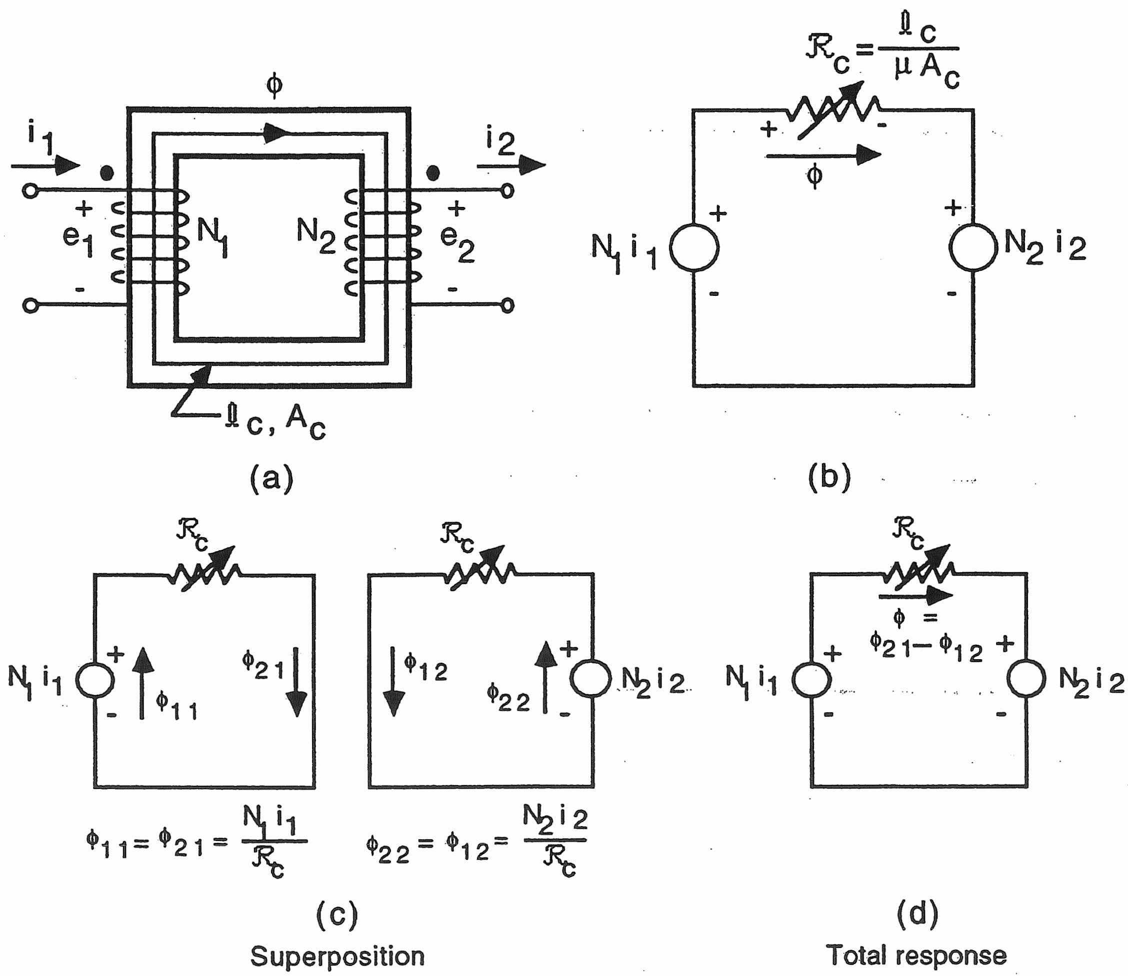


Figure 2.16 Two-Coil Magnetic Circuit (Subtractive MMFs)

The magnetic circuit is shown in (a), and its analog circuit in (b). Since linearity is assumed, the principle of superposition can be invoked which is shown in (c). The total response of both mmfs is shown in (d). Observe that the currents in (a) are assumed so that the mmfs are <u>opposing</u> or <u>subtractive</u>.The flux linkage of each coil can now be written,

$$+\lambda_1 = N_1\phi_1 = N_1\phi_{11} - N_1\phi_{12} = \frac{N_1^2}{\mathcal{R}_c}\, i_1 - \frac{N_1N_2}{\mathcal{R}_c}\, i_2 \qquad (2.24)$$

↑ <u>total</u> flux, ϕ, linking N1, is <u>in same direction as</u> mmf 1

$$-\lambda_2 = N_2\phi_2 = -N_2\phi_{21} + N_2\phi_{22} = -\frac{N_1N_2}{\mathcal{R}_c}\, i_1 + \frac{N_2^2}{\mathcal{R}_c}\, i_2$$

↑ <u>total</u> flux, ϕ, linking N2, <u>opposes</u> mmf 2

or,

$$\lambda_1 = L_{11}\, i_1 - |L_{12}|\, i_2 \qquad (2.25)$$

$$-\lambda_2 = -|L_{21}|\, i_1 + L_{22}\, i_2$$

where, $L_{11} = \frac{N_1\phi_{11}}{i_1} = \frac{N_1^2}{\mathcal{R}_c}$ and $L_{22} = \frac{N_2\phi_{22}}{i_2} = \frac{N_2^2}{\mathcal{R}_c}$ (2.26)

are the <u>self inductances</u> of coils 1 and 2, respectively ;

and $|L_{21}| = \frac{N_2\phi_{21}}{i_1} = |L_{12}| = \frac{N_1\phi_{12}}{i_2} = \frac{N_1N_2}{\mathcal{R}_c}$ (2.27)

are the <u>mutual inductances</u> between coils 1 and 2.

The coefficient of coupling between the two circuits is by definition,

$$k = \frac{|L_{12}|}{\sqrt{L_{11}L_{22}}} \qquad \text{where } 0 < k \leq 1 \qquad (2.28)$$

For the magnetic circuit in Fig. 2.16, the coefficient of coupling is unity, (by substituting the expressions for the inductances in the coupling equation), since <u>all</u> of the flux is assumed to link <u>all</u> of the turns of both coils. Only when this is true is,

$$L_{11} = \frac{N_1^2}{\mathcal{R}_c} \; ; \quad L_{22} = \frac{N_1^2}{\mathcal{R}_c} \; ; \quad |L_{12}| = |L_{21}| = \frac{N_1N_2}{\mathcal{R}_c} \quad \text{(H)} \quad (2.29)$$

The Faraday emfs generated across coils 1 and 2 are,

$$e_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} - |L_{12}|\frac{di_2}{dt} \qquad \text{(V)}$$

$$-e_2 = -\frac{d\lambda_2}{dt} = -|L_{21}|\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} \qquad \text{(V)}$$

The significance of, $-e_2$, in the above equation is indicated by the fact that, $L_{21}\frac{di_1}{dt}$, is the <u>driving</u> emf across coil 2 . (which is evident when $i_2 = 0$)

or,

$$L_{21}\frac{di_1}{dt} = L_{22}\frac{di_2}{dt} + e_2 \qquad \text{(V)}$$

then, in general,

$$
\begin{aligned}
e_1 &= L_{11}\frac{di_1}{dt} \pm |L_{12}|\frac{di_2}{dt} \\
\pm e_2 &= \pm|L_{21}|\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}
\end{aligned}
\qquad (2.30)
$$

where + signs are used for <u>additive mmfs</u>, and – signs are used for <u>subtractive</u> mmfs.

The <u>self</u> inductances in Eqns. (2.30), are <u>always positive</u> quantities.

By referring to Eqns. (2.24), the <u>mutual</u> inductance is <u>positive</u> if the coil mmfs are additive, or <u>negative</u> if the coil mmfs are subtractive.

Very often the sense of the coil windings is not given as in Fig. 2.16, but the coil polarity dots (positive ends of the windings as verified by Lenz's law), are given. If the coil currents, i_1 and i_2, are <u>both</u> chosen to flow into or away from the dots, the coil mmfs are additive and the mutual inductance is positive. If one of the currents is chosen to flow into the dot and the other to flow away from the dot, the coil mmfs are subtractive and the mutual inductance is negative.

The energy stored in the magnetic field of the circuit in Fig. 2.16 can now be found by calculating the instantaneous power <u>delivered</u> to both coils and realizing this energy <u>must be stored</u> in the field.

$$p = e_1 i_1 - e_2 i_2 = \frac{d\lambda_1}{dt} i_1 - \frac{d\lambda_2}{dt} i_2 = \frac{dW_f}{dt} \qquad (W) \qquad (2.31)$$

or, $$dW_f = d\lambda_1 i_1 - d\lambda_2 i_2$$

from Eqn. (2.25), $$= L_{11} i_1 di_1 - L_{12} i_1 di_2 - L_{21} i_2 di_1 + L_{22} i_2 di_2$$

$$= L_{11} i_1 di_1 + L_{22} i_2 di_2 - L_{12}(i_1 di_2 + i_2 di_1)$$

since, $$d(i_1 i_2) = i_1 di_2 + i_2 di_1 \qquad \text{(chain rule)}$$

then, $$dW_f = L_{11} i_1 di_1 + L_{22} i_2 di_2 - L_{12}\, d(i_1 i_2) \qquad (J)$$

and, $$W_f = L_{11}\int_0^{i_1} i_1\, di_1 + L_{22}\int_0^{i_2} i_2\, di_2 - L_{12}\int_0^{i_1 i_2} d(i_1 i_2) \quad \text{(J)}$$

or, in general, $$W_f = \frac{1}{2}L_{11}\, i_1^2 + \frac{1}{2}L_{22}\, i_2^2 \pm |L_{12}|\, i_1 i_2 \quad \text{(J)} \qquad (2.32)$$

Example 2.5

Consider the silicon-steel toroid of Example 2.1 with two coils of $N_1 = 200$ turns and $N_2 = 300$ turns. Coil 1 carries a current of 100 ma and coil 2 carries a current of 200 ma. The mean length of path of the core is 0.4 m and its cross-section area is 4×10^{-4} m^2.

$$\mathcal{F} = (200)(0.1) + (300)(0.2) = 80 \text{ A-t}$$

$$H_c = 80/0.4 = 200 \text{ A-t/m}$$

$$B_c = 1.0 \text{ T (Fig. 2.5)}$$

$$\phi = (1.0)(4 \times 10^{-4}) = 400\ \mu\text{Wb}$$

$$\mathcal{R}_c = \frac{\mathcal{F}}{\phi} = \frac{80}{400 \times 10^{-6}} = 200{,}000 \quad \text{(A-t/Wb)}$$

(a) What are the self and mutual inductances and the coefficient of coupling of these coils?

$$L_{11} = \frac{N_1^2}{\mathcal{R}_c} = \frac{(200)^2}{2 \times 10^5} = 0.2 \text{ H} \quad ; \quad L_{22} = \frac{N_2^2}{\mathcal{R}_c} = \frac{(300)^2}{2 \times 10^5} = 0.45 \text{ H}$$

$$L_{12} = L_{21} = \frac{N_1 N_2}{\mathcal{R}_c} = \frac{(200)(300)}{2 \times 10^5} = 0.3 \text{ H}$$

$$k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}} = \frac{0.3}{\sqrt{0.2 \times 0.45}} = 1.0$$

(b) What is the energy stored in the field?

$$W_f = \frac{1}{2}L_{11}\, i_1^2 + \frac{1}{2}L_{22}\, i_2^2 + L_{12}\, i_1 i_2$$

$$= (\frac{1}{2})(0.2)(0.1)^2 + (\frac{1}{2})(0.45)(0.2)^2 + (0.3)(0.1)(0.2)$$

$$W_f = 0.016 \text{ Joule}$$

c) What are the Faraday emfs generated across coils 1 and 2, if,

$$i_1 = 100 \sin 377\, t \qquad \text{(mA)}$$

$$i_2 = 200 \sin 377\, t \qquad \text{(mA)}$$

$$\lambda_1 = N_1\, \phi = L_{11}\, i_1 + L_{12}\, i_2 = (0.2)(0.1) \sin 377\, t + (0.3)(0.2) \sin 377\, t$$

$$= 0.08 \sin 377\, t \qquad \text{(Wb-t)}$$

$$\lambda_2 = N_2\, \phi = L_{21}\, i_1 + L_{22}\, i_2 = (0.3)(0.1) \sin 377\, t + (0.45)(0.2) \sin 377\, t$$

$$= 0.12 \sin 377\, t \qquad \text{(Wb-t)}$$

$$e_1 = \frac{d\lambda_1}{dt} = (0.08)(377) \cos 377\, t = 30.2 \cos 377\, t \qquad \text{(V)}$$

$$e_2 = \frac{d\lambda_2}{dt} = (0.12)(377) \cos 377\, t = 45.2 \cos 377\, t \qquad \text{(V)}$$

In like fashion the inductances L_{11}, L_{22}, L_{33}, L_{12}, L_{13}, L_{23} can be calculated for a three-coil magnetic circuit and extended to n-coil magnetic circuits.

LEAKAGE FLUX (Not all the flux generated by one coil links the other)

In Fig. 2.16 and Ex. 2.5, we considered a two-coil magnetic circuit where all of the flux generated by the mmfs of both coils, links all of the turns of each coil. In practice, this is not true, as is shown in Fig. 2.17, for additive mmfs,

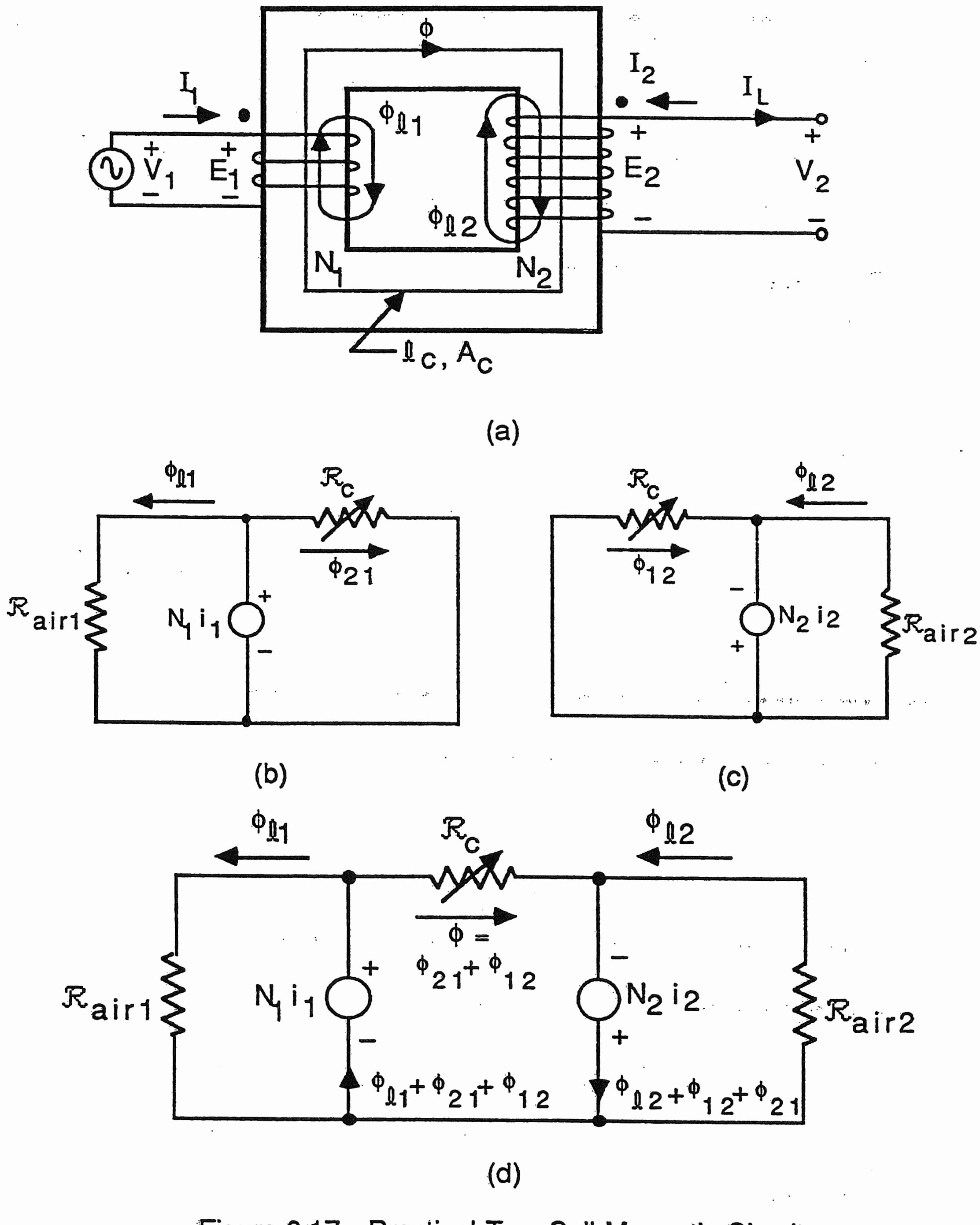


Figure 2.17 Practical Two-Coil Magnetic Circuit

In Fig. 2.17 (a), the difference in magnetic potential across each coil is large with the result that a small portion of the flux called the leakage flux, ϕ_ℓ, is driven around a high-reluctance path in air. The remaining large portion of flux, called the mutual flux is driven around the low-reluctance path in iron. Observe, that the leakage flux does not link the other coil, whereas the mutual flux does link both coils. The analog circuit is shown in (d) together with the total response of the two circuits shown in (b) and (c), using the principle of superposition. The flux linkage of each coil can now be written.

From Fig. 2.17 (b),

$$\phi_{\ell 1} = \frac{N_1 i_1}{\mathcal{R}_{air1}} \quad ; \quad \phi_{21} = \frac{N_1 i_1}{\mathcal{R}_c}$$

where, $\phi_{\ell 1} + \phi_{21}$ is the total flux generated by coil 1 that short circuits through air and links coil 2.

From Fig. 2.17 (c),

$$\phi_{\ell 2} = \frac{N_2 i_2}{\mathcal{R}_{air2}} \quad ; \quad \phi_{12} = \frac{N_2 i_2}{\mathcal{R}_c}$$

where, $\phi_{\ell 2} + \phi_{12}$ is the total flux generated by coil 2 that short circuits through air and links coil 1.

From Fig. 2.17 (d),

$$\lambda_1 = N_1 \phi_1 = N_1(\phi_{\ell 1} + \phi_{21}) + N_1 \phi_{12} = \left(\frac{N_1^2}{\mathcal{R}_{air1}} + \frac{N_1^2}{\mathcal{R}_c}\right) i_1 + \frac{N_1 N2}{\mathcal{R}_c} i_2 \tag{2.33}$$

$$\lambda_2 = N_2 \phi_2 = N_2 \phi_{21} + N_2(\phi_{\ell 2} + \phi_{12}) = \frac{N_1 N_2}{\mathcal{R}_c} i_1 + \left(\frac{N_2^2}{\mathcal{R}_{air2}} + \frac{N_2^2}{\mathcal{R}_c}\right) i_2$$

or,

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \tag{2.34}$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

where, $$L_{11} = \frac{N_1^2}{\mathcal{R}_{air1}} + \frac{N_1^2}{\mathcal{R}_c} \quad \text{and} \quad L_{22} = \frac{N_2^2}{\mathcal{R}_{air2}} + \frac{N_2^2}{\mathcal{R}_c} \tag{2.35}$$

and $$|L_{21}| = |L_{12}| = \frac{N_1 N_2}{\mathcal{R}_c} \tag{2.36}$$

The coefficient of coupling can now be given physical meaning, let,

$$k_1 = \frac{\phi_{21}}{\phi_{\ell 1} + \phi_{21}} \quad ; \quad k_2 = \frac{\phi_{12}}{\phi_{\ell 2} + \phi_{12}}$$

i.e., k_1 is the fraction of the total flux generated by coil 1 that links coil 2 and, k_2 is the fraction of the total flux generated by coil 2 that links coil 1.

From Eqn. (2.33), $N_2 \phi_{21} = L_{21} \, i_1 \; ; \; N_1(\phi_{\ell 1} + \phi_{21}) = L_{11} \, i_1,$ etc.,

$$k_1 = \frac{\frac{1}{N_2} L_{21} \, i_1}{\frac{1}{N_1} L_{11} \, i_1} = \frac{N_1 \, L_{21}}{N_2 \, L_{11}} \quad ; \quad k_2 = \frac{\frac{1}{N_1} L_{12} \, i_2}{\frac{1}{N_2} L_{22} \, i_2} = \frac{N_2 \, L_{12}}{N_1 \, L_{22}}$$

then,

$$k_1 \, k_2 = \frac{L_{21} \, L_{12}}{L_{11} \, L_{22}} = \frac{(L_{12})^2}{L_{11} \, L_{22}}$$

and,

$$k = \sqrt{k_1 \, k_2} = \frac{|L_{12}|}{\sqrt{L_{11} \, L_{22}}}$$

where, the coefficient of coupling, k , is the geometric mean of k_1 and k_2.

For example, if k_1 and k_2 were each 0.5, one half of the total flux generated by <u>each</u> coil would link the other coil, and the other half of the total flux generated by <u>each</u> coil would leak through an air path. The coefficient of coupling, which is a measure of the mutual coupling between <u>both</u> coils, is, then, one half. The coefficients, k_1 and k_2 , are seldom equal but depend on the number of turns on each coil and the circuit geometry. If all of the flux generated by each coil links all of the turns of each coil, then there is no leakage flux, and the coefficient of coupling is unity. The coefficient of coupling can be as high as 0.998 for magnetic circuits with ferrous cores but is less than or approximately 0.5 for air cores.

The Faraday emfs generated across both coils are given in Eqn. (2.30) and the energy stored in the magnetic field of the core and the air paths is given in Eqn. (2.32).

The reluctances of the leakage paths in air in Fig. 2.17 (a) are <u>not</u> generally known, so the self and mutual inductances <u>must</u> be <u>measured</u> directly by applying a sinusoidal source to the coils connected in series- mmfs additive, and then series- mmfs subtractive, as follows,

Measurement of self and mutual inductances

In Fig. 2.17 (a), if a sinusoidal voltage source, V_1, is connected to coil 1, using I_2 as the assumed current, the mmfs are additive, therefore,

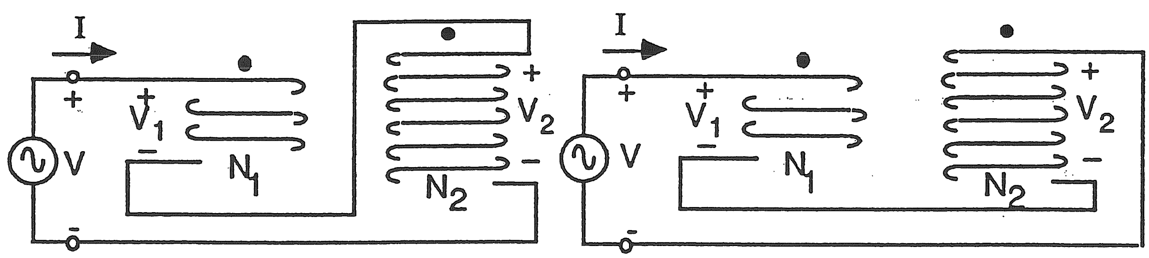
$$V_1 = j\omega L_{11} I_1 + j\omega L_{12} I_2 \qquad (2.37)$$

$$V_2 = j\omega L_{21} I_1 + j\omega L_{22} I_2$$

If the load current, $I_L = -I_2$, is used as the assumed current, the mmfs are subtractive, therefore,

$$\begin{aligned} V_1 &= \ \ j\omega L_{11} I_1 - j\omega L_{12} I_L \\ -V_2 &= -j\omega L_{21} I_1 + j\omega L_{22} I_L \end{aligned} \qquad (2.38)$$

If a sinusoidal source is applied across the two coils connected in series, mmfs additive, or subtractive,



→ L_1 mmfs additive → L_2 mmfs subtractive

now, $V = V_1 + V_2$ and $V = V_1 - V_2$

where, in Eqns. (2.37),(2.38) $I = I_1 = I_2 = I_L$ and $\dfrac{V}{I} = j\omega L_1$ or $j\omega L_2$

then, $L_1 = L_{11} + L_{22} + 2L_{12}$ $L_2 = L_{11} + L_{22} - 2L_{12}$

Example 2.6

If the two coils in Fig.2.17 (a) are identical and are connected in series, the inductance is measured as 137 mh. The connections to one of the coils is reversed and the inductance is measured as 43 mh. Find,

a) the mutual inductance.

b) the self inductance of each coil.

c) the coefficient of coupling.

From Eqns. (2.37),

$$V = V_1 + V_2 = j\omega L_{11} I + j\omega L_{22} I + 2j\omega L_{12} I$$

now,

$$j\omega L_1 = \frac{V}{I}$$

then,

$$L_1 = L_{11} + L_{22} + 2L_{12} = 137 \quad (\mathrm{mH})$$

From Eqns. (2.38),

$$V = V_1 - V_2 = j\omega L_{11} I + j\omega L_{22} I - 2j\omega L_{12} I$$

$$j\omega L_2 = \frac{V}{I}$$

$$L_2 = L_{11} + L_{22} - 2L_{12} = 43 \quad (\mathrm{mH})$$

since, $L_{11} = L_{22}$, the simultaneous solution of the above equations yields,

a) $L_{12} = 23.5$ (mH)

b) $L_{11} = 45$ (mH), $L_{22} = 45$ (mH)

c) $k = 0.522$

2-10 MAGNETIC-CIRCUIT MODELS

The mathematical models that can be used in this or other texts to represent one, two, or three-coil, time-varying, magnetic circuits are shown in Fig. 2.18.

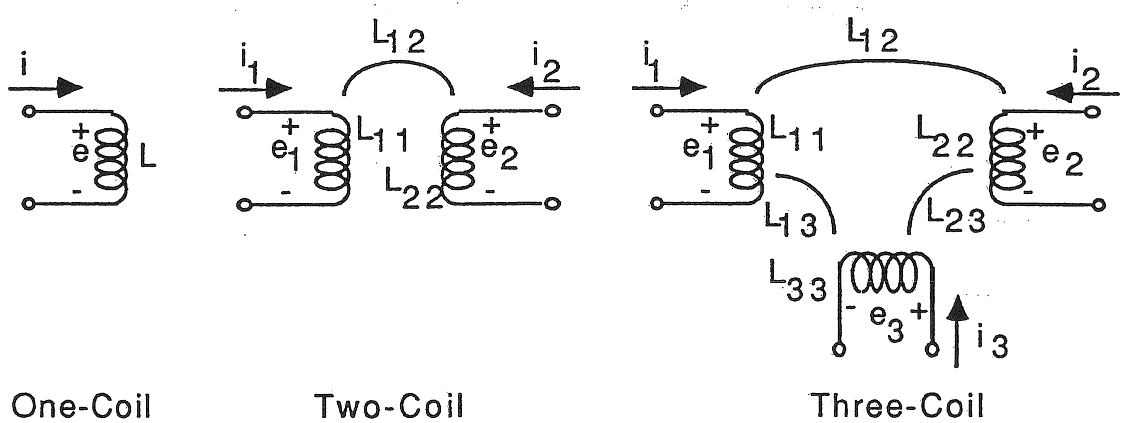


Figure 2.18 Magnetic-Circuit Models

2.11 SUMMARY

The basic laws for analyzing the <u>mmfs</u> of time-variant or time-invariant magnetic circuits are Maxwell's mmf law.

$$\oint_S \mathbf{J}\cdot d\mathbf{a} = \int_C \mathbf{H}\cdot d\boldsymbol{\ell} \qquad \text{(A)}$$

and his corollary law,

$$\oint_S \mathbf{B}\cdot d\mathbf{a} = 0 \qquad \text{(Wb)}$$

which, when single-dimensionalized are,

$$\Sigma\, N\, i \text{ (rises)} = \Sigma\, H\, \ell \text{ (drops)} \qquad \text{around any contour, and}$$

$$\Sigma \phi = 0 \qquad \text{at any magnetic node.}$$

Since these equations are analogous to Kirchhoff's voltage and current laws, the analog magnetic circuit becomes very important in the analysis of magnetic circuits.

Because of the nonlinearity of ferrous magnetic circuits, the recommended analytic procedure for analyzing these circuits, depending on whether the mmf or the flux is known, is,

$$\left\downarrow \begin{array}{c} F \\ H \\ B \\ \phi \end{array} \right\uparrow$$

The basic law for analyzing the emfs generated in time-variant, magnetic circuits is Maxwell's emf law,

$$\oint_C (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \cdot d\boldsymbol{\ell} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a} \qquad \text{(V)}$$

which, for stationary, coil-contours is,

$$e = \frac{d(N\phi)}{dt} = \frac{d\lambda}{dt} \qquad \text{(V)}$$

Valid only for linear magnetic circuits,inductance is defined,

$$L = \frac{N\phi}{i} = \frac{\lambda}{i} \qquad \text{(H)}$$

As a consequence of this definition, multiple-coil, magnetic circuits can be modeled with self and mutual inductances using both Maxwell's mmf and emf laws together with analog circuits corresponding to these device.

Also important in electromagnetic devices is the energy stored in the magnetic field, i.e.,

$$W_f = \int i \, d\lambda \qquad \text{(J)}$$

summed over all ports of the device.

PROBLEMS

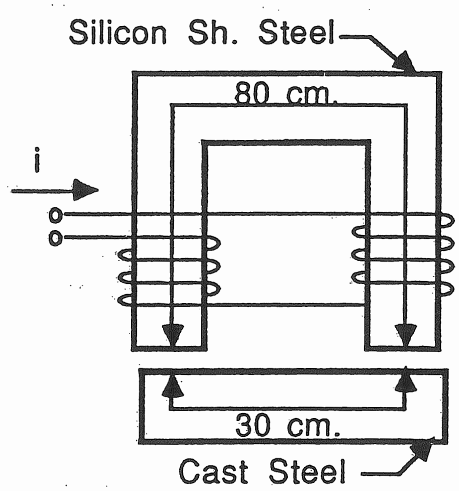
2.1 Calculate the relative permeability, for a flux density of 1.0T, for (a) Cast Steel, (b) Silicon Sheet Steel, (c) Nickel Iron Alloy.

2.2 Calculate the permeability and relative permeability for H = 200 A-t/m and H = 600 A-t/m for: (a) Cast Steel, (b)Silicon Sheet Steel. Is it possible to assume that the permeability is constant over the 200-600 A-t/m range for (a) or (b)?

2.3 In Fig. 2.7, find the air-gap flux if the coil carries 1.0 A. (Hint: Use a trial and error method to find the flux by initially assuming all the driving mmf is dropped across the gap).

2.4 A toroid made of cast steel has a mean length of path whose diameter is 30 cm, a cross-section area of 10 cm^2 and an air gap of length 0.25 cm. The toroid is wound with 1000 turns carrying a current of 2.0 A. Determine the average flux density in the air gap if fringing is negligible. (Use the hint in Problem 2.3)

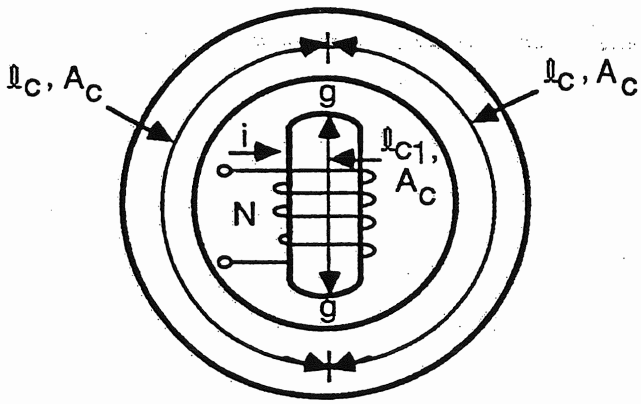
2.5 The cross-section area of 40 cm^2 is uniform throughout this electromagnet. On each leg is wound a 500-turn coil. Find the current that will establish a flux of 4 mWb in each 0.1 cm air gap.



2.6 The magnetic structure of a synchronous machine is shown in the figure. If the air-gap flux is 0.04π Wb, find the current if,

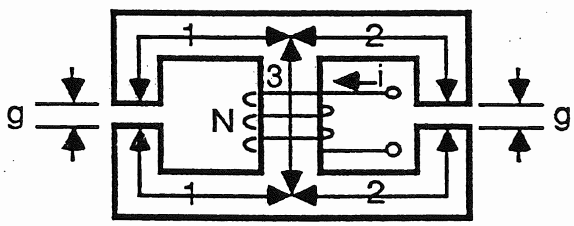
(a) the core is Cast Steel

(b) the core permeability is infinite.



$N = 1000$ turns
$g = 1.0$ cm
$A_c = 2000\ \text{cm}^2$
$\ell_c = 40$ cm
$\ell_{c1} = 10$ cm

2.7 Given the following magnetic circuit.



$\ell_1 = \ell_2 = \ell_3 = \ell$ meters

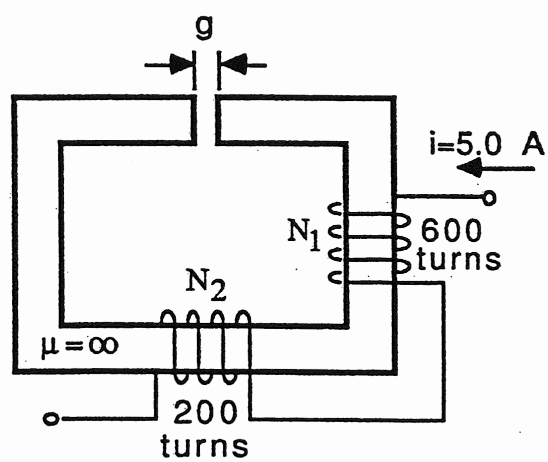
core area $= A_c$ (meter)^2

core permeability $= \mu_c =$ constant

neglect air-gap fringing

Find the expression for the flux in path 3 as a function of N, ℓ, g, A_c, ...

2.8 For the given magnetic circuit what is the air-gap flux?

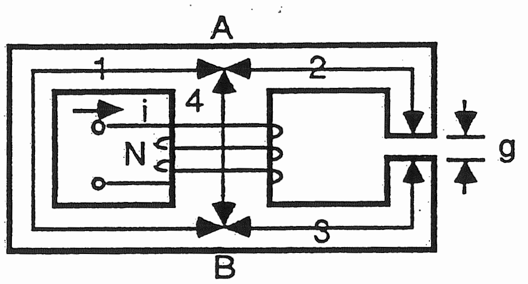


$g = 0.002$ m, $A_g = 0.004\ \text{m}^2$

2.9 The magnetic circuit shown is made of silicon sheet steel.

(a) What is the current in the coil?

(b) What is the magnetic potential difference across AB?



$\ell_1 = 50$ cm, 10 cm^2
$\ell_2 = 30$ cm, 10 cm^2
$\ell_3 = 20$ cm, 10 cm^2
$\ell_4 = 30$ cm, 16 cm^2
$\ell g = 0.1$ cm
$B_g = 0.8$ T
N = 1000 turns

2.10 What is the coil inductance, (H), in Problem 2.6? What is the energy stored in the field?

2.11 What is the coil inductance in Problem 2.7 as a function of N, ℓ, g, A_c, ...?

2.12 What are the self and mutual inductances (H) of the coils in Problem 2.5? What is the energy stored in the field?

2.13 What are the self and mutual inductances (H) of the coils in Problem 2.8? What is the energy stored in the field? If i = 5 sin 377t (A), what are the coil emfs?

2.14 A toroid is wound with a coil of N-turns. The radius of the mean length of path is R-meters. The cross-section of the toroid is circular of radius, a -meters, where a << R. Assume the coil resistance is negligible.

(a) Find the literal expression for the inductance of this toroid.

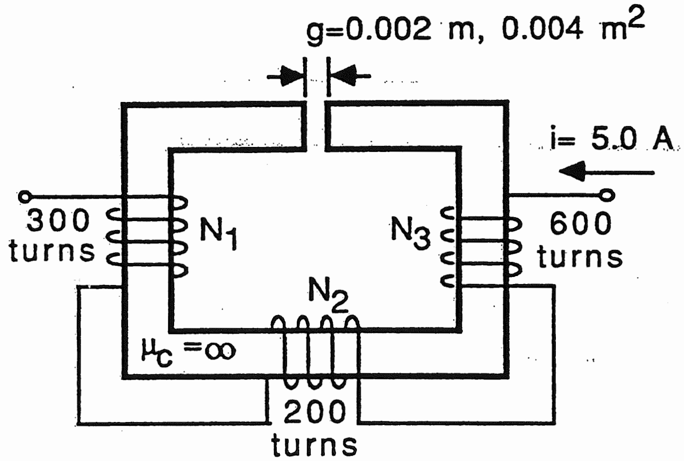
(b) The toroid is made of silicon sheet steel with R = 20 cm, a = 1.5 cm and N = 1000 turns. If the toroid field is to be increased at a uniform rate, i.e., di/dt = constant, to a maximum flux density of 0.7 T in 0.01 seconds and then decreased at a uniform rate to 0.0 T in 0.01 seconds cyclically, sketch,

i. the current-time waveform, i vs t.

ii. the applied voltage-time waveform, v vs t.

Show <u>numerical</u> values of all important magnitudes on the current, voltage, time axes.

2.15

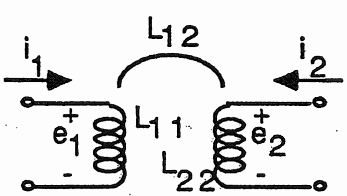


For the above magnetic circuit what are the self and mutual inductances of the coils?
If $i = 5 \sin 377\,t$, what are the coil emfs? What is the energy stored in the magnetic field?

2.16 There are ten times as many turns on coil 2 as on coil 1 in Fig. 2.17 (a). If the two coils are connected in series, the inductance is measured as 1.108 H. The connections to one of the coils is reversed and the inductance is measured as 0.9083 H. When coil 2 is open-circuited, the inductance of coil 1 is measured as 8.33 mH.

a) What are the self and mutual inductances and coefficient of coupling?
b) What fraction of the total flux generated by coil 1 is the mutual flux, ϕ_{21}?
   – is the leakage flux, $\phi_{\ell 1}$?
c) What fraction of the total flux generated by coil 2 is the mutual flux, ϕ_{12}?
   – is the leakage flux, $\phi_{\ell 2}$?

2.17 If two identical coils have the symbolic diagram,



where, $L_{11} = L_{22} = 1.0$ H ; $L_{12} = 0.81$ H

a) What is the coefficient of coupling?
b) What fraction of the total flux generated by coil 1 is the mutual flux, ϕ_{21}?
   – is the leakage flux, $\phi_{\ell 1}$?
c) What fraction of the total flux generated by coil 2 is the mutual flux, ϕ_{12}?
   – is the leakage flux, $\phi_{\ell 2}$?

CHAPTER 3

TRANSFORMERS

In the early 1800's, Michael Faraday applied a voltage to one coil and observed a consequent voltage across a second coil wound on the same iron core. With this very important, experimental observation, together with other experiments, the transformer has made possible the complex power grid that exists today. The transformer, then, is an important component of a power system and, it must be emphasized, it is not an energy converter but is used to transfer power at different voltage levels as indicated in the simple power system of Fig. 3.1.

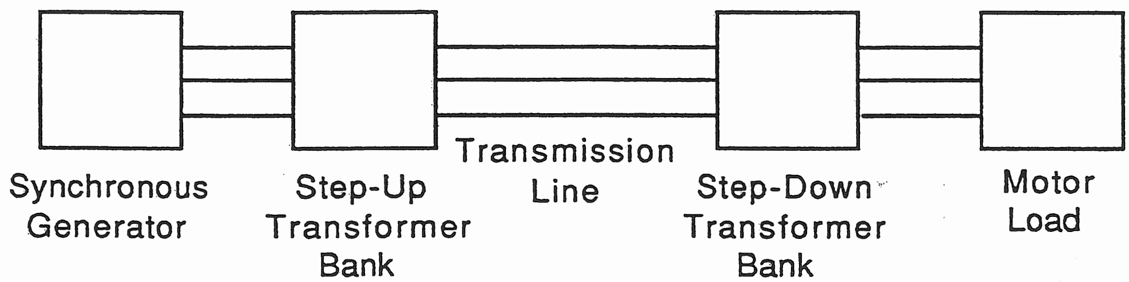


Figure 3.1 Elementary Power System

It is instructive to consider each transformer bank in Fig. 3.1 as a bank of three individual transformers, connected to transfer generated power at a low voltage level, to a transmission line at a high voltage level, and then to a load at a low voltage level. For safety and design considerations, power is always generated at the source and reconverted at the load, at low voltage levels, whereas power is transmitted over transmission lines at a high voltage level with correspondingly low line currents and line losses for a given transmitted power. The transformer will be considered first in single-phase circuits and then the analysis will be extended to three- phase circuits.

3-1 TRANSFORMER CONSTRUCTION

The transformer consists of two or more windings linked by a magnetic field. In addition to transferring power at different voltage levels, it is useful in matching impedances and isolating two or more circuits. The winding connected to an alternating source of voltage is called the primary and its variables carry the subscript, 1.

The winding connected to the load is called the secondary and its variables carry the subscript, 2. Since the transformer depends on a magnetic field that changes with time, a voltage is induced in the primary and secondary windings that is proportional to the number of turns on each winding. With the appropriate turns-ratio, a transformer will step-up or step-down a source voltage. Two types of magnetic circuits for transformers are shown in Fig. 3.2.

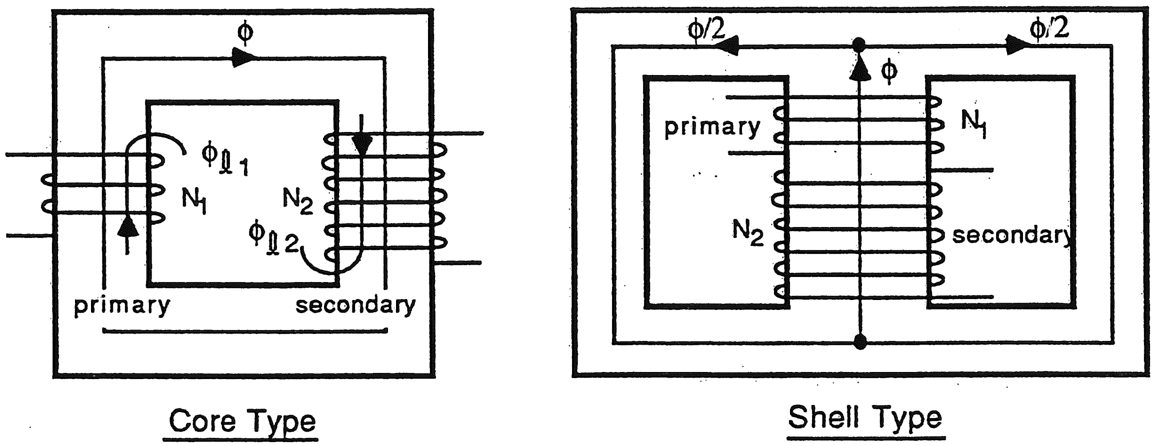


Figure 3.2 Transformer Magnetic Circuits

The core-type and the shell-type transformers in Fig. 3.2 are constructed of high-permeability steel with a low-reluctance path available between the windings so that the flux generated by one winding will link the other winding. Therefore, the path for the flux, ϕ, is primarily in steel. However, the high difference in magnetic potential across the coils of either transformer in Fig. 3.2 leads to leakage flux, $\phi_{\ell 1}$ and $\phi_{\ell 2}$, which is produced by each coil. This leakage flux does not link the other coil and its path is primarily in air. The leakage flux is relatively large in the core-type transformer, by the very nature of its construction, and the core-type transformer is used in fluorescent light fixtures as ballasts where the leakage flux and its reactance limits the starting current of the fixture. Without exception, the shell-type core is used in large power-transformers with megawatt ratings. Here the leakage flux is minimized by alternately placing a few turns of the primary and then a few turns of the secondary in a "pancake" arrangement on the center leg. The low leakage reactance of a power transformer will become important in its later analysis. The analysis of the transformer begins with the derivation of its model and the following section will set up the conventions adopted for this derivation.

3-2 THE IDEAL TRANSFORMER

The transformer voltage and current conventions shown in Fig. 3.3 will be consistently used throughout this chapter. The transformer is driven with a sinusoidal source, v_1, with a consequent voltage, v_2, across the load, and the dashed line indicates a mean path for the flow of flux in the ferrous core. The induced emfs across the coils, with the winding sense indicated, are assumed positive in the directions indicated and the currents are assumed positive into the primary winding and away from the secondary winding so that the mmfs are opposing. With these conventions in mind, the transformer will initially be considered ideal, i.e.,

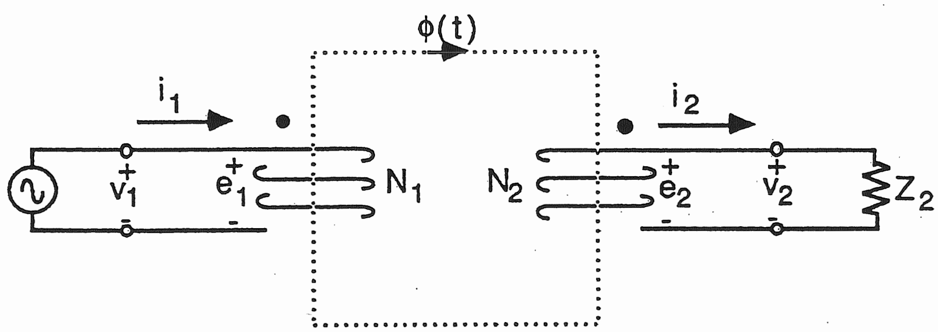


Figure 3.3 Transformer Voltage, Current Conventions

1. The windings have negligible resistance and leakage flux.
2. The flux is assumed to vary sinusoidally with time, is single valued and varies linearly with the current.
3. The core is assumed to be infinitely permeable, i.e., no mmf is dropped along the iron path.
4. The ferrous core losses are assumed zero.

Power system transformers approach these conditions and are, therefore, very suited to system applications.

Since time is an independent variable, time will be counted such that,

$$\phi = \Phi_m \sin \omega t \qquad \text{(Wb)} \qquad (3.1)$$

The coils are stationary contours and Faraday's law as expressed in Eqn. (2.17) applies.

Since all of the flux links all of the turns,

$$
\begin{aligned}
e_1 &= \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt} = \omega N_1 \Phi_m \cos \omega t \qquad \text{(V)} \\
e_2 &= \frac{d\lambda_2}{dt} = N_2 \frac{d\phi}{dt} = \omega N_2 \Phi_m \cos \omega t \qquad \text{(V)}
\end{aligned}
\tag{3.2}
$$

Using rms values, Equations (3.2) can be written in phasor form,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \tag{3.3}$$

Voltages are induced in the windings of an ideal transformer according to the turns-ratio a, which is defined as the source turns divided by the load turns. With the sense of the windings indicated in Fig. 3.3, Lenz's law predicts that the emfs e_1 and e_2 rise and fall together when measured from the undotted to the dotted ends of the coils, i.e., E_1 and E_2 are in time phase. The dots are called polarity markings and they become important later when these transformers are used in three-phase connections.

The magnetic field at each instant of time, in Eqn. (3.1) is produced by currents i_1 and i_2 according to Maxwell's mmf law. With the winding sense indicated in Fig. 3.3,

$$N_1 i_1 - N_2 i_2 = H_c \ell_c \qquad \text{(A-t)} \tag{3.4}$$

Since $B_c = \mu H_c$, and the iron is infinitely permeable, for a given flux density in the iron, $H_c = 0$. Equation (3.4) becomes,

$$N_1 i_1 - N_2 i_2 = 0 \qquad \text{(A-t)} \tag{3.5}$$

Using rms values, Eqn. (3.5) can be written in phasor form,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{3.6}$$

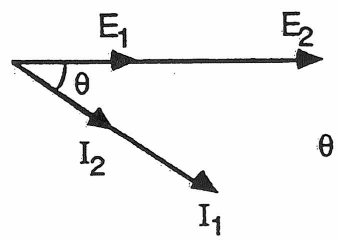
With subtractive mmfs in Fig. 3.3, the currents in Eqn. (3.6) rise and fall together, i.e., I_1 into a dot and I_2 away from a dot are in time phase. It therefore follows that if both currents are assumed positive into the dots they are 180° out of phase.

Since the losses in an ideal transformer are zero, the apparent power in equals the apparent power out.

$$E_1 I_1 = E_2 I_2 \qquad \text{(VA)} \qquad (3.7)$$

Equation (3.7) states that the current in the high-voltage winding is low while the current in the low-voltage winding is high. In summary, a load current or mmf in the secondary is exactly compensated by an equivalent current, or equal but opposite mmf, in the primary according to Maxwell's mmf law.

The characteristics of an ideal transformer are then summarized in the phasor diagram of Fig. 3.4, according to the conventions of Fig. 3.3.



θ = power factor angle of load.

Figure 3.4 Phasor Diagram of an Ideal Transformer

Since the winding resistance and leakage flux are assumed zero in an ideal transformer, the source and load-terminal voltages are equal to their respective Faraday emfs according to Kirchhoff's voltage law. The ratio of the <u>load</u> terminal voltage to current in the secondary is the load impedance.

$$\frac{V_2}{I_2} = \frac{E_2}{I_2} = Z_2 \qquad (\Omega)$$

Furthermore, the ratio of the source terminal voltage to the current in the primary is,

$$\frac{V_1}{I_1} = \frac{E_1}{I_1} = \frac{(N_1/N_2)(E_2)}{(N_2/N_1)(I_2)} = \left(\frac{N_1}{N_2}\right)^2 Z_2 = a^2 Z_2 \qquad (\Omega) \qquad (3.8)$$

The generator in Fig. 3.3 is not loaded with Z_2, but is loaded with an impedance, $a^2 Z_2$. This characteristic of a transformer is often used to match the generator and load impedances for maximum power transfer.

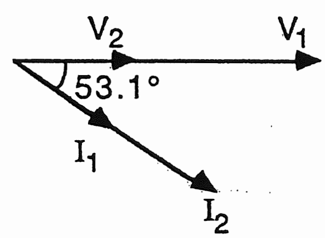
In summary, the voltage, current and impedance in the secondary of an ideal transformer can be referred to the primary as,

$$E_2' = \frac{N_1}{N_2}E_2 \;;\quad I_2' = \frac{N_2}{N_1}I_2 \;;\quad Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{3.9}$$

Equations (3.9) and their inverse will be useful later in the development of the transformer model.

Example 3.1

The ideal transformer of Fig. 3.3 has 400 turns on the primary and 100 turns on the secondary. The generator voltage is 120 V rms, 60 Hz and the load impedance is 3 + j4 ohms. Calculate the primary and secondary voltages and currents and sketch the phasor diagram. Also, what is the load on the generator?



$$V_2 = \frac{N_2}{N_1}V_1 = \frac{100}{400}(120\,\underline{/0^\circ}) = 30\,\underline{/0^\circ}\ \text{V}$$

$$I_2 = \frac{V_2}{Z_2} = \frac{30/0^\circ}{3+j4} = 6\,\underline{/-53.1^\circ}\ \text{A}$$

$$I_1 = \frac{N_2}{N_1}I_2 = \frac{100}{400}(6\,\underline{/-53.1^\circ}) = 1.5\,\underline{/-53.1^\circ}\ \text{A}$$

$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 = \left(\frac{400}{100}\right)^2 (5\,\underline{/53.1^\circ}) = 80\,\underline{/53.1^\circ}\ \Omega$$

3-3 THE PRACTICAL TRANSFORMER

The four restrictions of the ideal transformer cited earlier in Fig. 3.3 will now be removed to arrive at a practical transformer model. Consider, first, an unloaded transformer.

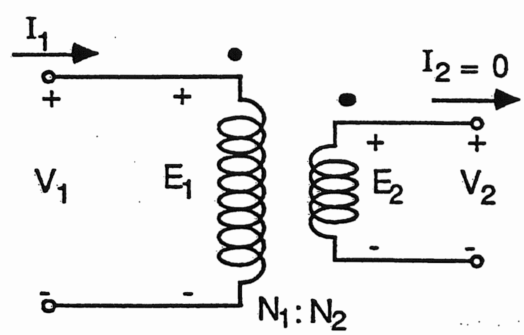


Figure 3.5 Open-Circuited Transformer

On open-circuit, the load current, I_2, is zero, and if the transformer were ideal, the input current, I_1, would be zero, according to Eqn. (3.6). In a practical, unloaded transformer, the input current, I_1, is not zero; in fact, even though there is no load on the transformer, watts and vars are delivered to its input. The reasons for this will follow, and the transformer model, which presently consists of the ideal transformer, will be changed accordingly.

Clearly, if watts and vars are delivered to this unloaded transformer, the input current I_1 must lag the input voltage, V_1, by an angle, θ_E, degrees. The input current, I_1, under these unloaded conditions is called the exciting current, I_E, and the phasor diagram is given in Fig. 3.6.

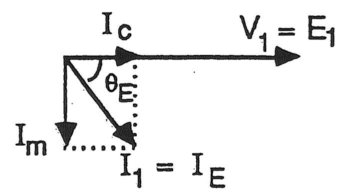


Figure 3.6 Transformer Exciting Current

In Fig. 3.6, the in-phase component of the exciting current is called the core-losses current, I_c, and the quadrature component of the exciting current is called the magnetizing current, I_m.

The exciting current, at no-load, then is,

$$I_1 = I_E = I_c - j\, I_m \qquad \text{(A)} \qquad (3.10)$$

With these defined variables, the watts delivered to this unloaded transformer is called the core loss, P_c, and the vars delivered is called the magnetizing power, Q_m. From Fig. 3.6,

$$P_c = E_1 I_E \cos\theta_E \qquad \text{(W)}$$

$$Q_m = E_1 I_E \sin\theta_E \qquad \text{(VAR)} \qquad (3.11)$$

where,

$$I_c = I_E \cos\theta_E \qquad \text{(A)}$$

$$I_m = I_E \sin\theta_E \qquad \text{(A)}$$

Consider, first, the vars delivered to this unloaded transformer. They can only be stored in the magnetic field set up in the iron core which implies that the permeability of the iron is not infinite. The magnetic intensity, H_c, is not zero, and a small mmf is dropped along the iron path reflecting this stored energy.

The var input can be determined by considering the voltage applied to the transformer as written in Eqn. (3.2)

$$v_1 = e_1 = \omega N_1 \Phi_m \cos \omega t \qquad (V) \qquad (3.12)$$

The rms value of Eqn. (3.12) is,

$$V_1 = \frac{2\pi f N_1 \Phi_m}{\sqrt{2}} = 4.44 f N_1 A_c B_m \qquad (V) \qquad (3.13)$$

where

A_c = cross-section area of the core, m^2
B_m = maximum flux density in the core, T
f = frequency of the source, Hz
N_1 = primary turns

It is evident from Eqn. (3.13), that the maximum flux density in the core is <u>not</u> a function of the input current, but is determined only by the input voltage, V_1. The parameters f, N_1 and A_c are usually chosen so that B_m occurs at the knee of the magnetization curve for rated voltage, V_1. Over voltage will, therefore, drive the core into saturation, and this is true for all electromagnetic devices.

From Maxwell's mmf law applied to Fig. 3.5, the quadrature component of the exciting current is proportional to the core magnetic intensity, H_m,

$$i_m = \frac{H_m \ell_c}{N_1} \qquad (A) \qquad (3.14)$$

The magnetizing current, then, is determined by the magnetization curve as shown in Fig. 3.7.

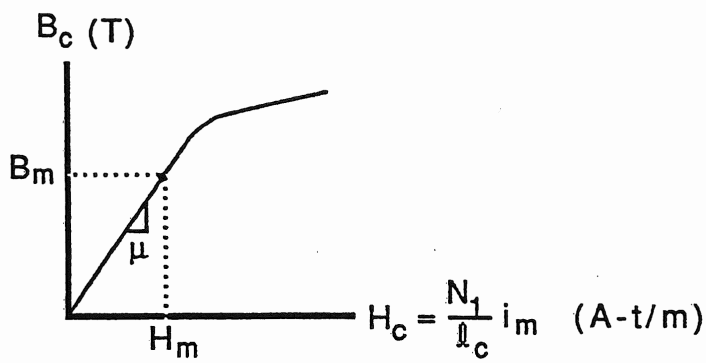


Figure 3.7 Magnetization Curve for Core with Finite Permeability

In summary, the rms voltage, V_1, determines the maximum flux density, B_m, in Fig. 3.7 and the slope of the magnetization curve determines the magnetizing current, I_m. The silicon steels of large power transformers have a very large permeability, resulting in a very small magnetizing ,I_m.

Consider, next, the watts delivered to this unloaded transformer. This power is converted, irreversibly, to heat within the core, because the core losses, hysteresis and eddy currents , are not zero.

The hysteresis loss occurs because the flux density is not single-valued at a given maximum flux density as shown in Fig. 3.8 (a).

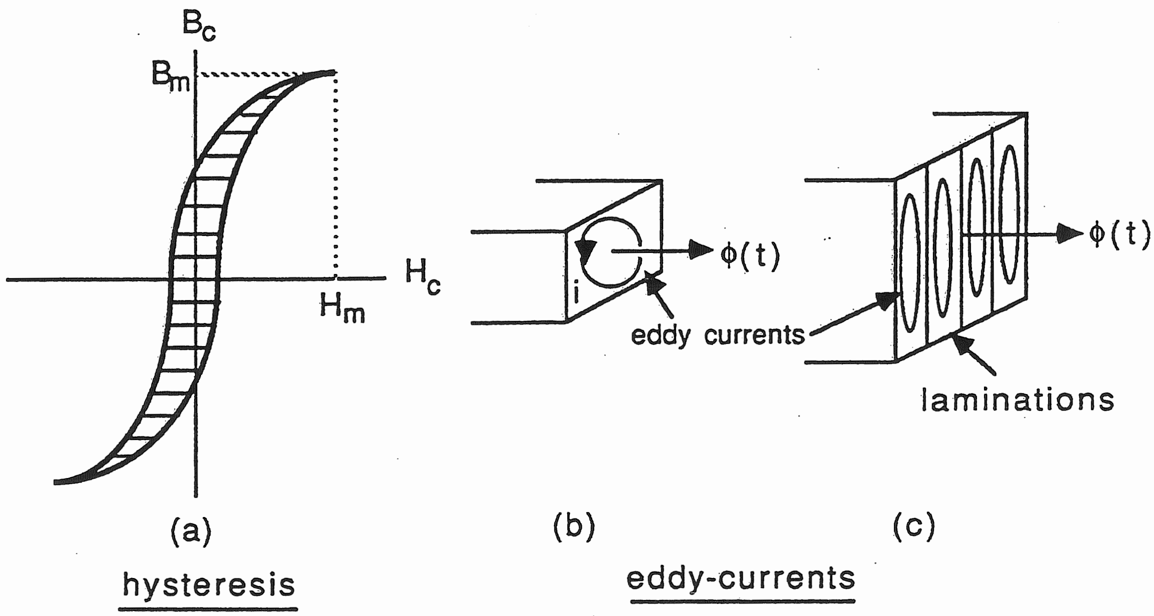


Figure 3.8 Hysteresis and Eddy Currents in an Iron Core

The magnetic state of the core traverses the hysteresis loop, counterclockwise, once each cycle of the source frequency. Ferrous cores consist of billions of current dipoles, each with a north and south pole, that can rotate when the magnetic intensity cyclicly reverses. The energy required to do this is converted to heat within the core and this energy is proportional to the area enclosed by the hysteresis loop. The hysteresis loss is given empirically as,

$$P_h = K_h f B_m^n \qquad \text{(W)}$$

The silicon steels used in power transformers have narrow hysteresis loops, so the hysteresis loss at 60 Hz is quite small.

The infinite number of stationary contours, one of which is shown in the Fig. 3.8 (b) core cross-section, is linked by the core flux which is changing with time. A Faraday emf is generated around each contour resulting in short-circuit currents called eddy-currents. Since the ferrous core is conductive, the eddy-current - i^2R losses are converted to heat, which is given empirically as,

$$P_{ec} = K_e f^2 B_m^2 \qquad \text{(W)}$$

These losses are extremely large unless the core is laminated as shown in Fig. 3.8 (c). The laminations are punched from hot-rolled silicon-steel sheets and stacked to form the core. The oxide-surface coating on both sides of each lamination serves as an excellent insulator between the laminations. The eddy-current paths are now broken up resulting in a quite-small eddy-current power loss. The sum of the hysteresis and eddy-current losses is the core loss, P_c.

In summary, the magnetizing vars, Q_m, of large power transformers is kept small by using high-permeability steels and the core losses, P_c, are kept small by laminating and using steels with narrow hysteresis loops. The transformer model can now be changed to reflect Eqns. (3.11) together with the phasor diagram in Fig. 3.6.

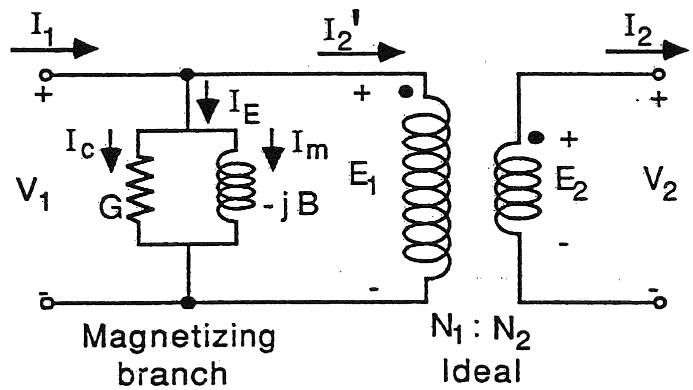


Figure 3.9 A Revised Transformer Model

The magnetizing branch of the model in Fig. 3.9 consists of a core-losses conductance G, mhos and a magnetizing susceptance B, mhos which are very small in power transformers made of laminated, high-permeability, narrow hysteresis-loop steel. In fact, the magnetizing branch of a power transformer is virtually an open circuit but is kept in the model to calculate transformer efficiency. The phasor diagram corresponding to the loaded transformer in Fig. 3.9 is given in Fig. 3.10.

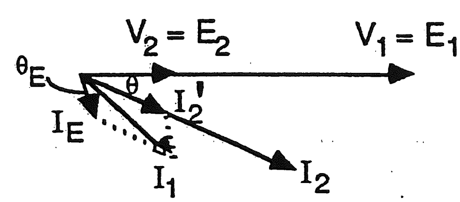


Figure 3.10 Revised Phasor Diagram

The exciting current, I_E, is the vector sum of the core losses current, I_c, and the magnetizing current, I_m, and lags V_1 by the angle, θ_E.

3-4 TRANSFORMER EXCITING CURRENT

The core flux in Fig. 3.9 is sinusoidal, $\phi = \Phi_m \sin \omega t$, because the generator voltage, v_1 and the Faraday emf, $e_1 = \omega N_1 \Phi_m \cos \omega t$, are sinusoidal. The exciting current, i_E, then, is not sinusoidal as shown in Fig. 3.11.

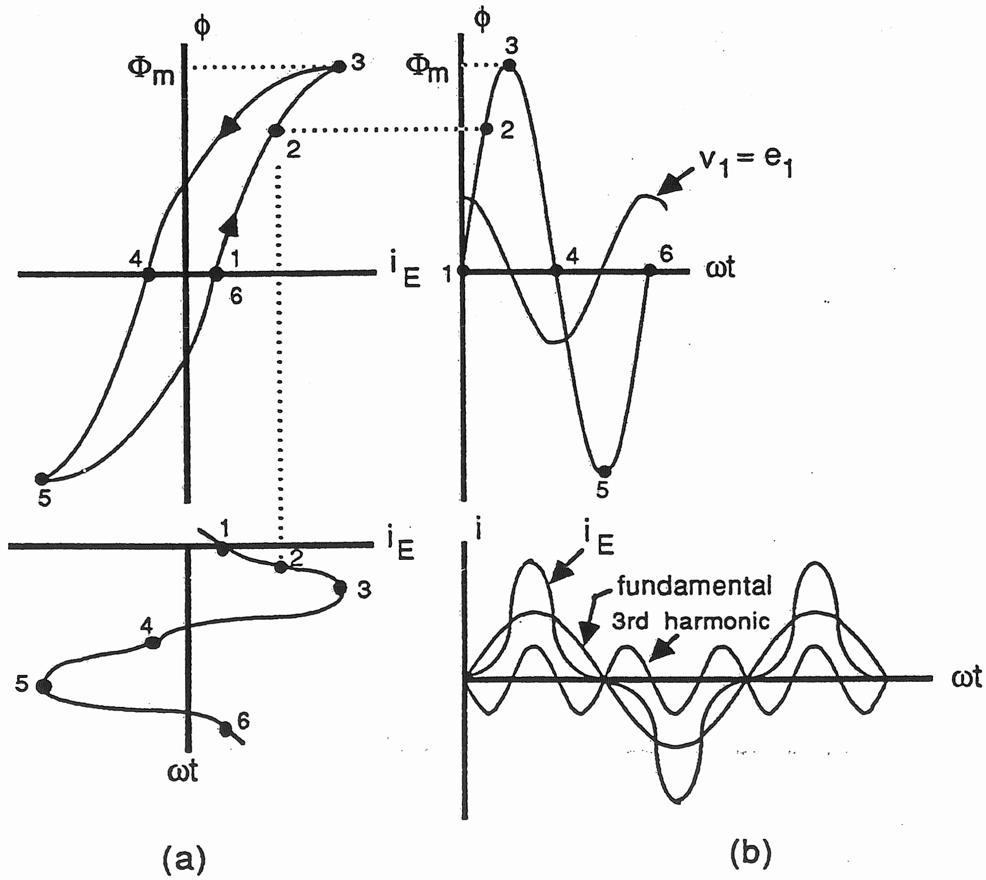


Figure 3.11 Harmonic Content of Transformer Exciting Current

Because of the double-valued hysteresis loop in Fig. 3.11, the exciting current, I_E, must be a nonsinusoidal variation with time to produce a sinusoidal variation of core flux with time. The Fourier series equivalent of the exciting current contains a fundamental whose period is that of the source frequency, and a strong third harmonic, plus an infinite number of odd harmonics whose amplitudes are successively smaller. That the exciting current contains a strong third harmonic is shown in Fig. 3.11 (b) where a third harmonic current is added to a fundamental current to yield a resultant current whose shape is very much like that of the actual exciting current. With this phenomenon in mind, the exciting-current phasor, I_E, in Fig. 3.10 represents the fundamental component of the exciting current, but the conductance, G, in Fig. 3.9, is sized to account for

the total core losses due to all the harmonics in the exciting current. The third harmonic content of the exciting current becomes very important when the transformers are connected three-phase, and a path for the third harmonic currents must be provided to obtain sinusoidally varying phase voltages in the output of the three-phase bank, as will be seen later in Section 3–15.

3-5 TRANSFORMER EQUIVALENT CIRCUITS

The remaining restrictions of the ideal transformer can now be removed. The primary and secondary windings of the loaded transformer inherently have resistance and leakage flux as shown in Fig. 3.12.

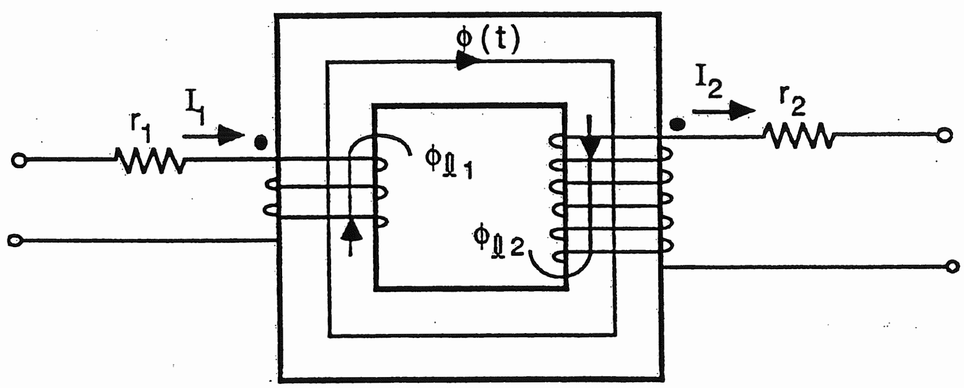


Figure 3.12 Winding Impedances

According to the principles of Chapter 2-8, the leakage flux, whose path is primarily air in Fig. 3.12, is represented in the transformer model as a series inductance, which is the ratio of the leakage flux linkages to the current that produced them. The exact transformer model is shown in Fig. 3.13.

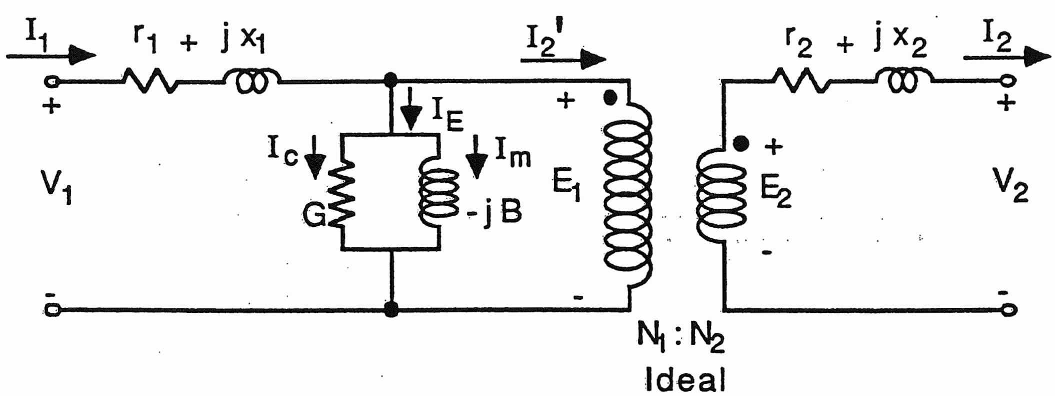


Figure 3.13 Exact Transformer Model

A word of caution is emphasized at this point. A model is only as good as the physical basis on which it is formed. The model in Fig. 3.13 does not include interturn capacitance, interwinding capacitance or winding-to-case capacitance. These capacitances are small, of the order of pico farads, and at power frequencies their reactances are so large that they do not affect transformer performance. For frequencies greater than 1.0 kHz this model fails completely, as Fig. 3.14 indicates.

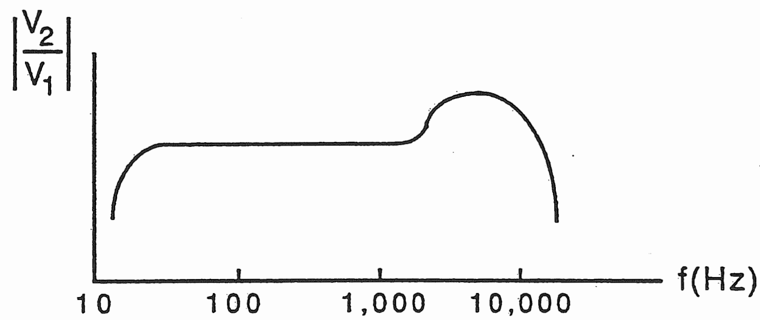


Figure 3.14 Transformer Frequency Response

The model in Fig. 3.13 contains three loops, and for purposes of analysis, it is unnecessarily complex, from a power engineering viewpoint. As pointed out earlier, the exciting current of a well-designed power transformer is very small, since the magnetizing branch is virtually an open-circuit. The primary impedance, $r_1 + j\,x_1$, is also small for "pancake" windings and large cross-section wire. The voltage drop across the primary impedance, therefore, even at full-load current, is small compared to the input voltage, V_1, so that the voltage across the magnetizing branch is essentially that of the input. With engineering validity, then, the magnetizing branch can be moved to the primary terminals, with small error, as shown in Fig. 3.15.

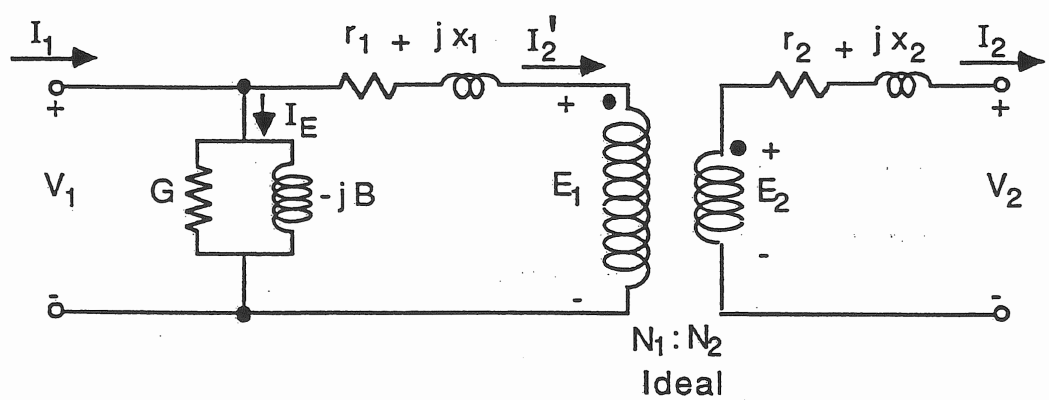
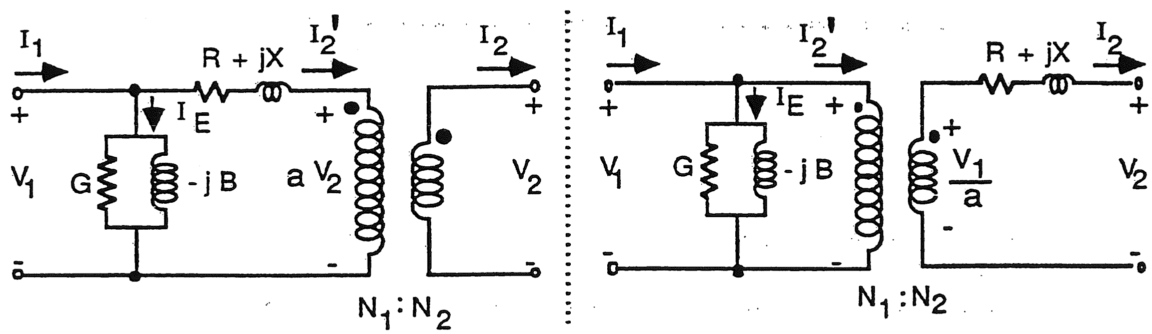


Figure 3.15 Approximate Equivalent Circuit

The actual winding impedances are shown in Fig. 3.15 and using the characteristics of the ideal transformer, the secondary impedance can be referred to the primary forming the primary equivalent circuit, or the primary impedance can be referred to the secondary forming the secondary equivalent circuit as shown in Fig. 3.16.



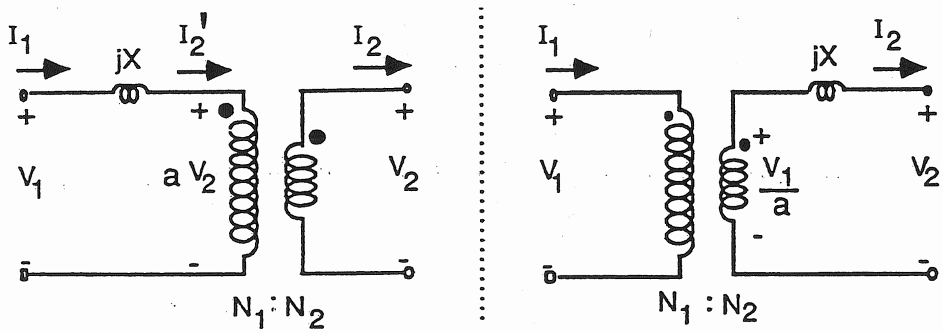
Primary Equivalent	Secondary Equivalent
R = Winding Equivalent Resistance	R = Winding Equivalent Resistance
$= r_1 + a^2 r_2 = r_1 + r_2'$	$= r_1/a^2 + r_2 = r_1' + r_2$
X = Leakage Equivalent Reactance	X = Leakage Equivalent Reactance
$= x_1 + a^2 x_2 = x_1 + x_2'$	$= x_1/a^2 + x_2 = x_1' + x_2$
$V_1 = (R + jX)(I_2/a) + aV_2$	$\frac{V_1}{a} = (R + jX)\, I_2 + V_2$

Figure 3.16 Approximate Primary and Secondary Equivalent Circuits

In Fig. 3.16, the equivalent series impedances, R + j X, are not the same for both equivalent circuits, but they are related by the turns-ratio squared. The magnetizing branch is placed at the primary terminals of both circuits, since the input voltage determines the core losses and the magnetizing vars. For a given load voltage, V_2, and load impedance, Z_2, the input voltage, V_1, can now be calculated, using Kirchhoff's voltage law around each loop of the equivalent circuits as indicated in Fig. 3.16. The voltage equations indicated are <u>vector</u> equations and all variables <u>must</u> be expressed in <u>complex</u> form. Either equivalent circuit may be used to calculate, V_1, with the same result, but the secondary equivalent circuit is used most often since the voltage equation involves the actual load voltage and current, thus minimizing error.

Very often, especially when large, power transformers are in a three-phase connection, the exciting branch is considered an open circuit, and very

often the equivalent winding resistance, R, is less than X/10, in which case it is neglected. The equivalent circuits for each transformer then becomes,



Primary Equivalent   Secondary Equivalent

Figure 3.17 Power Transformer Equivalent Circuits

where the equivalent leakage reactances, in Fig. 3.17, are not the same but are related by the turns-ratio squared.

3-6 SINGLE PHASE TRANSFORMER POWER FLOW

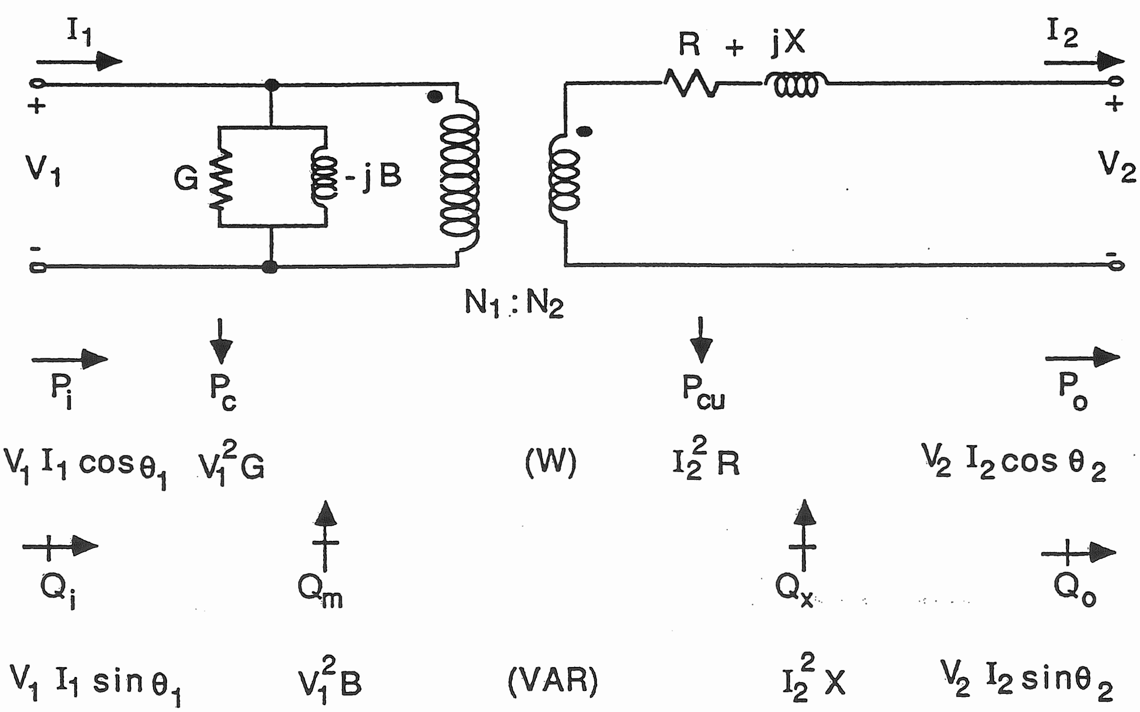


Figure 3.18 Transformer Power Flow

As shown in Fig. 3.18, the input power, P_i, must supply the core and copper losses plus the power required by the load. The reactive power input, Q_i, must supply the magnetizing vars stored in the field in the iron and the leakage vars stored in the field in air plus the vars required by the load.

3-7 TRANSFORMER RATING, EFFICIENCY AND VOLTAGE REGULATION

The characteristics of a transformer that are very important to its operation in a power system are its rating and its two figures of merit – efficiency and voltage regulation.

The rating of a transformer is usually written on its name plate as,

60 Hz, 10,000 kVA, 13.8 – 138 kV

This rating is important because it should not be exceeded. The transformer is designed to operate continuously for many years, as long as its rating is observed. The most important transformer rating, current, is usually not found on the label; rated winding current can be calculated from the ratio of rated volt-amperes to rated voltage. For the above rating, low-winding rated current is 725 A and high-winding rated current is 72.5 A. The transformer is designed to radiate continuously the I^2R losses at rated current, and the core losses at rated frequency and voltage, without destructive overheating. For short periods of time, current overload can be tolerated, provided there is no excessive temperature rise. Rated current is often called full-load current, and apparent power (MVA) is used as a rating rather than real power (MW), because a transformer could deliver more than rated current at zero power factor with no real power delivered.

Efficiency is a figure of merit that is a measure of the magnitude of the transformer internal losses, i.e., how much of the power delivered reaches the load. Efficiency is defined in terms of real power, MW.

$$\text{Eff.} = \eta = \frac{P_o}{P_i} = \frac{P_o}{P_o + \text{losses}} \tag{3.15}$$

Power transformers achieve almost 100% efficiency, since they use narrow-loop, laminated steel and an increased amount of copper.

Voltage regulation is another figure of merit for a transformer since it measures the effect of the internal impedance of a transformer on the voltage across the load. Loads are usually designed to withstand a variation of their rated voltage of ±5.0 %. Overvoltage or undervoltage beyond these limits may destructively jeopardize loads worth millions of dollars.

Voltage regulation is defined in terms of the magnitude of the secondary voltage as it varies from no-load to full-load.

$$V.R. = \frac{V_2\,(NL) - V_2\,(FL)}{V_2\,(FL)} \tag{3.16}$$

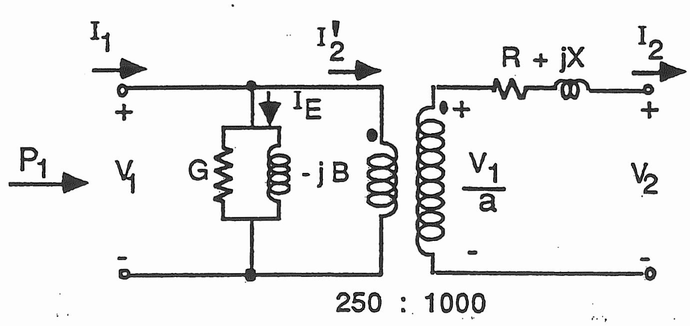
The regulation, as defined in Eqn. (3.16), can be positive or negative, depending on whether the load is inductive or capacitive. The desirable value of voltage regulation is zero percent, but acceptable values must lie approximately between ± 5.0 percent of rated voltage.

3-8 TRANSFORMER OPEN-CIRCUIT, SHORT-CIRCUIT TESTS

The parameters of the equivalent circuits in Fig. 3.16 are calculated from measurements made by open-circuiting or short-circuiting either winding, and placing the measuring instruments on the other winding. For either method, the secondary equivalent circuit is used to calculate the parameters.

Example 3.2

Open-circuit and short-circuit, voltmeter, ammeter and wattmeter measurements are made on a transformer rated 60 Hz, 5 kVA, 250–1000 V. The secondary equivalent circuit and its measured variables are indicated in the figure and below.



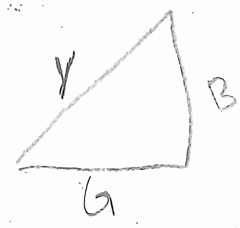
Secondary Open-Circuited	Secondary Short-Circuited
$V_1 = 250$ V	$V_1 = 7.9$ V
$V_2 = 1000$ V	$V_2 = 0$ V
$I_1 = 0.51$ A	$I_1 = 20$ A
$P_1 = 32.3$ W	$P_1 = 50$ W

On open circuit, V_1 = rated
$I_1 = I_E$ (exciting current)
$P_1 = P_c$ (core losses)

$P_1 = \frac{V^2}{R}$
$P_1 = V^2 G$

$$G = \frac{P_1}{V_1^2} = \frac{32.3}{(250)^2} = 517\ \mu\mho \quad (R_c = 1930\ \Omega)$$

$$Y = \frac{I_1}{V_1} = \frac{0.51}{250} = 2{,}040\ \mu\mho$$



$$B = \sqrt{Y^2 - G^2} = 1970\ \mu\mho \quad (X_m = 508\ \Omega)$$

$G^2 + B^2 = Y^2$
$B = \sqrt{Y^2 - G^2}$

$$a = \frac{V_1}{V_2} = \frac{250}{1000} = 0.25$$

On short circuit, V_1 is reduced (therefore $I_E << 0.51$ A) until $I_1 = I_2' = 20$ A (rated) and $P_1 = P_{cu}$ (copper losses).

$$R = \frac{P_1}{(aI_1)^2} = \frac{50}{(0.25 \times 20)^2} = 2.0\ \Omega$$

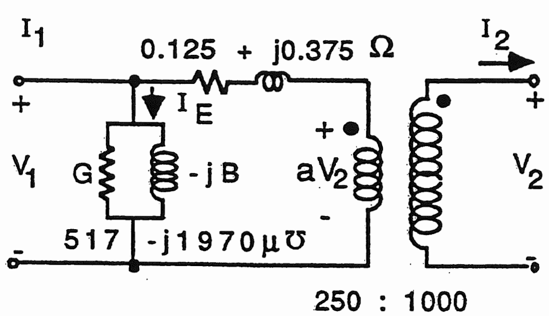
$P = I^2 R \rightarrow R = \frac{P}{I^2}$

$$Z = \frac{V_1/a}{a\ I_1} = \frac{(7.9)\ /\ (0.25)}{(0.25)(20)} = 6.32\ \Omega$$

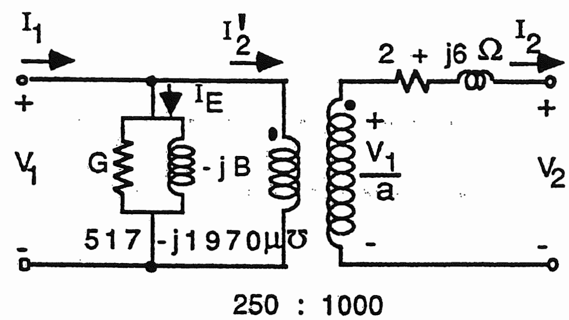
$Z = \frac{V}{I}$

$$X = \sqrt{Z^2 - R^2} = 6.0\ \Omega$$

a) Sketch the approximate primary and secondary equivalent circuits for this transformer.



Primary Equivalent



Secondary Equivalent

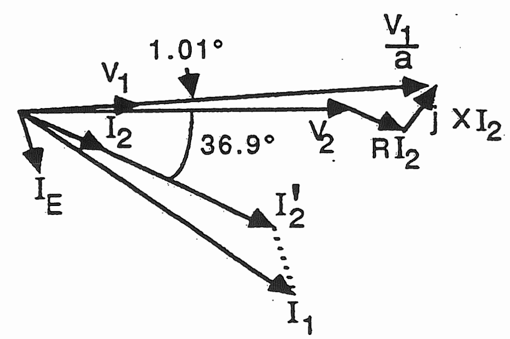
b) If the transformer is delivering rated kVA, 0.8 pf lagging, at rated voltage to a load, what is the primary voltage, and draw the phasor diagram.

Using the secondary equivalent circuit,

$$V_2 = 1000\underline{/0^\circ}\ \text{V} \quad ; \quad I_2 = \frac{5000}{1000}\underline{/-36.9^\circ} = 5.0\underline{/-36.9^\circ}\ \text{A}$$

$$\frac{V_1}{a} = (R + jX)I_2 + V_2 = (2 + j6)(5.0\underline{/-36.9^\circ}) + 1000\underline{/0^\circ} = 1026\underline{/1.01^\circ}\ \text{V}$$

$$V_1 = (0.25)(1026\underline{/1.01^\circ}) = 257\underline{/1.01^\circ}\ \text{V}$$



c) Calculate the transformer efficiency and voltage regulation for the load in part (b).

$$\eta = \frac{P_o}{P_o + \text{losses}} = \frac{5000\cos 36.9^\circ}{4000 + \underbrace{(257)^2(517\times10^{-6})}_{P_c} + \underbrace{(5)^2(2)}_{P_{cu}}} = 97.9\ \%$$

$$\text{V.R.} = \frac{V_2\,(\text{NL}) - V_2\,(\text{FL})}{V_2\,(\text{FL})} = \frac{1026 - 1000}{1000} = 2.6\%$$

(with $\frac{V_1}{a}$ written above V_2 (NL))

3-9 SINGLE-PHASE PER-UNIT SYSTEM

In an electric power system, transformers vary widely in their ratings and it is difficult to compare their internal impedances with respect to efficiency and voltage regulation. Furthermore, the equivalent circuit of the transformer is used in power system analysis and problems develop in remembering whether the equivalent circuit is referred to the high or low voltage side. As will be seen in subsequent analysis, the high and low equivalent circuits are the same in the per-unit system obviating the problems cited above. Manufacturers usually give the impedance and admittance of their transformers in per-unit values rather than ohmic values so that small and large transformers can be easily compared.

In the per-unit system, voltages, currents, impedances, and power are expressed as percentages of base values. The per-unit system will first be defined for single-phase quantities and then will be extended, later in the chapter, to three-phase quantities. A per-unit value of voltage, current, impedance or power is defined as follows,

$$\text{Quantity (pu)} = \frac{\text{actual value}}{\text{base value}} \tag{3.17}$$

where the quantity (pu) is dimensionless.

The per-unit system is well-known in other areas of electrical engineering, but the process is called "normalizing" variables. For example, consider the input impedance of the parallel resonant circuit in Fig. 3.19.

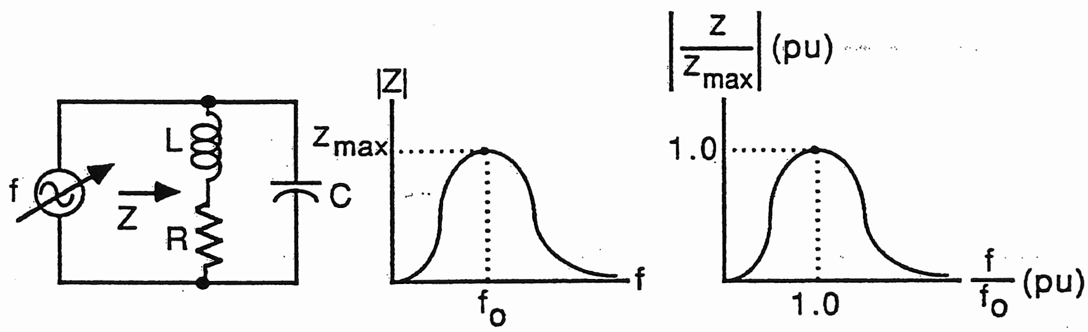
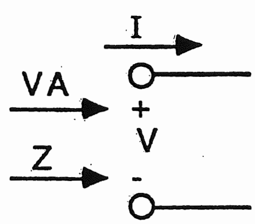


Figure 3.19 Parallel Resonant Circuit

In Fig. 3.19, for given values of R, L and C, the input impedance varies, where Z_{max} occurs at the resonant frequency, f_o. The center plot, however, is good only for the given values R, L, and C. A generalized plot, good for all values of R, L, and C is given in the right-hand sketch where the actual values of Z and f are "normalized" with respect to the arbitrary base values Z_{max} and f_o. In power-system parlance, the right-hand sketch would be characterized at resonance by saying the actual impedance is one per-unit or 100% of the base value, Z_{max},, at the actual frequency of one per-unit which is 100% of the base value, f_o.

All power engineers, without exception, specify the base quantities at any power-system node in the same way. Any single-phase node in a power system is completely specified when the four quantities in Fig. 3.20 are known.



1. voltage
2. volt-amperes } chosen
3. current
4. impedance } derived

Figure 3.20 Base Quantities at a Power System Node

Only two of the four quantities in Fig. 3.20 are independent; the other two can then be obtained from known laws in electrical engineering. Without exception, power engineers choose arbitrary values of voltage and volt-amperes as base values at a node, in which case, base values of current and impedance are no longer arbitrary but are derived.

The base value of current is then,

$$I_{base} = \frac{VA_{base}}{V_{base}} \quad (A) \qquad (3.18)$$

The base values of impedance and admittance are,

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(V_{base})^2}{VA_{base}} = \frac{(kV_{base} \times 1000)^2}{MVA_{base} \times 10^6} = \frac{(kV_{base})^2}{MVA_{base}} \quad (\Omega)$$

$$Y_{base} = \frac{1}{Z_{base}} \quad (\mho) \qquad (3.19)$$

Example 3.3

The 5 kVA, 250-1000 V transformer of Example 3.2 has two nodes- the low (primary) node and the high(secondary) node.

a) Express the low (primary) equivalent circuit and the high(secondary) equivalent circuit in per-unit, using the transformer rating as a base

Chosen

V_{base} (low) = 250 V
V_{base} (high) = 1000 V
VA_{base} = 5,000 VA (common to both sides)

Derived.

$$I_{base}(low) = \frac{VA_{base}}{V_{base}} = \frac{5000}{250} = 20.0 \text{ A}$$

$$I_{base}(high) = \frac{VA_{base}}{V_{base}} = \frac{5000}{1000} = 5.0 \text{ A}$$

$$Z_{base}(low) = \frac{(V_{base})^2}{VA_{base}} = \frac{(kV_{base})^2}{MVA_{base}} = \frac{(250)^2}{5000} = \frac{(0.25)^2}{0.005} = 12.5 \ \Omega$$

$$Z_{base}(high) = \frac{(V_{base})^2}{VA_{base}} = \frac{(kV_{base})^2}{MVA_{base}} = \frac{(1000)^2}{5000} = \frac{(1.0)^2}{0.005} = 200 \ \Omega$$

$$Y_{base}\,(low) = \frac{1}{Z_{base}} = \frac{1}{12.5} = 0.08\ \mho$$

$$Y_{base}(high) = \frac{1}{Z_{base}} = \frac{1}{200} = 0.005\ \mho$$

The per-unit values of the transformer, equivalent-circuit impedance and admittance can now be calculated,

$$Z(low) = \frac{Z}{Z_{base}} = \frac{0.125 + j\,0.375}{12.5} = 0.01 + j\,0.03\ \text{pu}$$

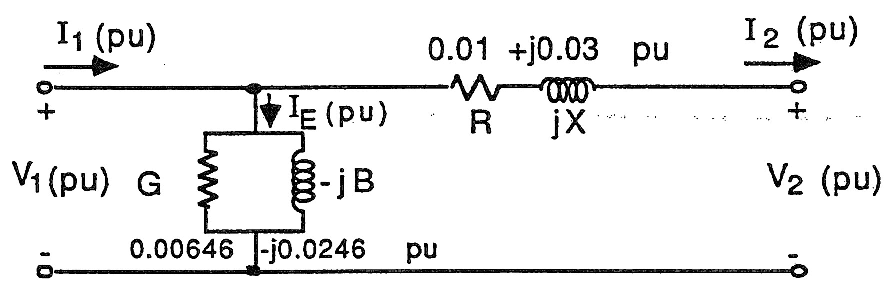
$$Z(high) = \frac{Z}{Z_{base}} = \frac{2 + j\,6}{200} = 0.01 + j\,0.03\ \text{pu}$$

$$Y(low) = \frac{Y}{Y_{base}} = \frac{(517 - j\,1970)(10^{-6})}{0.08} = 0.00646 - j\,0.0246\ \text{pu}$$

The ideal transformer turns-ratio is, in per-unit,

$$\frac{250}{250} : \frac{1000}{1000} = 1{:}1\ \text{pu}$$ (the ideal transformer in pu can be omitted).

Realize from the above, that both equivalent circuits in Example 3.2(a) have the same equivalent circuit in pu,



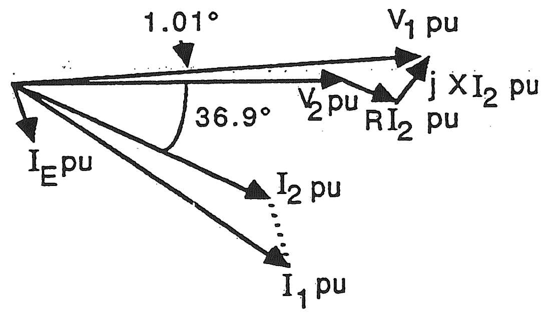
b) Calculate, V_1, per-unit, if the transformer is delivering rated kVA, 0.8 pf lagging at rated voltage to a load, using per-unit values only, and draw the phasor diagram.

$$V_2\,(pu) = \frac{V_2}{V_{base}} = \frac{1000\,\angle 0^\circ}{1000} = 1.0\,\angle 0^\circ\ \text{pu}$$

$$I_2\,(pu) = \frac{I_2}{I_{base}} = \frac{\frac{5000}{1000}\angle -36.9^\circ}{5.0} = 1.0\,\angle -36.9^\circ\ \text{pu}$$

$$V_1(\text{pu}) = (R + jX)\, I_2 + V_2 = (0.01 + j\,0.03)(1.0\,\underline{/-36.9^\circ}) + 1.0\,\underline{/0^\circ}$$

$$= 1.026\underline{/1.01^\circ}\ \text{pu}$$



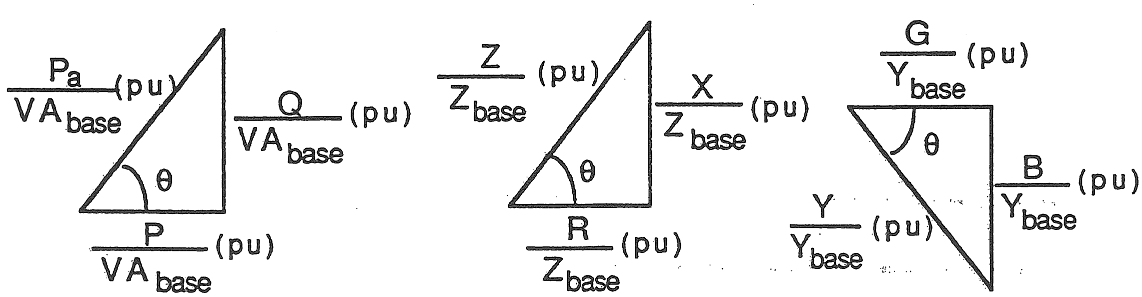
c) Calculate the efficiency and voltage regulation using per-unit values only,

$$P_o\,(\text{pu}) = \frac{P_o}{VA_{base}} = \frac{5000\cos 36.9^\circ}{5000} = 0.8\ \text{pu}$$

$$\eta = \frac{P_o}{P_o + \text{losses}} = \frac{0.8}{0.8 + \underbrace{(1.026)^2(0.00646)}_{Pc} + \underbrace{(1.0)^2(0.01)}_{Pcu}} = 97.9\ \%$$

$$VR = \frac{V_2\,(NL) - V_2\,(FL)}{V_2\,(FL)} = \frac{1.026 - 1.0}{1.0} = 2.6\ \%$$

From Example 3.3, it is seen that the power, impedance and admittance triangles remain invariant in angle and shape when placed in the per-unit system,



It is also important to realize that it is no coincidence that the low and high equivalent circuits are the same in the per-unit system. While the chosen values of V_{base} (low) and V_{base}(high) are arbitrary, they must be chosen in the transformer turns-ratio, which is necessarily true when the transformer rating is chosen as a base.

Remembering that a quantity in per-unit is,

$$Q\text{ (pu)} = \frac{\text{actual value}}{\text{base value}},$$

it may be instructive to consider the inverse analysis of Example 3.3.

Example 3.4

A single-phase transformer is rated,

$$\begin{gathered} 5\text{ kVA} \\ 250 - 1000\text{ V} \\ Z = 0.01 + j\,0.03\text{ pu} \\ Y = 0.00646 - j\,0.0246\text{ pu} \end{gathered}$$

What are the actual operating values of this transformer if it is delivering 1.0 pu VA, 0.8 lagging at 1.0 pu voltage to a load?

$$V_{base}\text{ (low)} = 250\text{ V} \quad ; \quad I_{base}\text{ (low)} = \frac{5000}{250} = 20\text{ A}$$

$$V_{base}\text{ (high)} = 1000\text{ V} \quad ; \quad I_{base}\text{ (high)} = \frac{5000}{1000} = 5.0\text{ A}$$

$$VA_{base} = 5\text{ kVA} \quad ; \quad Z_{base}\text{ (low)} = \frac{(0.25)^2}{0.005} = 12.5\ \Omega$$

$$Z_{base}\text{ (high)} = \frac{(1.0)^2}{0.005} = 200\ \Omega$$

$$I_2\text{ (pu)} = \frac{VA}{V} = \frac{1.0}{1.0}\underline{/-36.9^\circ} = 1.0\underline{/-36.9^\circ}\text{ pu} \qquad \begin{aligned} &\times 5\text{ (high)} = 5\underline{/-36.9^\circ}\text{ A} = I_2 \\ &\times 20\text{ (low)} = 20\underline{/-36.9^\circ}\text{ A} = I_2' \\ &\Uparrow \\ &I\text{ base} \end{aligned}$$

$$Z_2\text{ (load) pu} = \frac{V_2}{I_2} = \frac{1.0\underline{/0^\circ}}{1.0\underline{/-36.9^\circ}} = 1.0\underline{/36.9^\circ} \qquad \begin{aligned} &\times 200\text{ (high)} = 200\underline{/36.9^\circ}\ \Omega = Z_2 \\ &\times 12.5\text{ (low)} = 12.5\underline{/36.9^\circ}\ \Omega = Z_2' \\ &\Uparrow \\ &Z\text{ base} \end{aligned}$$

$$P_2\text{ (pu)} = V_2 I_2 \cos\theta_2 = (1.0)(1.0)\cos 36.9^\circ = 0.8\text{ pu} \qquad \times 5 = 4\text{ kW}$$

$$Q_2\text{ (pu)} = V_2 I_2 \sin\theta_2 = (1.0)(1.0)\sin 36.9^\circ = 0.6\text{ pu} \qquad \times 5 = 3\text{ kVAR}$$

$$\Uparrow$$
$$VA\text{ base}$$

$$V_1\,(pu) = (R + jX)\,I_2 + V_2 = (0.01 + j\,0.03)(1.0\,\underline{/-36.9^\circ}) + 1.0\,\underline{/0^\circ}$$

$$= 1.026\,\underline{/1.01^\circ}\ \text{pu} \quad \times\ 250 = 247\,\underline{/1.01^\circ}\ \text{V} = V_1$$

$$\times \underset{\Uparrow \atop V_{base}}{1000} = 1026\,\underline{/1.01^\circ}\ \text{V} = \frac{V_1}{a}$$

$$Z = 0.01 + j\,0.03\ \text{pu} \quad \times\ 12.5 = 0.125 + j\,0.375\ \Omega = Z\ \text{(referred to primary)}$$

$$\times \underset{\Uparrow \atop Z_{base}}{200} = 2 + j\,6\ \Omega = Z\ \text{(referred to secondary)}$$

$$Y = 0.00646 - j\,0.0246\ \text{pu} \quad \times\ 0.08 = 517 - j\,1970\ \mu℧ = Y\ \text{(primary side)}$$

$$\times \underset{\Uparrow \atop Y_{base}}{0.005} = 32.3 - j\,123\ \mu℧ = Y\ \text{(secondary side)}$$

$$I_E\,(pu) = V_1 Y = (1.026\,\underline{/1.01^\circ})(0.00646 - j\,0.0246)$$

$$= 0.0261\,\underline{/-74.3^\circ}\ \text{pu} \quad \times \underset{\Uparrow \atop I_{base}}{20} = 0.522\,\underline{/-74.3^\circ}\ \text{A (primary side)}$$

$$I_1\,(pu) = I_E + I_2 = 0.0261\,\underline{/-74.3^\circ} + 1.0\,\underline{/-36.9^\circ}$$

$$= 1.021\,\underline{/-37.8^\circ}\ \text{pu} \quad \times \underset{\Uparrow \atop I_{base}}{20} = 20.4\,\underline{/-37.8^\circ}\ \text{A (primary side)}$$

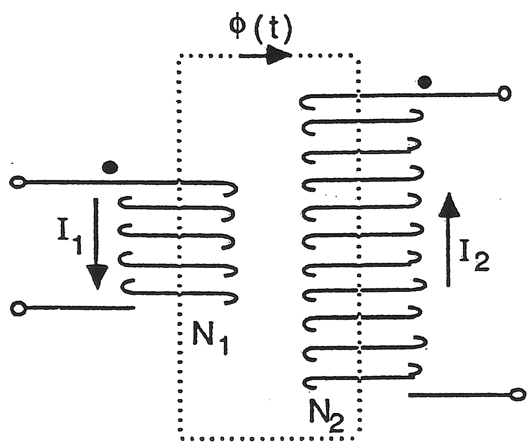
$$P_1\,(pu) = V_1 I_1 \cos\theta_1 = (1.026)(1.021)\cos(1.01^\circ + 37.8^\circ) = 0.816\ \text{pu} \quad \times\ 5 = 4.08\ \text{kW}$$

$$Q_1\,(pu) = V_1 I_1 \sin\theta_1 = (1.026)(1.021)\sin 38.8^\circ = 0.656\ \text{pu} \quad \times \underset{\Uparrow \atop VA_{base}}{5} = 3.28\ \text{kVAR}$$

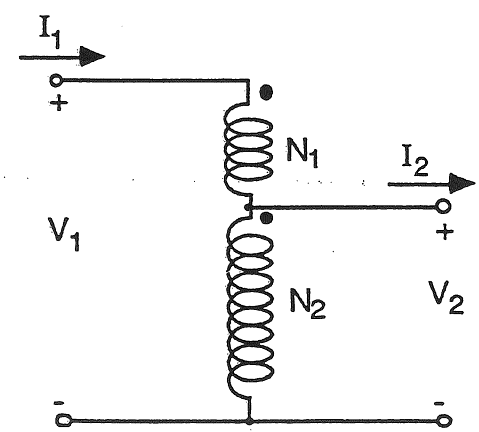
In summary, the internal impedance of 0.01 + j 0.03 pu and admittance of 0.00646 – j 0.0246 pu of this transformer can now be compared with other transformers of different ratings whose internal impedances are similarly expressed. While it may seem unnecessarily complex to place a transformer in the per-unit system, when a power system is analyzed with hundreds of thousands of transformers, the per-unit system will prove invaluable in later analysis.

3-10 THE AUTO TRANSFORMER

In electric power systems the auto transformer sometimes becomes a cost-effective savings in economy when compared to the ordinary transformer. Its disadvantage is that the input is no longer electrically isolated from the output as the ordinary transformer would be. The autotransformer can be considered as an ordinary transformer whose primary and secondary windings are connected in series as shown in Fig. 3.21.



(a)
Ordinary Transformer



(b)
Autotransformer

Figure 3.21 Autotransformer Diagram

As indicated in Fig. 3.21, the primary and secondary windings are always connected in series, resulting in no electrical isolation between the source and the load. The windings can be connected in series additive as shown in the figure or series subtractive; they can be connected in series across the source or across the load, resulting in many combinations as illustrated in Fig. 3.22.

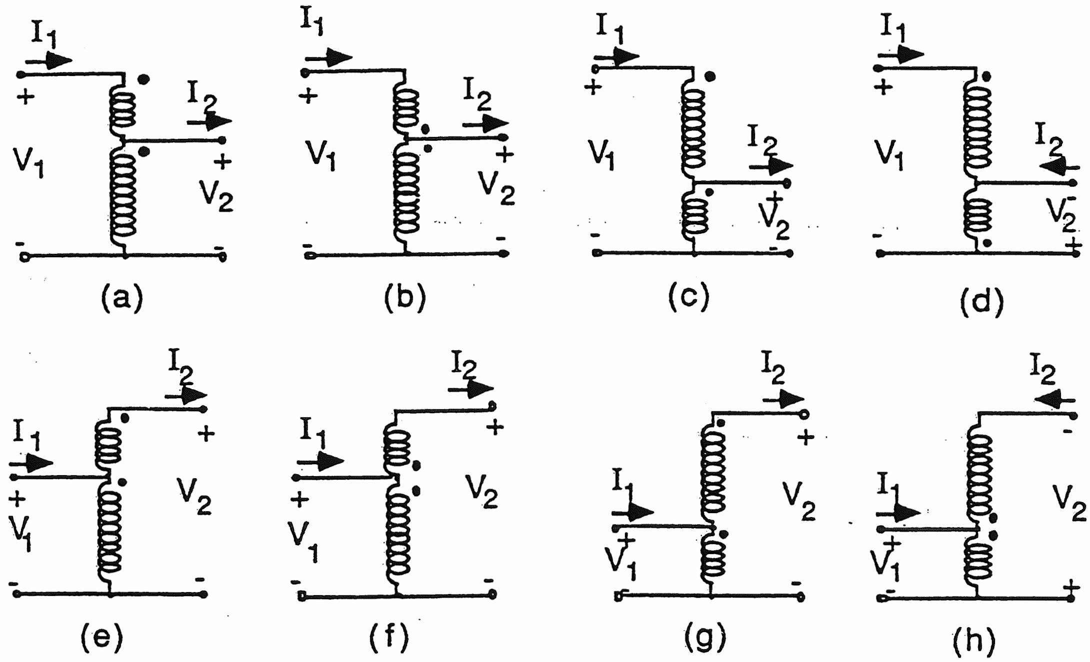


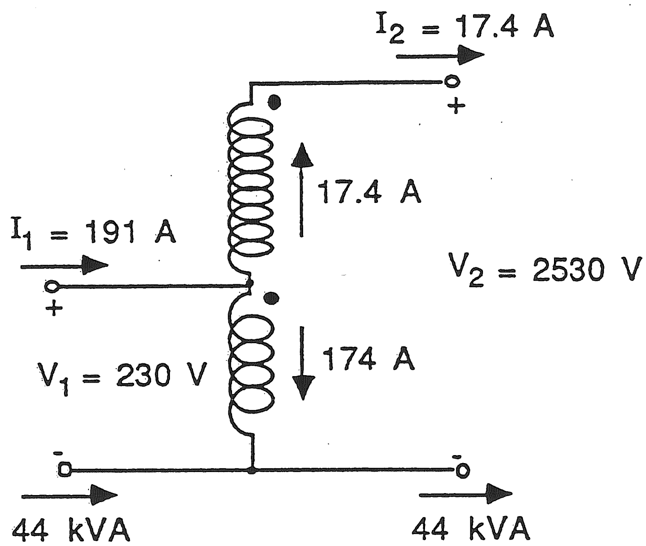
Figure 3.22 Autotransformer Connections

The analysis of the autotransformer is greatly facilitated by assuming the positive directions of the input and output voltages and currents as indicated in Fig. 3.22; power flow is then positive from left to right. Furthermore, the coil currents are always chosen so that they are in time-phase, then the currents can be added algebraically at the center node, i.e., one coil current is chosen flowing into a dot in which case the other coil current is chosen away from its dot as indicated in Fig. 3.21 (a) and explained in Eqn. 3.6). The autotransformers of Fig. 3.22 (g), (h) are analyzed in Example 3.5.

Example 3.5

A 230-2300 volt, 40 kVA transformer, with negligible losses, is to be connected as a step-up autotransformer from a 230 volt source to a 2530 volt load. Draw the autotransformer diagram showing all assumed voltage and current directions and polarity markings. Calculate all voltages and currents and the autotransformer kVA rating so that the coil ratings are not exceeded. Coil currents are assumed so that coil mmfs are subtractive with corresponding in-phase currents.

$$I_{low}\,(\text{rated}) = \frac{40000}{230} = 174 \text{ A} \quad ; \quad I_{high}\,(\text{rated}) = \frac{40000}{2300} = 17.4 \text{ A}$$



In the above diagram, the autotransformer is loaded so that rated coil currents flow.

$$V_2 = 230 + \left(\frac{2300}{230}\right) 230 = 2530 \text{ V}$$

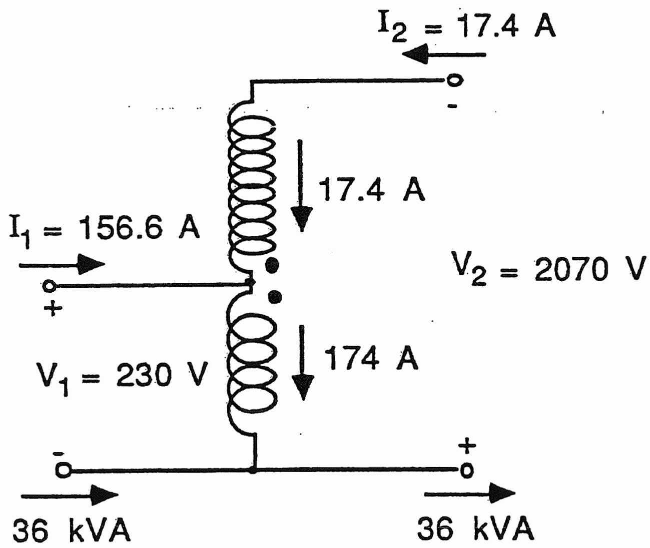
$$I_1 = 17.4 + \left(\frac{2300}{230}\right) 17.4 = 191 \text{ A}$$

$$\text{kVA}_{in} = (230)(191) = 44 \text{ kVA}$$

$$\text{kVA}_{out} = (2530)(17.4) = 44 \text{ kVA}$$

The results of the above analysis indicate that when the autotransformer is connected additive, it can be rated 4 kVA higher than the 40 kVA rating of its windings. This is true because, as the above diagram indicates, 40 kVA is transferred through the coil magnetic field and the remaining 4 kVA is conductively transferred directly from source to load. The additive, stepup autotransformer, then, has a cost advantage over an ordinary transformer with the same windings, except that the autotransformer windings must be insulated to case at a higher voltage. Autotransformers are extensively used in three-phase connections as step-up or down regulator transformer banks and as motor starters.

Repeat the above problem with the windings connected subtractive.



In the above diagram, the autotransformer is loaded so that rated coil currents flow.

$$V_2 = \left(\frac{2300}{230}\right) 230 - 230 = 2070 \text{ V}$$

$$I_1 = \left(\frac{2300}{230}\right) 17.4 - 17.4 = 156.6 \text{ A}$$

$$\text{kVA}_{in} = (230)(156.6) = 36 \text{ kVA}$$

$$\text{kVA}_{out} = (2070)(17.4) = 36 \text{ kVA}$$

3-11 THREE PHASE TRANSFORMER CONNECTIONS

While single-phase transformers are used extensively in residence-service inputs, three-phase transformer banks are used primarily in the high-voltage portion of a power system. Single-phase transformer theory will now be extended to three-phase transformer banks in the four most-used connections shown in Fig. 3.23.

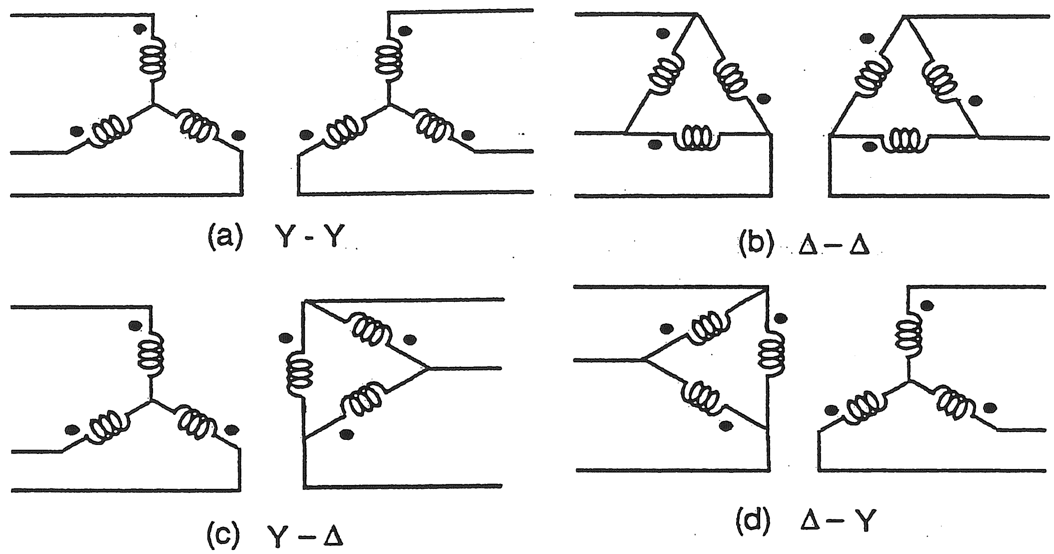


Figure 3.23 Transformer Bank Connections

Each of the four banks in Fig. 3.23 consists of three identical transformers whose primaries and secondaries are connected in wye or delta. The primary and secondary windings of each transformer that are magnetically coupled are drawn parallel to each other. The most-used banks are (b), (c), and (d) with the Δ– Δ bank having the advantage that one transformer can be removed for repair resulting in an open-delta, three-phase connection.

The voltage, current and kVA relationships of two of the transformer banks, Y-Y and Y-Δ. will be considered in Example 3.6.

Example 3.6

Each of the transformer banks in Fig. 3.23 consists of three, identical individual transformers whose ratings are,

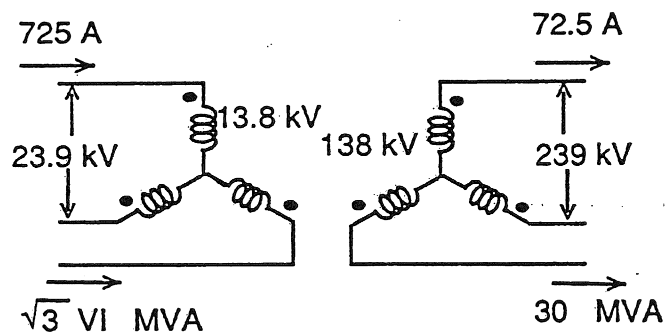
13.8 - 138 kV
10 MVA
$X = 1.9\ \Omega$ (low voltage side)

The magnetizing branch of each transformer is considered open, and the equivalent winding resistance is negligible compared to the leakage reactance of 1.9 Ω. Draw the transformer bank diagrams for the Y-Y and Y-Δ connections and label each diagram with voltages, currents and total MVA that would load the individual transformers at their rated values. From these values give the transformer bank ratings.

Rated current for each individual transformer is,

$$I_1 = \frac{10000}{13.8} = 725 \text{ A} \quad ; \quad I_2 = \frac{10000}{138} = 72.5 \text{ A}$$

a) Y-Y Transformer Bank

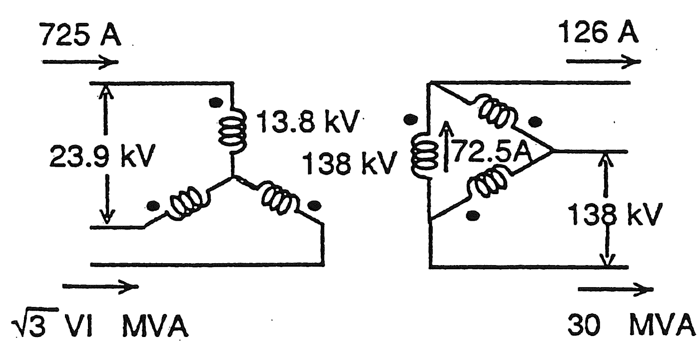


Bank Rating : 23.9 Y - 239 Y kV (line voltage and connection)

30 MVA (total MVA)

Figure 3.24 Y - Y Bank Rating

b) Y-Δ Transformer Bank



Bank Rating : 23.9 Y - 138 Δ kV (line voltage and connection)

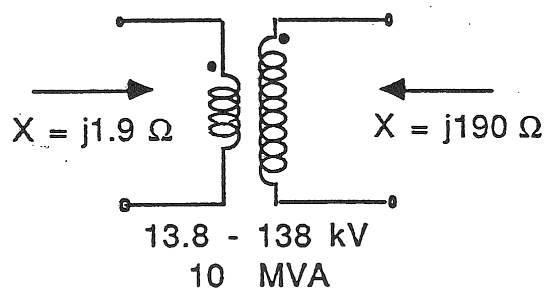
30 MVA (total MVA)

Figure 3.25 Y – Δ Bank Rating

The bank ratings are always given in terms of line voltages, connection and total MVA. The bank rating in this form will become very useful in determining its per-phase equivalent circuit and when the bank is made part of a power system where per-unit quantities are later used. Consider, now, the per-phase equivalent circuit of the Y-Y and Y-Δ banks in Example 3.7.

Example 3.7

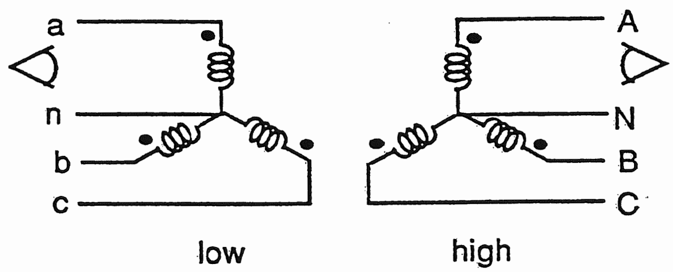
Derive the per-phase equivalent circuits of the Y-Y and Y-Δ transformer banks using the three individual transformers of Example 3.6. Each individual transformer can be visualized as follows,



With the assumptions concerning the magnetizing branch and the winding resistance made in Example 3.6, the impedance looking into the low winding is the leakage reactance referred to the primary. The impedance looking into the high winding is the leakage reactance referred to the secondary and they are related by the turns-ratio squared.

The transformer bank, consisting of three of these individual transformers, is a three-phase device with three ports, each port defined from line to neutral. Therefore, the per-phase equivalent circuit is found, line to neutral, and phase a will be selected. Phases b and c will have identical equivalent circuits.

Y-Y Equivalent Circuit



Bank Rating

23.9 Y - 239 Y kV
30 MVA
X = 1.9 Ω (low side)

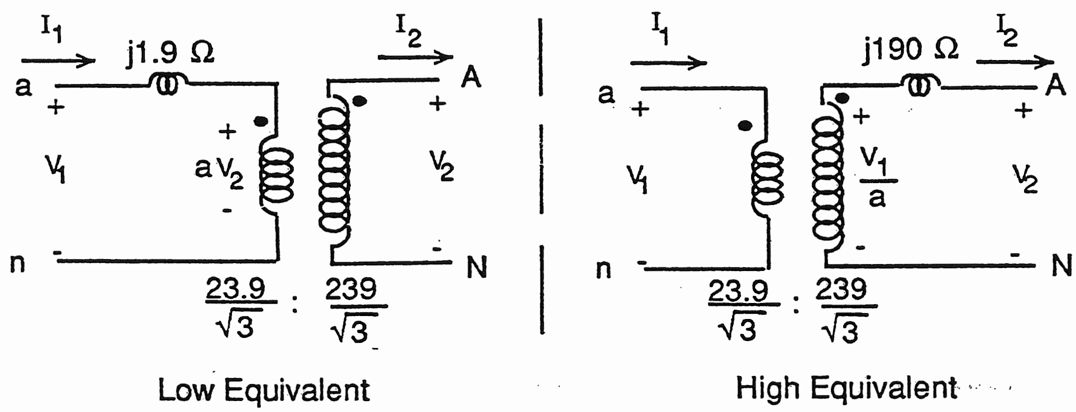


Figure 3.26 Y - Y Per-Phase Equivalent Circuit

The low and high per-phase equivalent circuits in Fig. 3.26 are found by looking into ports a and A and observing the impedance seen. The effective turns-ratio, in this case, is the ratio of the rated voltage of each coil ($V_{line}/\sqrt{3}$). Since the $\sqrt{3}$ cancels in the turns-ratio, the effective turns-ratio is the ratio of the rated line voltages. All voltages, currents, and impedances are referred to either side by this turns-ratio.

Y-Δ Equivalent Circuit

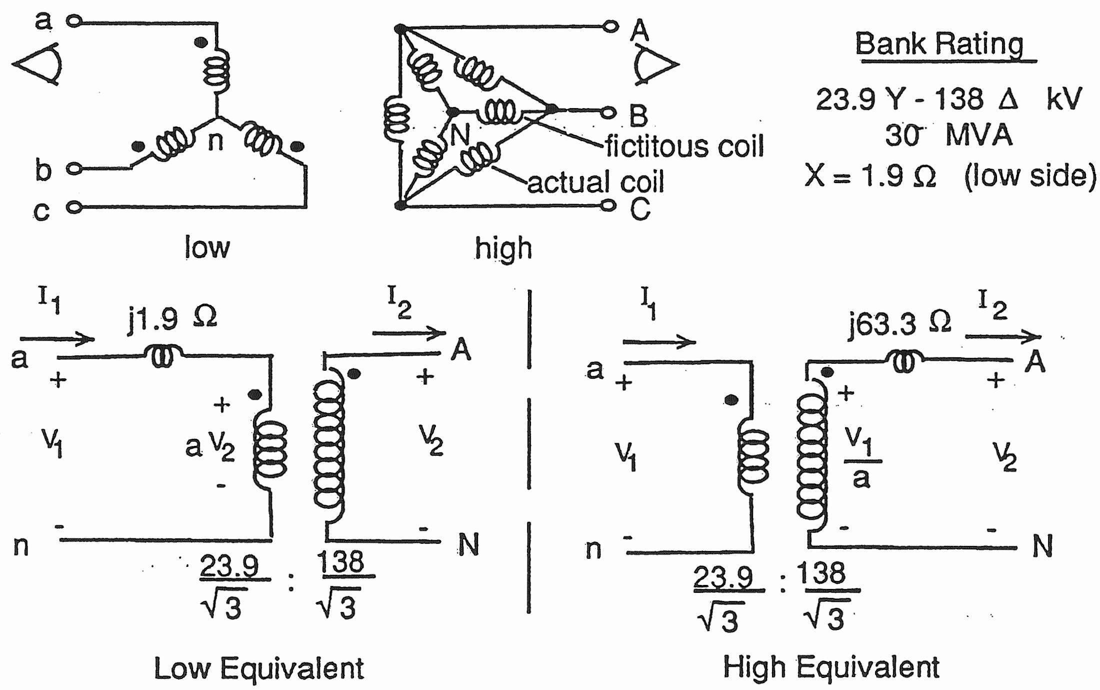


Figure 3.27 Y – Δ Per-Phase Equivalent Circuit

The low-side equivalent circuit in Fig. 3.27 is found by looking into port a. Here, the actual primary coil of the transformer is seen, line to neutral, with its leakage reactance of 1.9 Ω. The high-side equivalent circuit is found by looking into port A. Here, the neutral does not physically exist since the secondaries are connected in delta. The neutral point, N, is established by replacing the delta with an equivalent wye-set of fictitious windings as indicated in Fig. 3.27. Port A is now established by looking into the fictitious winding, A-N. Before we can look into the fictitious winding, however, we must look into each actual secondary winding and we see leakage reactance, 190 Ω. Therefore, the impedance looking into each fictitious coil is 190/3 = 63.3 Ω, and is so-indicated in the high-side equivalent circuit of Fig. 3.27. The effective turns-ratio is the ratio of the rated voltage across the primary coil and the fictitious secondary coil ($V_{line}/\sqrt{3}$) which is not the actual turns ratio of the individual transformers. Since the $\sqrt{3}$ cancels in the turns-ratio, the effective turns-ratio is the ratio of the line voltages. In fact, for all four of the bank connections in Fig. 3.23, the effective turns-ratio of each equivalent circuit is the ratio of the line voltages; the bank rating then becomes very important in establishing this ratio.

3-12 THREE PHASE TRANSFORMER BANK POWER FLOW

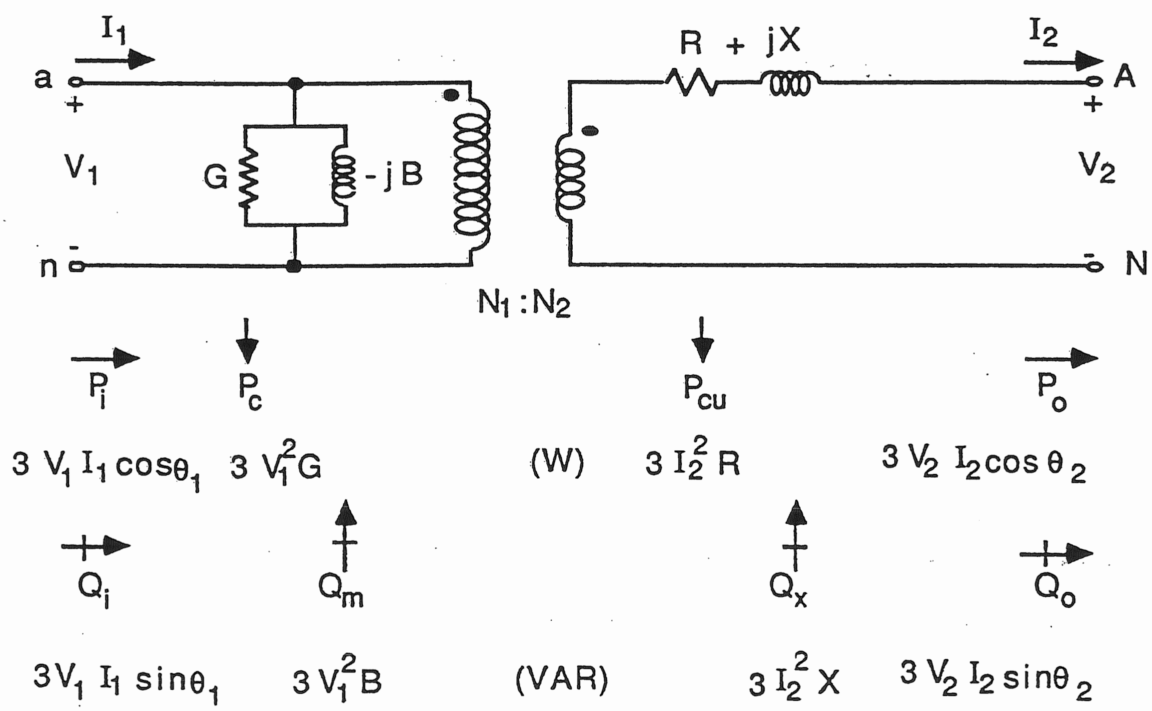


Figure 3.28 Transformer Bank Power Flow

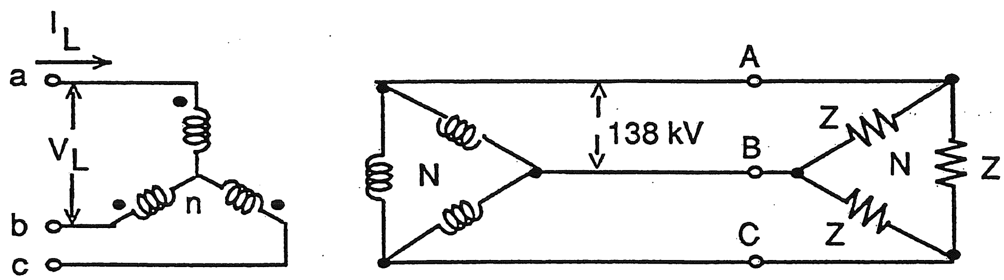
In general the total power input must supply the core and copper losses, plus the power required by the load. The total reactive power input must supply the magnetizing vars stored in the field in the iron, and the leakage vars stored in the field in air, plus the vars required by the load.

3-13 TRANSFORMER BANK ANALYSIS

Once the per-phase equivalent circuits are obtained for each transformer bank in Fig. 3.23, the banks can be analyzed, under load, as in Example 3.8.

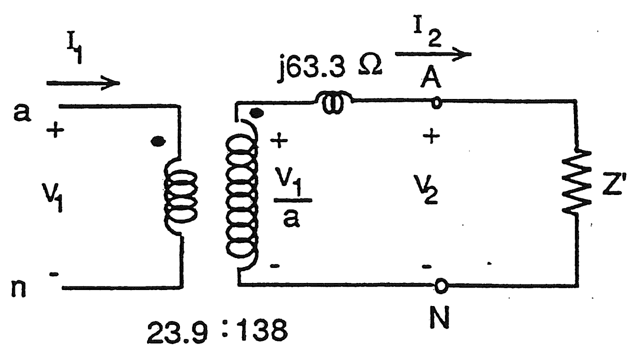
Example 3.8

The Y-Δ transformer bank of Example 3.7 is delivering rated MVA, 0.8 pf lagging, at rated voltage to a delta-connected balanced load. Calculate the input line voltage, line current, load impedance, and voltage regulation.



BANK
23.9 Y - 138 Δ kV
30 MVA

LOAD
30 MVA
0.8 pf lagging
138 kV



The high-side, per-phase equivalent circuit is used in these calculations.

$$V_2 = \frac{138}{\sqrt{3}} \angle 0^\circ = 79.7 \angle 0^\circ \text{ kV} \quad ; \quad I_2 = \frac{30000}{\sqrt{3}\,(138)} \angle -36.9^\circ = 126 \angle -36.9^\circ \text{ A}$$

$$\frac{V_1}{a} = j\,X\,I_2 + V_2 = (63.3 \angle 90^\circ)(126 \angle -36.9^\circ) + 79{,}700 \angle 0^\circ$$

$$= 84{,}729 \angle 4.32^\circ \text{ V } (\phi)$$

$$V_1 = \left(\frac{23.9}{138}\right)(84{,}729 \angle 4.32^\circ) = 14{,}674 \angle 4.32^\circ \text{ V } (\phi)$$

$$= \sqrt{3}\,(14{,}674) = 25.4 \text{ kV (line)}$$

$$I_1 = \frac{I_2}{a} = \left(\frac{138}{23.9}\right)(126\ \underline{/-36.9^\circ}) = 728\ \underline{/-36.9^\circ}\ \text{A (line)}$$

$$Z' = \frac{79700\ \underline{/0^\circ}}{126\ \underline{/-36.9^\circ}} = 633\ \underline{/36.9^\circ}\quad \Omega/\phi$$

$$Z = (3)(633\ \underline{/36.9^\circ}) = 1899\ \underline{/36.9^\circ}\quad \Omega$$

$$\text{V.R.} = \frac{V_2\,(NL) - V_2\,(FL)}{V_2\,(FL)} = \frac{84729 - 79700}{79700} = 6.3\,\%$$

3.14 THREE PHASE PER UNIT SYSTEM

Modern electric power systems consist of many three-phase transformer banks. Each bank has a high and low, per-phase equivalent circuit, and it would be most difficult to keep track of which circuit is used in a complex system. This problem is eliminated by converting all equivalent-circuit parameters to per-unit values, in which case the high and low equivalent circuits are the same in the three-phase per-unit system.

For any three-phase device, including the transformer bank, power engineers, without exception, choose rated <u>line</u> voltage and rated <u>total</u> kVA as base values, in which case, base current and base impedance are derived.

The base value of current is, then,

$$I_{base} = \frac{kVA_{base}}{\sqrt{3}\,kV_{base}} \qquad \text{(A)} \qquad (3.20)$$

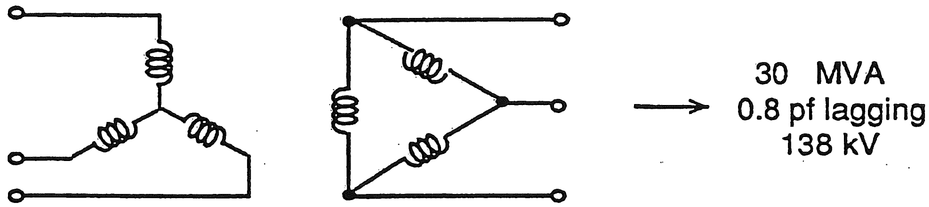
The base values of impedance and admittance are,

$$Z_{base} = \frac{V_{base(\phi)}}{I_{base}} = \frac{kV_{base}/\sqrt{3} \times 1000}{\dfrac{kVA_{base}}{\sqrt{3}\,kV_{base}}} = \frac{(kV_{base})^2 \times 1000}{MVA_{base} \times 1000} = \frac{(kV_{base})^2}{MVA_{base}} \quad (\Omega)$$

$$Y_{base} = \frac{1}{Z_{base}} \qquad (\mho) \qquad (3.21)$$

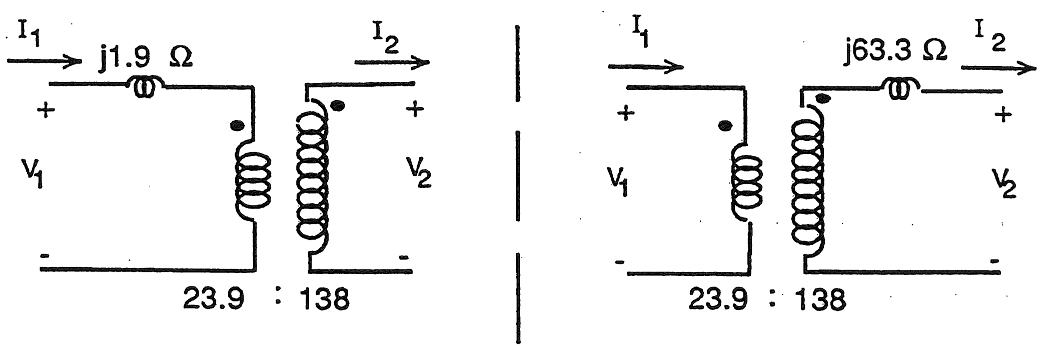
Example 3.9

Convert the equivalent circuits of the Y-Δ transformer bank in Example 3.7 to the per-unit system using the bank rating as a base. Calculate the input voltage, current, load impedance and voltage regulation, using per-unit values only, if the bank is delivering rated kVA, 0.8 pf lagging at rated voltage to a three-phase load.



23.9 Y - 138 Δ kV
30 MVA
X = 1.9 Ω (low side)

From Example 3.7, the equivalent circuits are given,



Low Equivalent

High Equivalent

Using the bank rating as a base,

Chosen,

kV_{base} (low) = 23.9 kV (line)

kV_{base} (high) = 138 kV (line)

MVA_{base} = 30 MVA (total - common to both sides)

Derived.

$$I_{base}\,(\text{low}) = \frac{kVA_{base}}{\sqrt{3}\,kV_{base}} = \frac{30000}{\sqrt{3}\,(23.9)} = 725\ \text{A}$$

$$I_{base}\,(\text{high}) = \frac{kVA_{base}}{\sqrt{3}\,kV_{base}} = \frac{30000}{\sqrt{3}\,(138)} = 126\ \text{A}$$

$$Z_{base}\,(\text{low}) = \frac{(kV_{base})^2}{MVA_{base}} = \frac{(23.9)^2}{30} = 19\ \Omega$$

$$Z_{base}\,(\text{high}) = \frac{(kV_{base})^2}{MVA_{base}} = \frac{(138)^2}{30} = 633\ \Omega$$

The equivalent-circuit impedance, in per-unit, is,

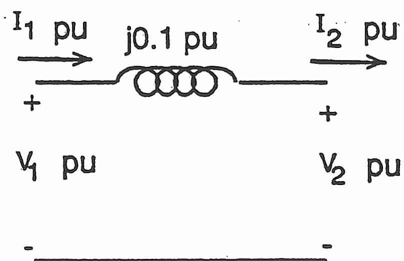
$$Z\,(\text{low}) = \frac{Z}{Z_{base}} = \frac{j1.9}{19} = j\,0.1\ \text{pu}$$

$$Z\,(\text{high}) = \frac{Z}{Z_{base}} = \frac{j163.3}{633} = j\,0.1\ \text{pu}$$

The ideal transformer turns-ratio is, in per-unit,

$$\frac{23.9}{23.9} : \frac{138}{138} = 1{:}1$$ (the ideal transformer in pu can be omitted)

The equivalent circuit, in per-unit, is the same for both sides and is,



V_1, pu, can now be calculated for rated load, 0.8 pf lagging.

$$V_2\,(\text{pu}) = \frac{V_2}{V_{base}} = \frac{138{,}000/\sqrt{3}\,\angle 0^\circ}{138{,}000/\sqrt{3}} = 1.0\,\angle 0^\circ\ \text{pu}$$

$$I_2\,(\text{pu}) = \frac{I_2}{I_{base}} = \frac{\dfrac{30{,}000}{\sqrt{3}\,(138)}\,\angle -36.9^\circ}{126} = 1.0\,\angle -36.9^\circ\ \text{pu}$$

$$V_1\,(pu) = j\,X\,I_2 + V_2 = (0.1\ \angle 90^\circ)(1.0\ \angle -36.9^\circ) + 1.0\ \angle 0^\circ = 1.063\ \angle 4.32^\circ \text{ pu}$$

$$I_1 = I_2\,(pu) = 1.0\ \angle -36.9^\circ \text{ pu}$$

$$Z_2\,(\text{load})\ pu = \frac{V_2}{I_2} = \frac{1.0\ \angle 0^\circ}{1.0\ \angle -36.9^\circ} = 1.0\ \angle 36.9^\circ \text{ pu}$$

$$V.\,R. = \frac{V_2\,(NL) - V_2\,(FL)}{V_2\,(FL)} = \frac{1.063 - 1.0}{1.0} = 6.3\ \%$$

Also,

$$P_2\,(pu) = V_2 I_2 \cos\theta_2 = (1.0)(1.0)\cos 36.9^\circ = 0.8 \text{ pu}$$

$$Q_2\,(pu) = V_2\,I_2 \sin\theta_2 = (1.0)(1.0)\sin 36.9^\circ = 0.6 \text{ pu}$$

$$P_1\,(pu) = V_1\,I_1 \cos\theta_1 = (1.063)(1.0)\cos(4.32^\circ + 36.9^\circ) = 0.8 \text{ pu}$$

$$Q_1\,(pu) = V_1 I_1 \sin\theta_1 = (1.063)(1.0)\sin(41.2^\circ) = 0.7 \text{ pu}$$

With the above quantities known in per-unit, it may be instructive to consider the inverse analysis of this example.

$I_2\,(pu) = 1.0\ \angle -36.9^\circ$ $\quad \times\ 126$ (high) $= 126\ \angle -36.9^\circ$ A $= I_2$ (line)

$\quad \times\ 725$ (low) $= 725\ \angle -36.9^\circ$ A $= I_2'$ (line)

$\Uparrow$

I_{base}

Z_2 (load) pu $= 1.0\ \angle 36.9^\circ$ $\quad \times\ 633$ (high) $= 633\ \angle 36.9^\circ\ \Omega/\phi = Z_2$

$\quad \times\ 19$ (low) $= 19\ \angle 36.9^\circ\ \Omega/\phi = Z_2'$

$\Uparrow$

Z_{base}

$X\,(pu) = 0.1\ \angle 90^\circ$ $\quad \times\ 633$ (high) $= 63.3\ \angle 90^\circ\ \Omega/\phi = X$ (high side)

$\quad \times\ 19$ (low) $= 1.9\ \angle 90^\circ\ \Omega/\phi = X$ (low side)

$\Uparrow$

Z_{base}

$V_1\,(pu) = 1.063\ \angle 4.32^\circ$ $\quad \times\ 23.9$ (low) $= 25.4$ kV (line)

$\Uparrow$

V_{base}

$P_2 = 0.8$ pu $\quad \times \quad 30 = 24$ MW (total)

$Q_2 = 0.6$ pu $\quad \times \quad 30 = 18$ MVAR (total)

$\Uparrow$

VA_{base}

$P_1 = 0.8$ pu $\quad \times \quad 30 = 24$ MW (total)

$Q_1 = 0.7$ pu $\quad \times \quad 30 = 21$ MVAR (total)

$\Uparrow$

VA_{base}

As the above calculations indicate, once you are in the per-unit system, never divide or multiply per-unit values by $\sqrt{3}$, since this factor has already been considered in obtaining the base values.

Furthermore, this Y-Δ transformer bank can now be completely specified,

23.9 Y- 138 Δ kV
30 MVA
X = 10% (on bank rating as a base)

The rating of each individual transformer that makes up this bank is,

13.8 - 138 kV
10 MVA
X = 10% (on transformer rating as a base)

It can be shown, in general, that the impedance of the bank, in per-unit, on the bank rating as a base, is always the same as the impedance, in per-unit, of each individual transformer on its rating as a base.

3-15 EXCITING CURRENT OF THREE-PHASE TRANSFORMER BANKS

When three-phase transformer banks are used in power systems, balanced loads are connected line-to-line or, more often, line-to neutral. It is extremely important, therefore, to have sinusoidal line-to-neutral voltages especially if the loads are communication or electronic devices. It will be shown in this section that three-phase, transformer-exciting currents adversely affect the sinusoidality of the line-to-neutral output voltages of a transformer bank, and that they also cause unwanted interference with communication lines running parallel with the transmission lines supplying the transformer bank. It will also be shown how these adverse effects can be avoided with the use of Δ-connected windings or the use of Δ-connected tertiary windings. From section 3.4 and Fig. 3.11, for the output voltage of an individual transformer to be sinusoidal, the exciting current <u>must</u> be nonsinusoidal, i.e.,

$$i_E = I_1 \cos \omega t + I_3 \cos 3 \omega t + \cdots \qquad \text{(A)}$$

The exciting current contains a strong third harmonic which becomes important when three transformers are connected in a three-phase bank. The input line currents of an unloaded bank are the exciting currents, as shown in Fig. 3.29.

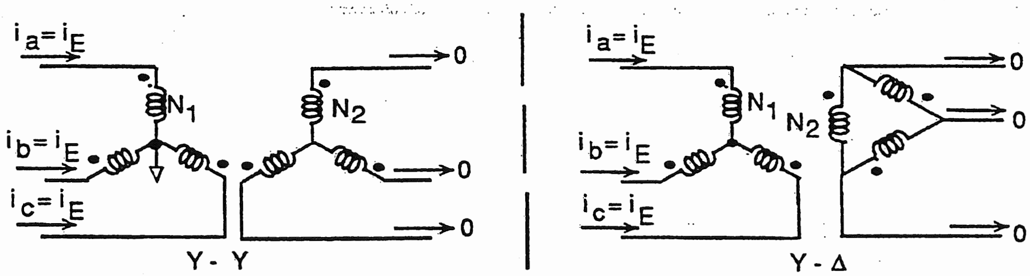


Figure 3.29 Unloaded Transformer Banks

In Fig. 3.29, the input line currents of the unloaded, Y-Y transformer bank are,

$$i_a = I_1 \cos \omega t + I_3 \cos 3 \omega t + \cdots \qquad \text{(A)}$$

$$i_b = I_1 \cos (\omega t - 120°) + I_3 \cos (3 \omega t - 360°) + \cdots \qquad \text{(A)} \qquad (3.22)$$

$$i_c = I_1 \cos (\omega t + 120°) + I_3 \cos (3 \omega t + 360°) + \cdots \qquad \text{(A)}$$

The phasor diagrams for the fundamentals and third-harmonics of Eqns. (3.22) are shown in Fig.3.30.

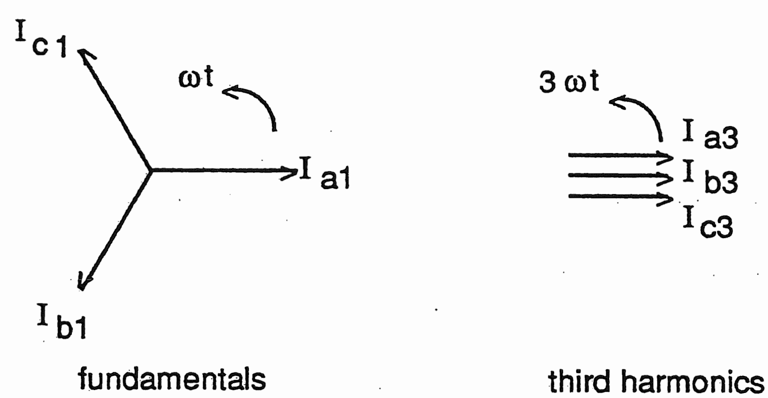


Figure 3.30 Harmonic Content of Y - Y Bank, Exciting Currents

Observe from Fig. 3.30, the three fundamentals of the input line currents add to zero at all instants of time and, therefore, no fundamental exciting line current flows to ground through the primary neutral. Observe, also, that the three third-harmonics of the input line currents are in time-phase; they do not sum to zero and therefore a third-harmonic current, of magnitude three times the single harmonic, flows to ground through the primary neutral.

In the Y-Y bank of Fig. 3.29, if the neutral of the primary windings is not grounded, there is no path for the third-harmonic currents and they cannot exist. Therefore, the exciting current must be sinusoidal with consequent nonsinusoidal iron flux and nonsinusoidal secondary phase voltages. (Refer to Fig. 3.11). Since many loads are connected line to neutral on the secondary of a Y-Y bank, the primary neutral is always grounded to provide a path to the grounded, Y-connected generator driving the bank for the third-harmonic exciting currents, thus assuring sinusoidal phase voltages across the loads. However, while the grounded Y-Y bank is satisfactory from a sinusoidal-voltage viewpoint, third-harmonic currents flow in the transmission lines connecting the primary to the generator. Since the third-harmonic line currents are in time-phase, they produce fields which are directly additive and the inductive effect of these fields on communication circuits parallel to the transmission lines is objectionable, to say the least, and for this and other reasons grounded Y-Y transformer banks are seldom-used, or if they are used, Δ–connected, tertiary windings must be provided in the bank secondaries as will be presently explained.

The ungrounded Y-Δ, bank in Fig. 3.29 is much more satisfactory since the ungrounded primary neutral assures no third-harmonic content in the line currents to the generator. The delta-connected secondaries now become important in this discussion. Kirchhoff's voltage law around a closed loop states that the sum of the secondary line voltages (which are the secondary-coil voltages) must be zero. Therefore, there can be no third harmonic voltages in the secondary line voltages since third harmonic voltages cannot sum to zero. If, therefore, the secondary coil-voltages must be sinusoidal and there is no path for third harmonic currents in the primary coils, there must be an alternate path for the third-harmonic exciting currents to flow. This path exists in the closed delta secondaries and a small (compared to the load) third-harmonic current circulates (three times the coil harmonic current) thus assuring sinusoidal (fundamental) exciting currents in the primary windings and sinusoidal voltages across the secondary loads. Now we can see that the neutral of the Y-primaries in a Y-Δ transformer bank can be grounded or ungrounded and there will be no harmonic currents in the primary supply lines as long as there is a path for harmonic current flow in the Δ–secondaries. We conclude,

1. Third harmonic currents can flow in transformer-bank lines only if grounded neutrals exist.

2. In delta-connected windings, third-harmonic currents can circulate about the delta.

3. There can be third-harmonic content in line to neutral voltages but there cannot be third-harmonic content in line to line voltages.

In high-voltage portions of electric power systems, autotransformers can be connected in a grounded or ungrounded wye and a provision is made for a third-harmonic path by adding a tertiary winding to each autotransformer secondary as indicated in Fig. 3.31.

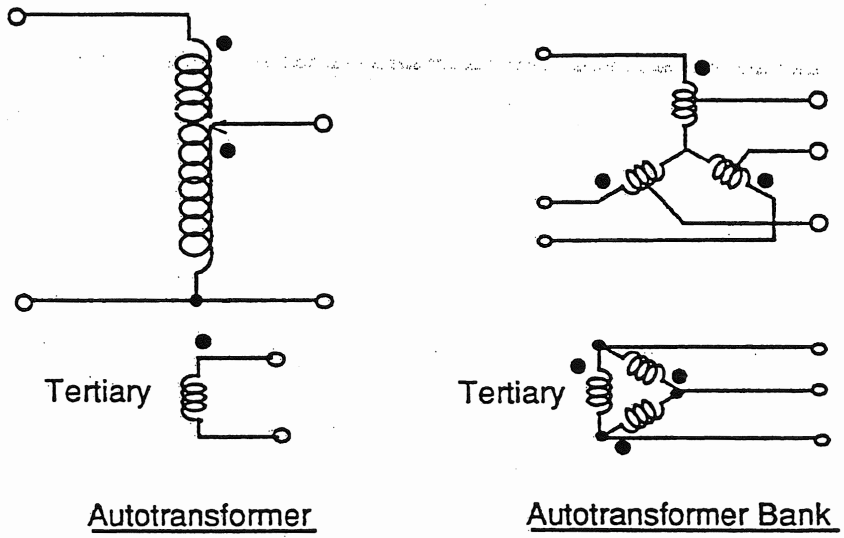


Figure 3.31 Three Phase Autotransformer Bank

In Figure 3.31, the three phase autotransformer bank serves the high-voltage portion of the power system with no third-harmonic current content in the input lines and the delta-connected tertiary windings serve as a third-harmonic exciting-current path assuring no third harmonic content in the output line to neutral voltages, and at the same time serve as a low-voltage source to local manufacturing plants. With this in mind, it can now be seen that the Y-Y bank in Fig. 3.29, grounded or ungrounded, will have no harmonic line currents if Δ-connected tertiary windings are provided in the bank secondaries.

3-16 SUMMARY

The ideal transformer is characterized by its voltages and currents,

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = a \quad ; \quad \frac{I_1}{I_2} = \frac{1}{a} \qquad \text{where } a = \frac{N_1}{N_2}$$

The induced voltages are in time-phase when measured toward the polarity dots and the currents are in time-phase when the mmfs are subtractive.

The practical transformer is characterized by its equivalent circuits where the input voltage determines the maximum flux density in the iron,

$$V_1 = 4.44\, f\, N_1\, A_c\, B_{max} \qquad \text{(V)}$$

and the slope of the magnetization curve determines the magnetizing current,

$$I_m = \frac{H_m \ell_c}{N_1} \qquad \text{(A)}$$

The exciting current has as its two components - the core-losses current, I_c, due to hysteresis and eddy-currents, and the magnetizing current, I_m. The exciting current, then, is minimized by using high-permeability, narrow-hysteresis loop, laminated steel for the core.

Equivalent circuits for single-phase transformers can be simplified by moving the magnetizing branch to the input terminals as is the case for large power transformers. This results in equivalent winding impedance R + j X, referred to the high or low side. This impedance and the magnetizing admittance, G - j B, can be measured and calculated from the open-circuit - short-circuit test. With the equivalent circuits, the figures of merit - efficiency and voltage-regulation, can be determined. The equivalent circuits can be further simplified by placing them in the single-phase per-unit system, where the transformer rating is used as a base. A single-phase transformer can be connected as an autotransformer with a consequent increase in MVA rating and increased efficiency since the losses are the same for constant load. The disadvantage of an autotransformer is the loss of isolation between the generator and the load.

Three identical single-phase transformers can be connected as a three-phase transformer bank in four ways - Y-Y, Δ–Δ, Y-Δ and Δ-Y. The bank rating is <u>always</u> in terms of <u>line</u> voltage and <u>total</u> volt-amperes. The per-phase equivalent circuits are always determined from line to neutral looking into the high and low sides of the bank. This always necessitates replacing delta-connected coils with fictitious, equivalent, Y-connected coils. The equivalent circuit, effective turns-ratio can be shown to be the ratio of the rated line voltages for all four bank connections. The bank, equivalent circuits can be simplified by placing them in the three-phase per-unit system using the bank rating as a base. The impedance and the admittance of the transformer bank, in per-unit, using the bank rating as a base, is always equal to the admittance and impedance of each individual transformer, in per-unit, using the transformer rating as a base.

Exciting currents of three-phase transformer banks can adversely affect the sinusoidality of output, line to neutral voltages and Δ–connected secondary or tertiary windings must be used to insure output sinusoidality and eliminate communication interference.

This chapter serves as an excellent basis for further power system analysis and for subsequent analysis of power system components.

PROBLEMS

3.1 An ideal transformer has 400 turns on the primary and 100 turns on the secondary. The generator current is 50 A - rms and lags the generator voltage of 120 V- rms by an angle of 30°. What are the primary and secondary voltages and currents (polar form)? What is the load impedance (polar form)? Draw the phasor diagram and label completely with numerical values.

3.2 An ideal transformer has 400 turns on the primary and 100 turns on the secondary. The load voltage is 120 V- rms. The load impedance is $5\,\underline{/53.1^\circ}\ \Omega$. What are the primary and secondary voltages and currents (polar form)? Sketch the phasor diagram and label completely with numerical values. What impedance (polar form) does the generator see?

3.3 A core-type transformer has a mean length of 0.6 m and a cross-section of 0.005 m^2. The core is made of silicon sheet steel and has primary and secondary windings with 150 and 450 turns, respectively. The source voltage is 200 V- rms at 60 Hz. On open circuit, estimate the exciting current,

(a) when the 150-turn winding is placed across the source, 450-turn winding across the load.

(b) when the 450-turn winding is placed across the source, 150-turn winding across the load.
(neglect core-loss current, I_c, in parts (a),(b)).

What is the source current for each of the above connections when a resistance load of 120 Ω is placed across the remaining winding?

3.4 A transformer is rated 10 kVA, 1000 - 500 V, 60 Hz.
The primary impedance is $r_1 = 2.0\ \Omega$, $x_1 = 6.0\ \Omega$.
The secondary impedance is $r_2 = 0.5\ \Omega$, $x_2 = 1.5\ \Omega$.

The exciting admittance is $G = 100\ \mu\mho$, $B = 400\ \mu\mho$

a) What is the primary voltage if the transformer delivers rated kVA, 0.8 pf leading, at rated voltage to a load?

b) Draw the phasor diagram

c) Calculate the <u>net</u> hysteresis and eddy-current loss.

d) Calculate the vars required to set up the core field.

e) Calculate the vars required to set up the leakage field.

3.5 For the transformer in Problem 3.4, calculate the efficiency and the voltage.regulation at rated load, 0.8 pf leading.

3.6 The open-circuit, short-circuit tests for a 50 kVA, 2400 - 240 V, 60 Hz transformer are given,

On open-circuit :	On short -circuit :
$V_2 = 240$ V	$V_1 = 48$ V
$I_2 = 5.41$ A	$I_1 = 20.8$ A
$P_2 = 186$ W	$P_1 = 617$ W

a) Draw and label completely, with numerical values, the high-side and low side approximate equivalent circuits.

b) Determine the efficiency and voltage regulation at full load, 0.8 pf lagging, rated voltage.

3.7 In Problem 3.6, using the transformer rating as a base, calculate parts (a) and (b) with per-unit values only.

3.8 A 300 kVA, 12,000 : 4000 V, 60 Hz transformer has open-circuit test readings: 4000V, 1.1 A, 2000 W, and short-circuit test readings: 300 V, 25 A, 2800 W.

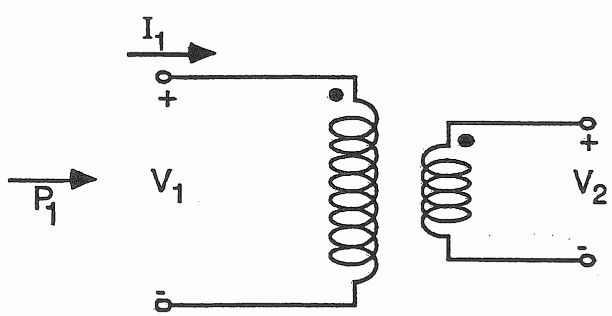
a) Which side of the transformer was open-circuited?, short-circuited?

b) What is the primary voltage, if the transformer delivers rated kVA at 0.8 pf lagging, rated voltage?

c) Calculate the efficiency and voltage regulation at the above load.

3.9 In Problem 3.8, using the transformer rating as a base, calculate parts (a), (b) and (c) with per-unit values only.

3.10



12 - 4 kV

300 kVA

$Z \text{ (low)} = 0.5 + j1.24 \ \Omega$

$Y \text{ (high)} = 13.8 - j27.3 \ \mu℧$

A single-phase transformer is given in the figure, with its internal impedance referred to the low-side and its magnetizing admittance referred to the high side. What are the oc - sc meter values indicated on the figure?

3.11 A 480: 120 volt, 5 KVA, 2-winding transformer is to be used as an auto-transformer to supply a 480-volt circuit from a 600-volt source. When tested as a 2-winding transformer at rated load, 0.8 pf lagging, its efficiency is 0.965.

a) Show a diagram of connections as an autotransformer. On the the diagram show polarities and current directions.

b) Determine its kVA rating as an autotransformer.

c) Find its efficiency as an autotransformer at rated load, 0.8 pf lagging.

3.12 A single-phase transformer is rated 500 kVA, 33 – 2.4 kV, with internal impedance $Z = j\ 1.0\ \Omega$ (low-side). The transformer is driving a load of 200 kW, 0.9 pf lagging at 2.25 kV.

a) What is the voltage and current at the primary terminals of the transformer?

b) What is the transformer internal impedance in per-unit?

c) If three of the above transformers are connected in a three-phase transformer bank, wye on the high side, delta on the low side, find the transformer bank rating.

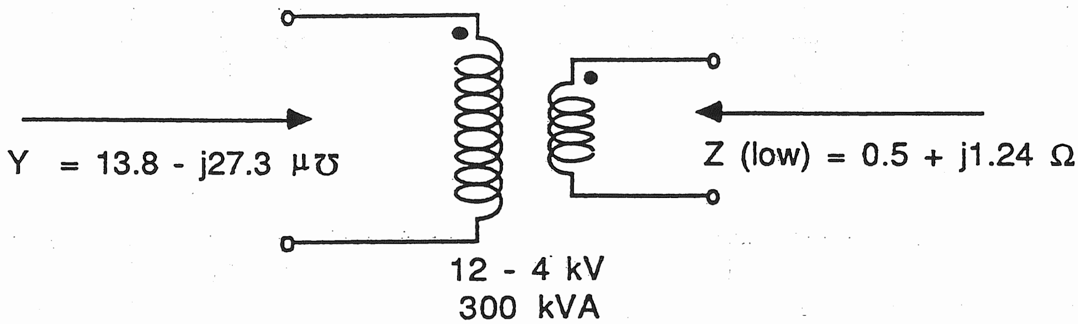
3.13. A single-phase transformer is rated 1000 kVA, 24 –2.4 kV, with an internal impedance referred to the low side of $Z = j\ 2.0\ \Omega$. The transformer is driving a load of 400 kW, 0.85 pf lagging, at 2.3 kV.

a) What is the voltage and current at the primary terminals of the transformer?

b) What is the transformer internal impedance in per-unit?

c) If three of the above transformers are connected in a three-phase transformer bank, wye on the high side, wye on the low side, find the transformer bank rating.

3.14. A $\Delta - \Delta$ transformer bank consists of three individual transformers <u>each</u> with its rating given in the figure.



a) Give the bank rating.

b) Draw and completely label with numerical values, the per-phase equivalent circuits referred to the high-side and the low-side.

c) If the transformer bank is delivering rated kVA at rated voltage, 0.8 pf lagging, what is the input line voltage?

d) What is the load impedance (Ω)?

e) What is the bank efficiency and voltage regulation at this load?

3.15 In Problem 3.14, using the bank rating as a base,

a) What is the bank impedance and admittance in per-unit?

b) Do parts (b), (c), (d) and (e) using per-unit values only.

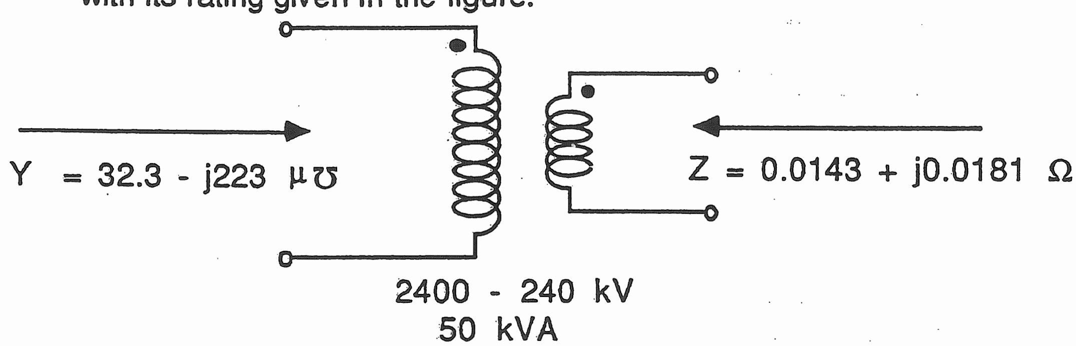
3.16 Do Problem 3.14 for a Δ-Y transformer bank.

3.17 In Problem 3.16, using the bank rating as a base,

a) What is the bank impedance and admittance in per-unit?

b) Do parts (b), (c), (d) and (e) using per-unit values only.

3.18 A Δ – Δ transformer bank consists of three individual transformers <u>each</u> with its rating given in the figure.



a) Give the bank rating.

b) Draw and completely label with numerical values, the per-phase equivalent circuits referred to the high-side and the low-side.

c) If the transformer bank is delivering half-rated kVA at rated voltage, 0.8 pf leading, what is the input line voltage?

d) What is the load impedance?

e) What is the bank efficiency and voltage regulation at this load?

3.19 In Problem 3.18, using the bank rating as a base,

a) What is the bank impedance and admittance in per-unit?

b) Do parts (b), (c), (d) and (e) using per-unit values only.

3.20 Do Problem 3.18 for a Δ-Y transformer bank.

3.21 In Problem 3.20, using the bank rating as a bank,

a) What is the bank impedance and admittance in per-unit?

b) Do parts (b), (c), (d) and (e) using per-unit values only.

3.22 Three identical autotransformers, as specified in Problem 3.11, are connected in wye as a transformer bank in a manner similar to Figure 3.31. The tertiary windings are unloaded.

a) Show a diagram of connections as an auto transformer bank. On the diagram show polarities and current directions.

b) Give the bank rating.

c) Find its efficiency as an autotransformer bank at rated load, 0.8 pf lagging, rated voltage.

CHAPTER 4

ELECTROMECHANICAL ENERGY CONVERSION

Literally billions of dollars have been invested in the power grid of the United States and other countries, where, for the foreseeable future, large blocks of power will continue to be converted from fossil fuels and nuclear sources. Each power plant in the grid takes the energy stored in fossil or nuclear fuels and converts it into heat energy stored in steam. The steam is delivered to a high-speed turbine which converts the heat energy to mechanical energy. The turbine drives a generator which converts the mechanical energy to electrical energy. The electrical energy is then carried over transmission and distribution lines to load centers, whose motor loads reconvert the electrical energy to useful mechanical energy. This chapter is concerned with the basic principles that underlie the conversion of energy from mechanical to electrical or electrical to mechanical form, together with some simple applications. These basic principles will be emphasized and continuously used throughout the remainder of this text.

Since electric power systems consist of devices that convert energy from electrical to mechanical form or from mechanical to electrical form, these devices must have movable iron portions of the path in which a magnetic field exists. Furthermore, a magnetic force must act on the movable iron to convert energy to mechanical form as is evident from the fact, that

$$W_m = \int f \, dx \qquad (J)$$

where, W_m = mechanical energy converted

f = magnetic force acting on the movable iron

x = displacement of the movable iron

The magnetic force generated in devices that convert energy from one form to another is broadly divided into two classes -

Lorentz - force, energy-conversion devices that employ moving, current-carrying conductors placed in a magnetic field whose motion is constrained to linear or rotary form.

Reluctance-force, energy-conversion devices, where portions of the iron path are movable, and the moving iron is constrained to linear or rotary motion.

Devices that employ the magnetic force in the form of the Lorentz force or the Reluctance force or both, include loudspeakers, solenoids, electromagnets,

microphone, generators, motors, etc. The subsequent analysis will be simplified, at first, when Lorentz-force devices are considered, and generalized when Reluctance-force devices are considered.

Throughout this text, device response is limited to steady state, and device transient response is considered in other texts such as automatic control, etc. Consider now, the energy balance that underlies both classes of devices.

4-1 ENERGY BALANCE

The energy flow through all electromechanical devices can be basically modeled with three blocks as in Fig. 4.1

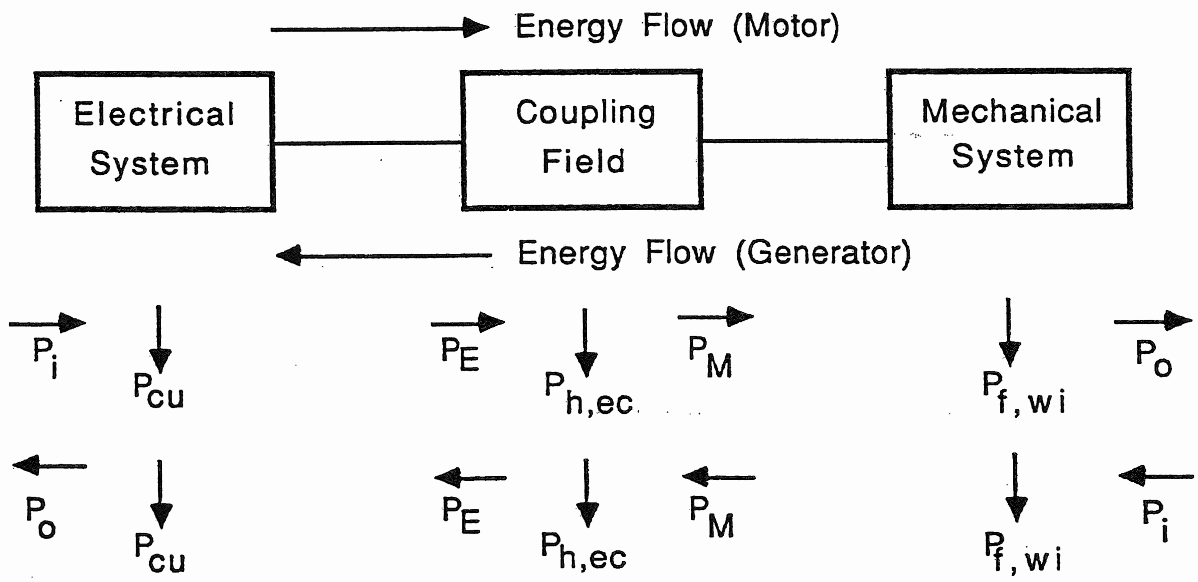


Figure 4.1 Basic Device, Energy and Power Flow

All electromechanical devices, whether translational, rotational or a combination of the two, can be broken down into three basic parts. The electrical system of the device can be identified and separated. The mechanical system of the device can be identified and separated. The electric or magnetic field that couples these two systems can be ascertained. Only devices that use the magnetic field to couple the electrical and mechanical systems will be considered in this text. To date, there are no dielectrics with permittivities for the electric field as high as the permeabilities of ferrous materials for the magnetic field.

The direction of energy flow for motor or generator action in a device is given in Fig. 4.1 The magnitude of energy flow, however, is not as important as its time rate of change – power. Energy can only be changed in form; it cannot be created or destroyed. The power input to any device, whether a motor or generator, must be accounted for. As is evident in Fig. 4.1 power is converted from electrical to mechanical form in a motor. Here the electrical system is a <u>source</u> of energy and the mechanical system is a <u>sink</u> of energy. The opposite is true for a generator – here the mechanical system is a <u>source</u> of energy and the electrical system is a <u>sink</u> of energy.

For a <u>motor</u> , the rate at which energy is delivered to the device by the electrical system is called the power input, P_i. Some of this energy is lost irreversibly as copper losses, P_{cu}. The rest of this energy is delivered to the coupling magnetic field as P_E. Some of this energy is irreversibly lost in the field as hysteresis and eddy-current losses. The rest of this energy is delivered by the coupling field to the mechanical system as P_M. Some of this energy is irreversibly lost as friction and windage, $P_{f,wi}$. The rest of this energy is usefully consumed as mechanical power output, P_o. The opposite is true for a generator. Throughout this text the basic power-flow model in Fig. 4.1 will be adapted to energy-conversion devices and this model must be well understood. Observe from Fig. 4.1 that the following equations can be written.

MOTOR:

$$P_i = P_{cu} + P_E \qquad \text{(W)} \tag{4.1}$$

$$P_E = P_{h,ec} + P_M \qquad \text{(W)} \tag{4.2}$$

$$P_M = P_{f,wi} + P_o \qquad \text{(W)} \tag{4.3}$$

GENERATOR:

$$P_i = P_{f,wi} + P_M \qquad \text{(W)} \tag{4.4}$$

$$P_M = P_{h,ec} + P_E \qquad \text{(W)} \tag{4.5}$$

$$P_E = P_{cu} + P_o \qquad \text{(W)} \tag{4.6}$$

The energy-balance equations (4.1) through (4.6) then form the basis for creating and analyzing models for all electromechanical energy conversion devices.

4-2 LORENTZ-FORCE ENERGY CONVERSION

The two requisites for this type of electromagnetic energy conversion are,

1. A magnetic field must exist in space (coupling field).

2 Current-carrying conductors must exist that are capable of moving through this magnetic field.

Such a device is the translational generator illustrated in Fig. 4.2.

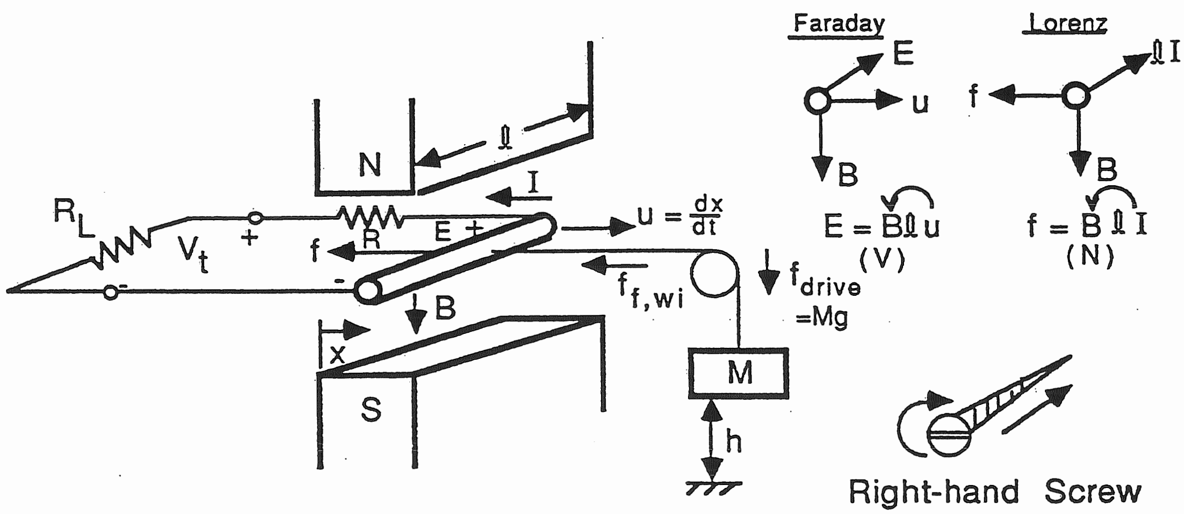


Figure 4.2 Translational Generator

The device in Fig. 4.2 has the two requisites for energy conversion. The magnetic field is created by placing an air gap in a magnetic circuit. The flux density in the gap is constant and uniform, and gap fringing is neglected. Flux always flows from North to South within the gap, and the strength of this field is determined from the principles of chapter 2. The current-carrying conductor of length ℓ, meters is oriented perpendicular to the field, and is constrained to have linear motion within the field by means of a cable running over a pulley connected to an ascending or descending mass, M.

It is clear that if this device is to be a generator, the mechanical system must be a source of energy and the electrical system must be a sink of energy. To satisfy this requirement, the mass must descend so that its potential energy Mgh, (J), diminishes. This change in mechanical energy must be accounted for, and the conclusion reached is that it is delivered to the device. As a consequence of the descending mass, the conductor is pulled to the right through the magnetic field at a velocity u, m/sec by the gravitational force, f_{drive}, (in this case, called the driving force) acting on the mass M.

The assumption is made that the mass of the conductor is much smaller than the mass M, in which case the conductor reaches a steady state velocity, $u = dx/dt$, m/sec, very quickly. At this point, the laws of Maxwell and Lorentz are invoked.

4-3 MAXWELL'S EMF LAW AND LORENTZ'S FORCE LAW

From Chapter (2-7), Maxwell's emf law for a moving contour (the electric circuit in Fig. 4.2) through a constant field is,

$$\oint_C (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \cdot d\boldsymbol{\ell} = 0 \qquad (\mathrm{V}) \tag{4.7}$$

now,

$$\mathcal{E} \triangleq \int \mathbf{E} \cdot d\boldsymbol{\ell}$$

or,

$$\mathcal{E} = -(\mathbf{u} \times \mathbf{B})\,\boldsymbol{\ell} \tag{4.8}$$

then,

$$\mathcal{E} = -B\,\ell\,u \qquad (\mathrm{V}) \tag{4.9}$$

($\mathbf{u}$, $\mathbf{B}$, $d\boldsymbol{\ell}$ are mutually perpendicular)

where $\mathcal{E}$ is the total emf induced across the conductor.

The velocity, flux density and conductor orientation are almost always mutually perpendicular in practical energy-conversion devices. To avoid confusion concerning the minus sign in Eqn. (4.9), the induced emf will be defined as,

$$|\mathcal{E}| \triangleq B\,\ell\,u \qquad (\mathrm{V}) \tag{4.10}$$

and its direction (from – to +) will be determined by the right-hand screw rule, i.e., its direction will be the advance of a right-hand screw when the velocity vector is turned towards the flux density vector as illustrated in Fig. 4.2. This Faraday emf is caused by motion of the conductor through the magnetic field and will always be present, whether the electric circuit is open (no current flow) or closed (current flow). If the circuit is closed, a current will flow in the direction of the emf, for generator action, resulting in a drop of potential, V_t, across the electrical sink, R_L.

Because current flows through the conductor in the magnetic field of Fig. 4.2, a force will be exerted on the conductor as predicted by Lorentz's Law,

$$df = I\,B\,d\ell\,\sin\phi \quad (N) \qquad (4.11)$$

where ϕ is the angle between current direction and magnetic field. When Eqn.(4.11) is integrated over the conductor length, Lorentz's law becomes,

$$f = I\,(\boldsymbol{\ell} \times \mathbf{B}) \quad (N) \qquad (4.12)$$

Because the current-carrying conductor is oriented perpendicular to the field, the magnitude of the Lorentz force is,

$$f = B\,\ell\,I \quad (N) \qquad (4.13)$$

and its direction will be the advance of a right-hand screw when the ℓI vector is turned towards the flux density vector as illustrated in Fig. 4.2.

While it is possible to have only a Faraday emf generated in a conductor moving through a magnetic field (open circuit), or it is possible to have only a Lorentz force acting on a current carrying conductor (stationary) in a magnetic field, it is the simultaneous presence of a Faraday emf and a Lorentz force on a moving, current-carrying conductor that results in energy conversion.

4-4 TRANSLATIONAL GENERATOR

The three basic energy-conversion blocks of Fig. 4.1 are readily discernable in the translational generator of Fig. 4.2.

The electrical system consists of the conductor whose lumped resistance R, and Faraday emf E, are taken out in series, according to Thevenin's Theorem, together with the electrical sink, R_L.

The coupling field is the constant, uniform field B, without which, there would be no Faraday emf, or Lorentz force, no matter how fast the conductor is moving or how large the current through the conductor.

The mechanical system consists of the descending mass M, whose potential mechanical energy is being delivered to the device.

These three blocks are adapted to the translational generator and presented in Fig. 4.3.

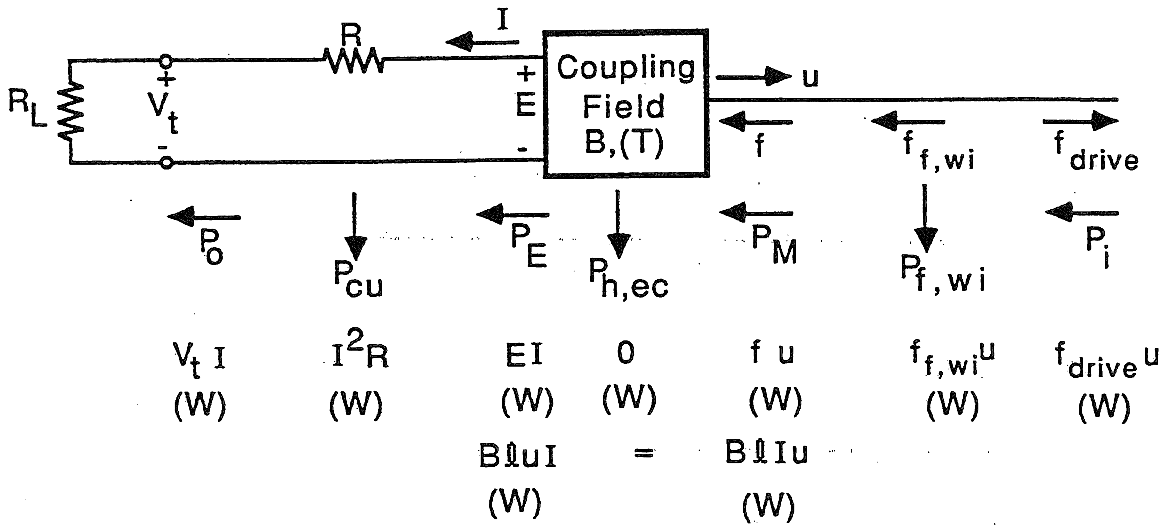


Figure 4.3 Translational-Generator, Power Flow Diagram

The mechanical source of the generator in Fig. 4.3 is the descending mass M whose potential energy is being delivered to the device at a time-rate f_{drive} (N) multiplied by the conductor velocity, u (m/sec). This input power has the dimensions of mechanical watts and since this device is a generator the gravitational force, f_{drive}, determines the direction of motion of the conductor. The drive force, according to Newton's law for steady state equilibrium, must at all times be equal to the back forces of friction and windage of the pulley by moving the conductor and mass through air and the Lorentz force, f. Some of the mechanical power input is then lost irreversibly as heat in friction and windage; the rest of the mechanical power is delivered to the coupling field as the Lorentz force f, (N) multiplied by the conductor velocity u, (m/sec) or P_M watts. Since the coupling field is constant with time, the hysteresis and eddy-current losses, (explained in detail in the chapter on transformers), are zero. The mechanical power input to the coupling field is then found in electrical form as the Faraday emf, E, across the conductor multiplied by the conductor curent, I or P_E, watts. Some of this power is irreversibly converted to heat as copper losses, and the rest is usefully converted as power output, $V_t I$, watts. The power flow through this generator is then summarized in Equations 4.14 – 4.16.

$$f_{drive}\,u = f_{f,wi}\,u + f\,u \qquad \text{(W)} \qquad (4.14)$$

$$f\,u = E\,I \qquad \text{(W)} \qquad (4.15)$$

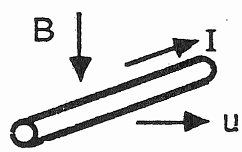
$$E\,I = I^2 R + V_t I \qquad \text{(W)} \qquad (4.16)$$

In summary, generator action, as illustrated in Fig. 4.3, is, without exception, characterized as follows:

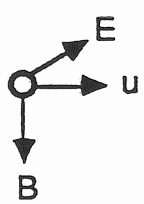
1. The mechanical drive force always determines the direction of motion in a generator.
2. In the electrical system the Faraday emf is always a forward emf (– to +) in the direction of current flow.
3. In the mechanical system the Lorentz force is always a back force opposing conductor motion.

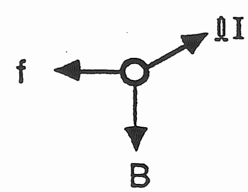
Example 4.1

A conductor with resistance, 0.2 Ω is oriented perpendicular to a uniform magnetic field of 0.8 T. It is observed to be moving to the right at 50 m/sec with negligible mechanical losses. The 20 cm-long conductor carries a current of 5.0 A directed into this page.



a) Compute the magnitude and direction of the Faraday emf and Lorentz force generated on this conductor.





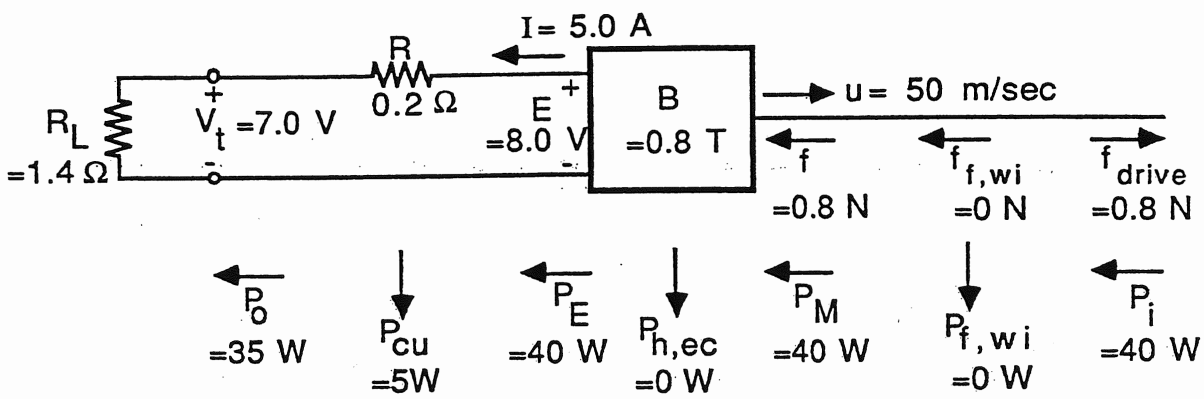
$$E = B\ell u$$
$$= (0.8)(20\text{x}10^{-2})(50)$$
$$= 8 \text{ V (into page)}$$

$$f = B\ell I$$
$$= (0.8)(20\text{x}10^{-2})(5)$$
$$= 0.8 \text{ N (to left)}$$

b) Is this device a motor or generator?

The device is a generator since the Faraday emf is a forward emf and the Lorentz force is a back force.

c) Draw and label, with numerical values, the power flow diagram for this device.



4-5 TRANSLATIONAL MOTOR

Since energy conversion is reversible, the device in Fig. 4.2 can be operated as a motor.

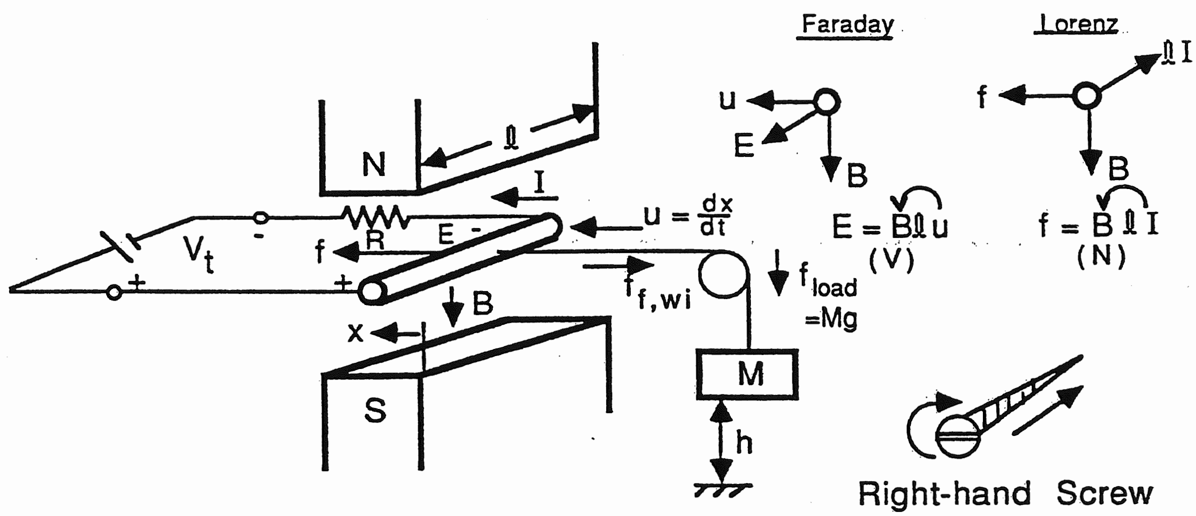


Figure 4.4 Translational Motor

The motor in Fig. 4.4 requires a source of electrical energy which is a properly oriented emf, whose terminal voltage is V_t, and a mechanical sink which is an ascending mass, M, whose mechanical, potential–energy is increasing.

The source now delivers a counter-clockwise current which creates a forward Lorentz force, f, which pulls the conductor through the magnetic field to the left at a steady state velocity, u, m/sec. The gravitational force is now a load force against which the Lorentz force is pulling the conductor. The back force of friction and windage always opposes conductor motion. A Faraday emf is generated across the conductor as a back emf whose direction (– to +) opposes the direction of the current. Because the Faraday emf is a back emf, electrical energy is delivered to the coupling field, and because the Lorentz force is a forward force, mechanical energy is taken from the coupling field and stored in the mass, M, as potential energy. This is summarized in the motor power flow diagram.

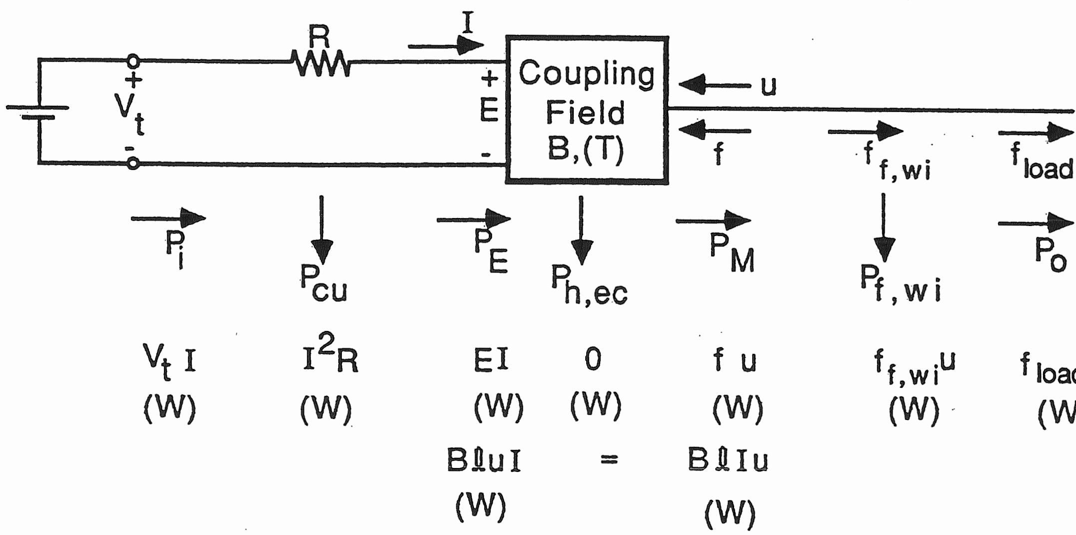


Figure 4.5 Translational-Motor, Power Flow Diagram

The power flow through the motor is then given by Equations 4.17 – 4.19.

$$V_t I = I^2R + EI \qquad \text{(W)} \tag{4.17}$$

$$EI = f\,u \qquad \text{(W)} \tag{4.18}$$

$$f\,u = f_{f,wi}\,u + f_{load}\,u \qquad \text{(W)} \tag{4.19}$$

In summary, <u>motor</u> action, as illustrated in Fig. 4.4, is without exception, characterized as follows:

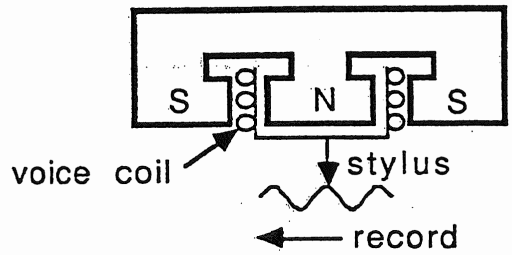
1. The Lorentz force always determines the direction of motion in a motor.

2. In the electrical system, the Faraday emf is always a <u>back</u> emf (– to +) opposing the direction of current.

3. In the mechanical system, the Lorentz force is always a <u>forward</u> force in the direction of conductor motion.

Example 4.2 Generator action –

A magnetic pickup consists of a 20-turn coil, whose length per turn is 1.0 cm, placed in a uniform field, B = 0.2 T. If the maximum allowable recording amplitude is 0.02 mm, predict the voice-coil voltage at 1000 Hz.



If the displacement is sinusoidal, then,

$$x(t) = 2 \times 10^{-5} \sin 2\pi f t \qquad (m)$$

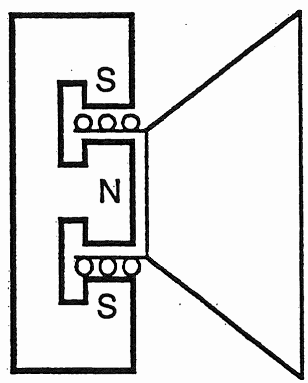
$$u = dx/dt = (2\pi)(1000)(2 \times 10^{-5}) \cos 2\pi f t = 0.126 \cos \omega t \qquad (m/sec)$$

$$v = e = B \ell u = (0.2)(20 \times 0.01)(0.126) \cos \omega t = 0.005 \cos \omega t \qquad (V)$$

$$V\,(rms) = \frac{0.005}{\sqrt{2}} = 3.54 \quad (mV) \ @ \ 1000 \text{ Hz}$$

Example 4.3 Motor action –

The voice coil of a permanent-magnet loudspeaker has 20 turns with each turn of length, 1.0 cm, placed in a uniform magnetic field of 0.5 T. What is the force acting on the cone if the coil current is 0.313 cos ωt (A) at 1000 Hz.



$$f = B \ell i = (0.5)\,(0.2)(0.313) \cos 6280\, t \qquad (N)$$

$$f = 31.4 \cos 6280\, t \qquad (mN)$$

4-6 ROTATIONAL GENERATOR

Electric power systems consist primarily of Lorentz-force devices, which include dc, synchronous motors and generators and induction motors whose analysis will be considered in detail in later chapters. This section, however, will consider a very simple, rotational, Lorentz-force device whose basic, power-flow principles underlie the machines listed above.

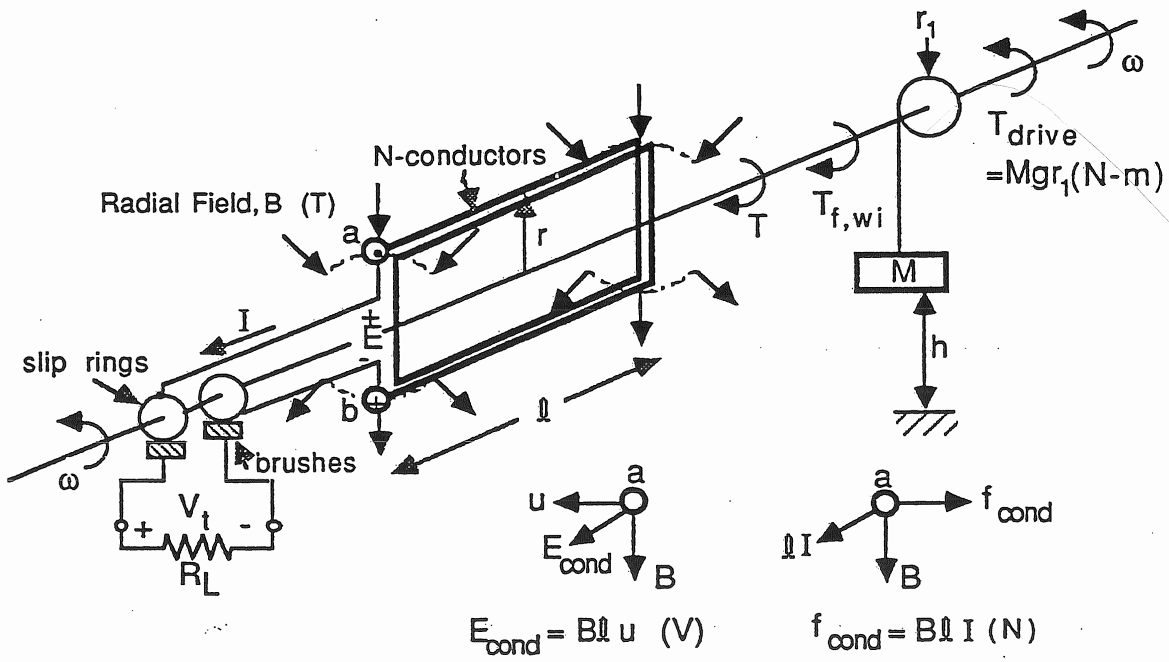


Figure 4.6 Rotational Generator

Consider an N-turn coil placed in a radial magnetic field as illustrated in Fig. 4.6. The magnetic field is constant, uniform and radial along the shaft of the machine over a distance of ℓ, meters. The direction lines of the flux density converge radially to each point on the shaft and then diverge radially from this point. A practical method for obtaining this field will be considered in the later chapter on dc machines. The N-turn coil has two coil sides a and b, each consisting of N-effective conductors as indicated in Fig. 4.6. These conductors are mutually perpendicular to the field and motion, whereas the end-conductors of the coil are always aligned with the field, and will therefore have negligible generated Faraday emfs and Lorentz forces.

The mechanical source for this generator consists of a descending mass, M, connected by a cable to a pully of radius, r_1 meters, mounted on the shaft. The gravitational force on the mass, acting at a radius, r_1, then constitutes a drive torque which turns the shaft at a steadystate angular velocity of ω, rad/sec. The potential energy of the descending mass is then delivered to the device.

The electrical sink for this generator consists of a load resistance, R_L, connected to stationary carbon brushes touching copper slip rings, which are insulated from each other and the shaft. The two ends of the N-turn coil are connected to the slip rings, which allows rotary motion and a continuous path for current flow.

Facing the slip rings, the descending mass causes counter-clockwise shaft rotation. At this point, the basic properties of circular motion will be reviewed.

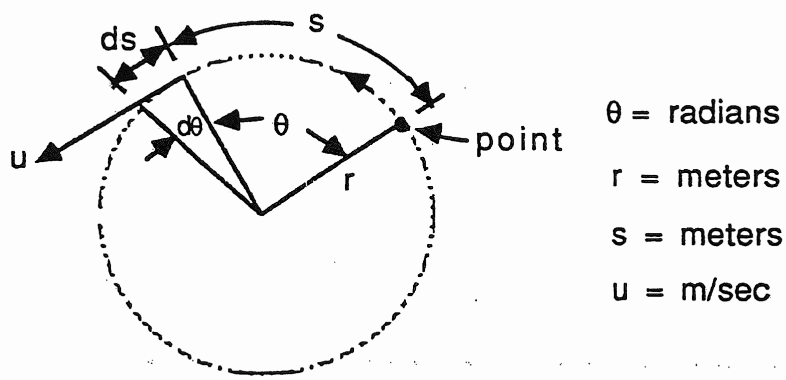


Figure 4.7 Circular Motion

In Fig. 4.7, a point in circular motion moving through an angle, θ, sweeps out a length of arc,

$$s = r\theta \qquad (m) \qquad (4.20)$$

The tangential velocity of the point is, then,

$$u = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega \qquad (m/sec) \qquad (4.21)$$

where ω is the angular velocity of the point in rad/sec.

In Fig. 4.6 the conductors in coil sides a and b are moving counter clockwise through a radial field with a constant angular velocity of ω, rad/sec. The tangential velocity u, the field B, and the conductors are always mutually perpendicular, resulting in a Faraday emf generated on each conductor,

$$E_{cond} = B \ell u = B \ell r \omega \qquad (V) \qquad (4.22)$$

The direction of these emfs (– to +) is towards coil terminal, a, in the top coil side and away from coil terminal, b, in the bottom coil side. These emfs are additive, in series, resulting in a coil-terminal emf,

$$E = 2 N B \ell r\omega \qquad (V) \qquad (4.23)$$

with the polarity indicated in Fig. 4.6.

Since the electric circuit is closed via the slip rings, brushes and load resistance, a current, I will flow towards coil terminal, a, in the top coil side and away from coil terminal, b, in the bottom coil side. Facing the slip rings, this produces a clockwise Lorentz force on all the conductors in coil sides a and b,

$$f_{cond} = B \ell I \qquad (N) \qquad (4.24)$$

This force acts at a radius r, meters resulting in a conductor-Lorentz torque,

$$T_{cond} = B \ell I r \qquad (N\text{-}m) \qquad (4.25)$$

Since all of the conductors have the clockwise torque of Eqn. (4.25) the total, back Lorentz torque on the coil is,

$$T = 2N B \ell I r \qquad (N\text{-}m) \qquad (4.26)$$

The power flow through this generator can now be calculated by realizing that torque (N-m) times ω (rad/sec) has the dimension of watts. The power flow diagram of this generator is given in Fig. 4.8.

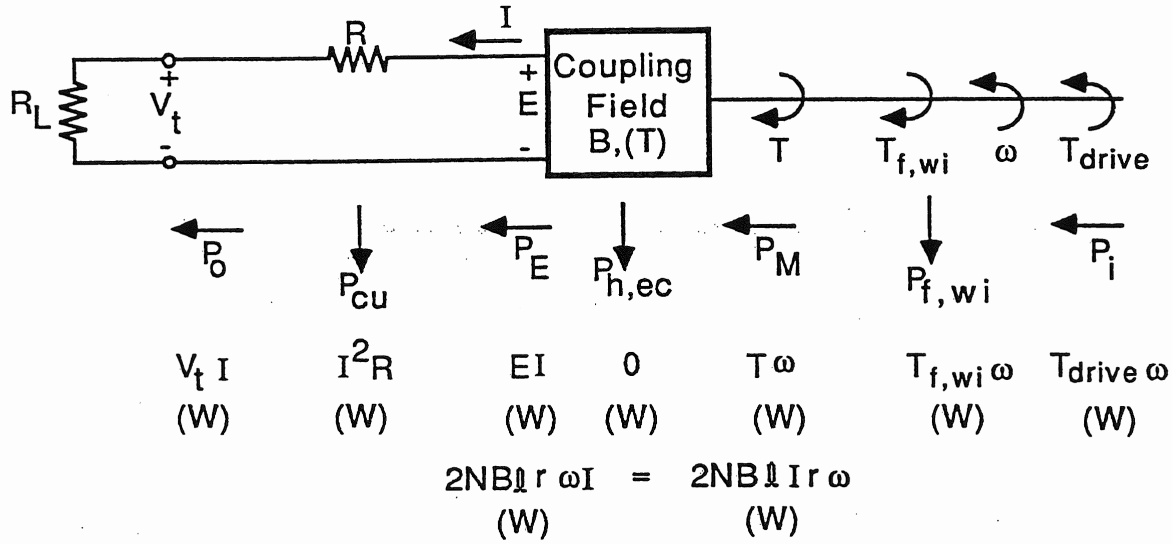


Figure 4.8 Rotational-Generator, Power Flow Diagram

From Fig. 4.8, the equations for generator power flow are,

$$T_{drive}\,\omega = T_{f,wi}\,\omega + T\,\omega \qquad \text{(W)} \qquad (4.27)$$

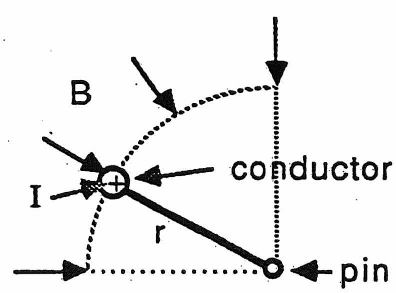
$$T\omega = EI \qquad \text{(W)} \qquad (4.28)$$

$$EI = I^2R + V_tI \qquad \text{(W)} \qquad (4.29)$$

Generator action is then characterized by the fact that the drive torque determines the direction of rotation, the Lorentz torque is always a back torque, and the Faraday emf is always a forward emf.

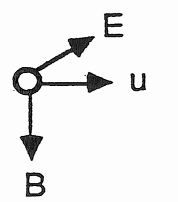
Observe, also, from Fig. 4.6, that the voltage across the load, V_t, reverses every half revolution at a constant value. This voltage can be made continuous by replacing the slip rings with a commutator, as will be seen in the later chapter on dc machines.

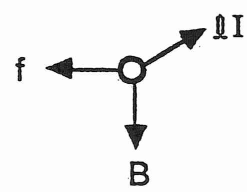
Example 4.4



A conductor, 20 cm in length, and resistance, 0.2 Ω, is placed perpendicular to a uniform, radial magnetic field of B = 1.0 T. The conductor is constrained to circular motion with a radius arm of 0.1 m, and the conductor carries a current of, I = 5.0 A, into this page. It is observed to move clockwise at 450 rad/sec. The back force of pin friction and windage is 0.1 N.

a) What is the generated Faraday emf and Lorentz torque?





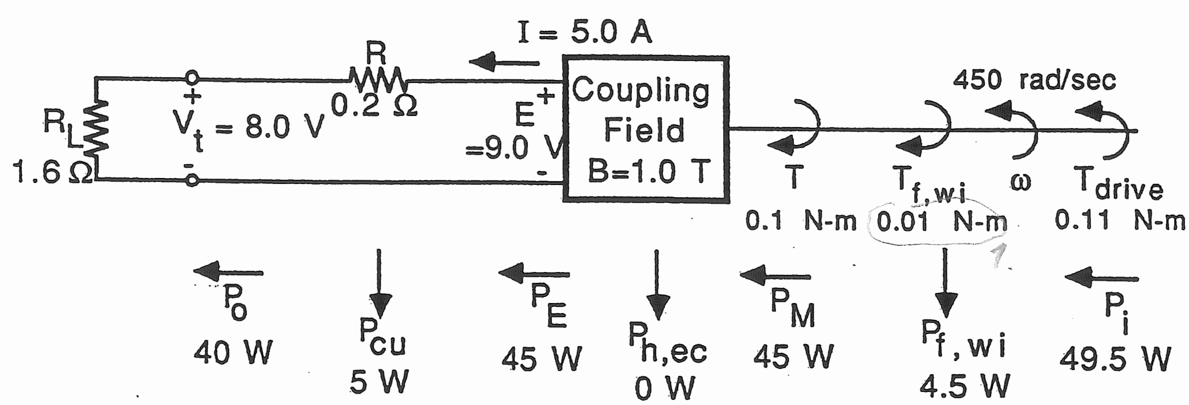
$E = B\,\ell\,r\,\omega = (1.0)(0.2)(0.1)(450)$; $T = B\,\ell\,I\,r = (1.0(0.2)(5.0)(0.1)$

$= 9.0$ V (into page) $= 0.1$ N-m (counter clockwise)

b) Is this device a motor or generator?

<u>generator</u> (forward emf, back torque)

c) Draw and label with numerical values, the power flow diagram for this device.



4.7 ROTATIONAL MOTOR

Since energy conversion is reversible, the device in Fig. 4.6 can be made a motor,

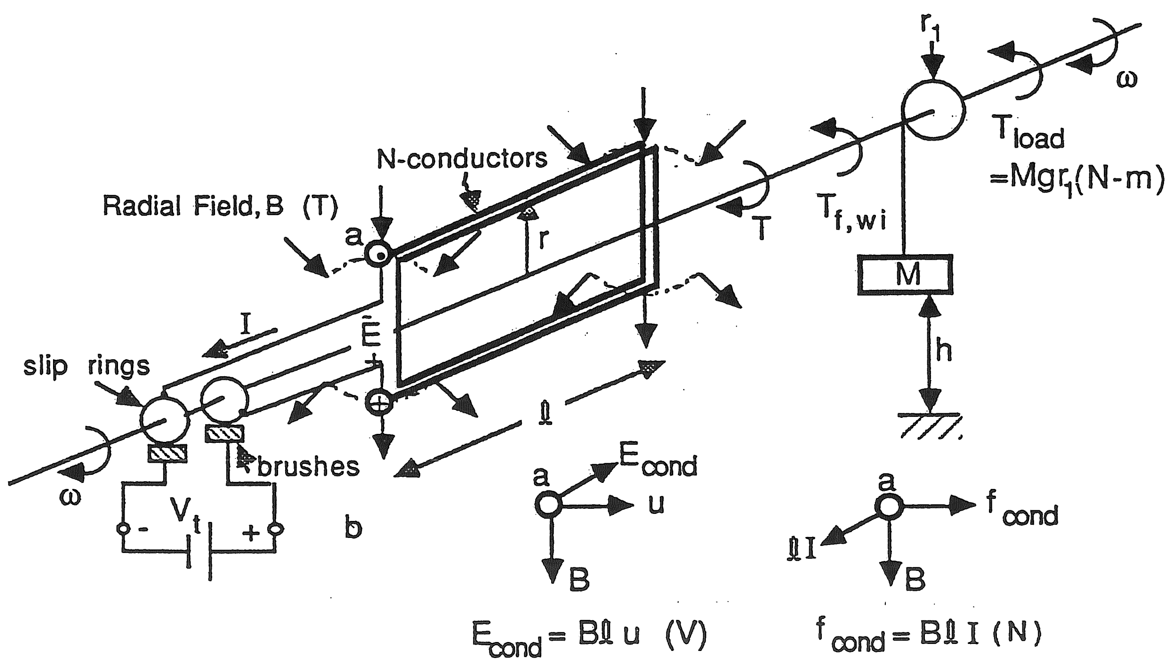


Figure 4.9 Rotational Motor

In Fig. 4.9, an N-turn coil is placed in a constant, uniform, magnetic, radial– field which exists for ℓ, meters along the motor shaft. The coil terminals a and b are connected to slip rings, which allow rotary motion and a continuous path for current flow. The mechanical sink for this motor is a mass, M connected by a cable to a pulley of radius, r_1, meters mounted on the shaft. The mass, M, must ascend against the force of gravity so that its potential energy increases, indicating that mechanical energy is being delivered to the mechanical system. Facing the slip rings, this implies clockwise rotation of the shaft and the force of gravity times r_1 constitutes a back torque called the load torque, T_{load}.

The conductors in coil sides a and b must, then, move clockwise through the field. A Faraday emf is induced in each conductor,

$$E_{cond} = B \ell u = B \ell r \omega \qquad (V) \qquad (4.30)$$

The direction of the conductor emf (– to +) is away from coil-terminal, a, in the top coil side and towards coil-side, b, in the bottom coil side. These emfs are additive, in series, resulting in a coil-terminal emf,

$$E = 2 N B \ell r \omega \qquad (V) \qquad (4.31)$$

with polarity indicated in Fig. 4.9. For a motor the Faraday emf in Eqn. (4.31) must be a back emf so current must flow in the direction indicated in Fig. 4.9. This is accomplished by orienting an electrical source across the brushes, establishing a terminal voltage, V_t, that delivers electrical energy to this device.

Current, then, flows towards coil-terminal, a, in the top coil side and away from coil-terminal, b, in the bottom coil side. Facing the slip rings this produces a clockwise Lorentz force on all the conductors in coil sides a and b.

$$f_{cond} = B \ell I \qquad (N) \qquad (4.32)$$

This force acts at a radius r, meters resulting in a conductor torque,

$$T_{cond} = B \ell I r \qquad (N\text{-}m) \qquad (4.33)$$

Since all of the conductors have the clockwise torque of Eqn. (4.33), the total, forward Lorentz torque is,

$$T = 2 N B \ell I r \qquad (N\text{-}m) \qquad (4.34)$$

From Fig. 4.9, it is seen that the Lorentz torque determines the direction of rotation and is equal to the back torques of friction and windage and the load torque for constant angular velocity, ω.

It is also seen from Fig. 4.9, that the coil will rotate 90° from its present position and then stop, since further rotation will produce a reverse Lorentz torque on the winding. This undesirable condition will be eliminated in the later chapter on dc machines, by replacing the slip rings with a commutator allowing the Lorentz torque to be continuous in one direction for any position of the winding. However, the power flow diagram for this motor, during the interval of rotation, is given in Fig. 4.10.

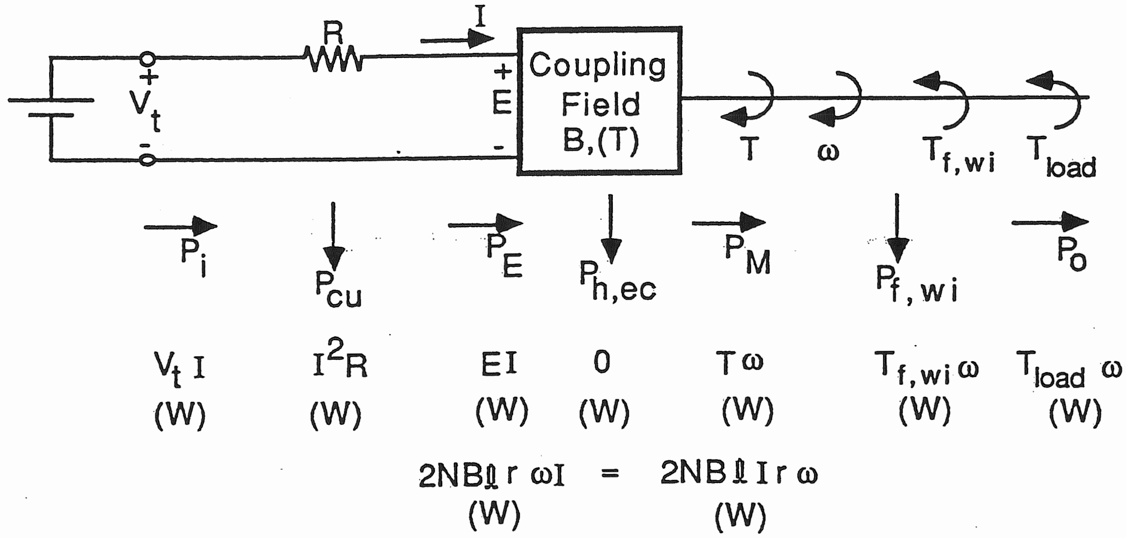


Figure 4.10 Rotational-Motor, Power Flow Diagram

From Fig. 4.10, the equations for motor power flow are,

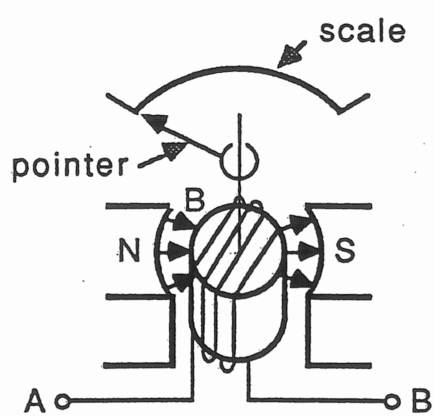
$$V_t I = I^2 R + E I \qquad \text{(W)} \qquad (4.35)$$

$$E I = T\omega \qquad \text{(W)} \qquad (4.36)$$

$$T\omega = T_{f,wi}\,\omega + T_{load}\,\omega \qquad \text{(W)} \qquad (4.37)$$

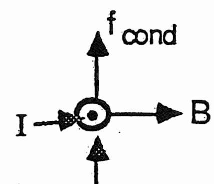
Motor action is then characterized by the fact that the Lorentz torque determines the direction of rotation, the Lorentz torque is always a forward torque and the Faraday emf is always a back emf.

Example 4.5



A permanent-magnet, D'Arsonval meter movement is illustrated in the figure. The flux density, B = 0.4 T, is radial and uniform around the air gap. A 10-turn coil is wound on a soft-iron, cylindrical rotor whose diameter is 1.0 cm and length is 1.0 cm. The coil carries a current of 0.5 mA and is rotating at 1.57 rad/sec.

a) What direction must the current flow to produce up scale deflection of the pointer?



top of left-hand
vertical conductors

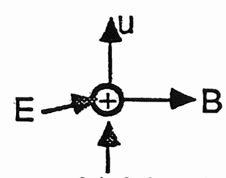
I must flow from A to B for clockwise rotation.

b) What is the Lorentz torque acting on the rotor?

$$T = 2\,N\,B\,\ell\,I\,r = (2)(10)(0.4)(0.01)(0.5 \times 10^{-3})(0.01/2)$$

$$= 0.2\ \mu\text{N-m (clockwise)}$$

c) What is the magnitude and direction of the Faraday emf generated in the coil?



top of left-hand
vertical conductors

$$E = 2\,N\,B\,\ell\,r\,\omega = (2)(10)(0.4)(0.01)(0.01/2)(1.57)$$

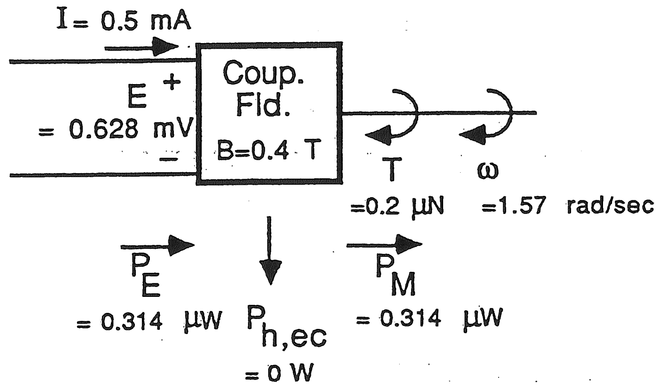
$$= 0.628\ \text{mV}$$

directed (+ to –) from A to B

d) Is this device a motor or generator?

<u>motor</u> (back emf, forward torque)

e) What is the power input and output of the coupling field?



4-8 RELUCTANCE-FORCE ENERGY CONVERSION

SINGLY-EXCITED, MOVABLE-IRON DEVICES

In the foregoing sections, we considered Lorentz-force, translational or rotational devices that consisted of movable, current-carrying conductors placed in an air-gap, magnetic field. We shall now consider electromagnetic devices in which there are movable portions of the iron path. Such devices are relays, solenoids, reluctance motors, etc. Consider the electromagnet in Fig.4.11.

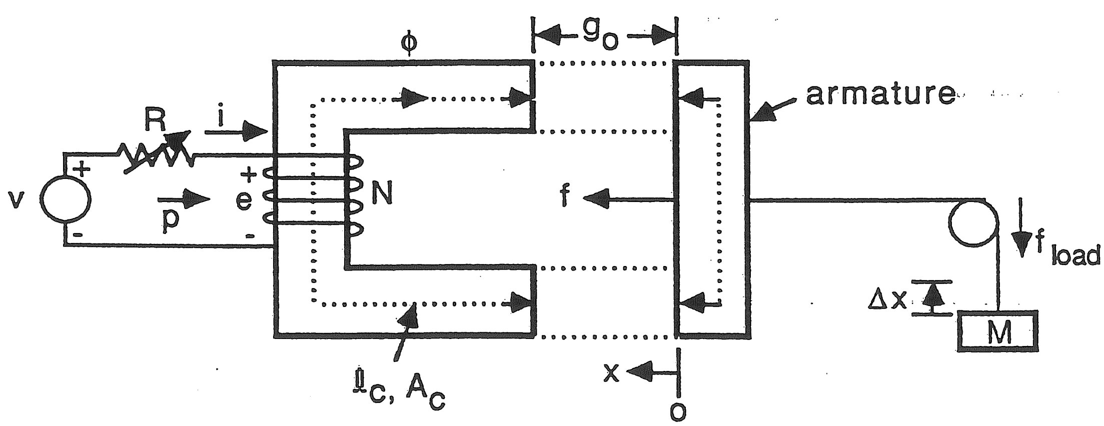


Figure 4.11 Electromagnet

In Fig. 4.11, the movable portion of the iron path is called the armature. Observe that there are no current-carrying conductors placed in the airgap of this device.

A magnetic force, called a Reluctance force, will act on all movable portions of the iron path, in such a direction as to reduce the reluctance of the device to a minimum or increase the energy stored in the magnetic field of the device to a maximum.

It is this force that accounts for the 60-cycle hum that is commonly heard from transformers and other similar electromagnetic devices.

This reluctance force cannot be explained by Lorentz's force law in Eqn. (4.12), but must be explained by a general, energy-analysis of the device. This is implemented by first applying Maxwell's mmf law to the device, from which the energy stored in the magnetic field can be calculated. Then, as the armature moves, because of the reluctance force acting on it, the energy distribution throughout the device will change, and this change in energy must be conserved. From this change in energy distribution we will then calculate the reluctance force that acts on the movable iron in the device.

Later in this chapter, when we consider doubly-excited devices, we shall find that this general energy analysis will predict both the Reluctance and the Lorentz forces acting on the magnetic circuit.

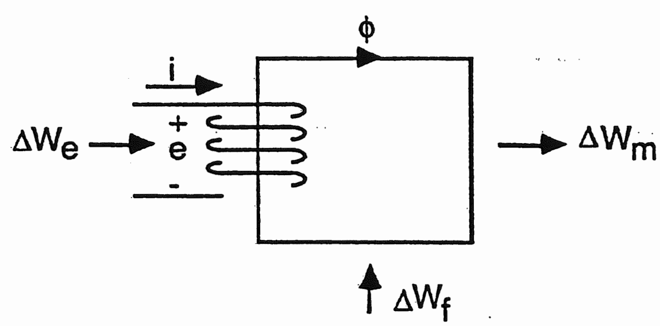
Redistribution of energy through any movable-iron device

Consider first, the change in energy stored in the magnetic field of the electromagnet in Fig. 4.11, as the armature moves through a displacement, Δx.

For a given mmf, as the armature moves, the circuit reluctance and flux linkage with the coil will change, resulting in a Faraday emf generated across the coil. A change in electric energy delivered to the coil then occurs, and this energy must be conserved, i.e.,

$$\Delta W_e = \Delta W_f + \Delta W_m \qquad \text{(J)} \qquad (4.38)$$

The change in electric energy delivered to the coil can only be stored as a change in field energy, and as a change in mechanical energy delivered to the mass in Fig. 4.11.



The change in electric energy is calculated as,

$$\Delta W_e = \int e i dt = \int_{\lambda_1}^{\lambda_2} i d\lambda \qquad (J)$$

where, $$e = \frac{d\lambda}{dt} \qquad (V)$$

The change in energy stored in the field, from Chapter 2, Eqn. (2.22), is evaluated as the final energy stored minus the initial energy stored,

$$\Delta W_f = \underset{\text{final}}{\int_{o}^{\lambda_2} i d\lambda} - \underset{\text{initial}}{\int_{o}^{\lambda_1} i d\lambda} \qquad (J)$$

The change in energy converted to mechanical form is,

$$\Delta W_m = f\Delta x \qquad (J)$$

Example 4.6

The armature of the electromagnet in Fig. 4.11 is held open at x = 0. In this position, the coil inductance is measured as, L = 8.0 H. With the armature held stationary in this position, the current is decreased from 2.0 to 1.0 A. Calculate the change in energy throughout this device as the current is decreased.

$L = \frac{\lambda}{i} = 8.0 \text{ H}$; $\lambda = 8i$ (Wb-t) @ $x = 0$, and is plotted in Fig. 4.12,

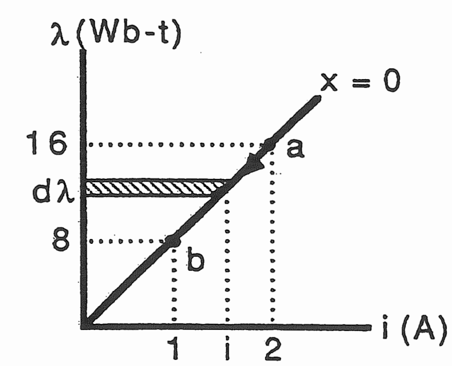


Figure 4.12 Magnetization Curve

$$\Delta W_e = \int_{\lambda_1}^{\lambda_2} i\,d\lambda = \int_{16}^{8} \frac{1}{8}\lambda\,d\lambda = \frac{1}{8}\left[\frac{\lambda^2}{2}\right]_{16}^{8} = -12 \text{ (J)}$$

$$\Delta W_f = \int_{0}^{\lambda_2} i\,d\lambda - \int_{0}^{\lambda_1} i\,d\lambda = \int_{0}^{8} \frac{1}{8}\lambda\,d\lambda - \int_{0}^{16} \frac{1}{8}\lambda\,d\lambda$$

final − initial

$$= \frac{1}{8}\left[\frac{\lambda^2}{2}\right]_{0}^{8} - \frac{1}{8}\left[\frac{\lambda^2}{2}\right]_{0}^{16} = 4-16 = -12 \text{ (J)}$$

$$\Delta W_m = f\Delta x = 0 \text{ (J)}$$

$$\Delta W_e = \Delta W_f + \Delta W_m$$

$$-12 = -12 + 0$$

The decrease in energy stored in the field is returned to the electrical source, since no energy is converted to mechanical form.

Derivation of the reluctance force acting on the movable iron

Consider, now, a small increase in armature displacement, Δx, as indicated in Fig. 4.11. This increase in displacement results in a change of energy as predicted by Eqn. (4.38) and can take place in many ways, two of which are shown in Fig. 4.13.

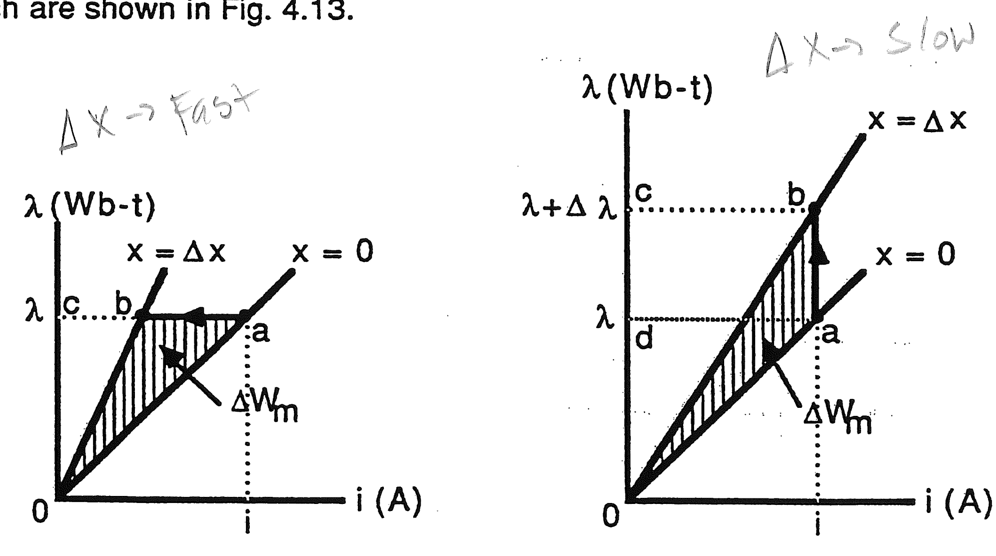


Figure 4.13 Trajectories in $\lambda - i$ Space

As the armature is allowed to move through displacement, Δx, the magnetic state of the device moves along the two paths, a-b, called trajectories,during the transient interval of armature motion. During this transient interval, the flux is changing and an emf, e, is induced across the coil, which is a function of armature speed.

In Fig. 4.13 (a), the armature is allowed to move very quickly and the resistance, R, in Fig. 4.11, is increased so that motion takes place before the flux can change, and λ remains constant along the path a-b.

In Fig. 4.13 (b), the armature is allowed to move very slowly with constant resistance, R, so that the induced emf, e, is negligible and the current remains constant along the path a-b.

The actual trajectory of a given device is a curved path between points a and b in Fig. 4.13 (b), and can be found by writing and solving the differential equations for the device. However, regardless of the path used, the energy equation (4.38) is valid since this system is conservative, i.e., lossless (hysteresis and eddy currents are considered negligible) and the net energy change is independent of the path followed. We shall now apply Eqn. (4.38) to the two paths in Fig. 4.13 to evaluate the reluctance force on the armature.

In Fig. 4.13 (a), Eqn. (4.38) becomes,

$$\Delta W_e = 0 = \Delta W_f + f\,\Delta x$$

then,

$$0 = (obc - oac) + f\,\Delta x$$

$$f\,\Delta x = -(obc - oac) = -\Delta W_f = oab$$

and,

$$f = \lim_{\Delta x \to 0} -\frac{\Delta W_f}{\Delta x} = -\frac{\partial W_f(\lambda, x)}{\partial x}\bigg|_{\lambda = \text{constant}} \quad (N) \qquad (4.39)$$

Eqn. (4.39) states that for a positive displacement, at constant flux, no electric energy is input to the device and the energy delivered to the mechanical system is taken from the energy stored in the magnetic field. The magnetic force, then, is equal to the space rate of change at which energy can be taken from the field when the flux is kept constant. Observe, from Fig. 4.13(a),

ΔW_e is the area to the left of the trajectory.

ΔW_f is the difference in areas to the left of the magnetization curves.

ΔW_m is the shaded area which is bounded by the magnetization curves and the trajectory.

In Fig. 4.13 (b), Eqn. (4.38) becomes,

$$\Delta W_e = dabc = \Delta W_f + f\,\Delta x$$

then,
$$f\,\Delta x = \Delta W_e - \Delta W_f = dabc - (obc\text{-}oad)$$
$$= dabc + oad - obc = oab$$

and
$$f\,\Delta x = oab = \underset{obc}{W_{f2}} - \underset{oad}{W_{f1}} = \Delta W_f$$

or,
$$f = \lim_{\Delta x \to 0} \frac{\Delta W_f}{\Delta x} = + \left. \frac{\partial W_f\,(i,x)}{\partial x} \right|_{i=\text{constant}} \quad \text{(N)} \qquad (4.40)$$

Eqn. (4.40) states that for a positive displacement, at constant current, the energy delivered to the mechanical system is equal to the increase in energy stored in the magnetic field. Observe from Fig. 4.13(b),

ΔW_e is the area to the left of the trajectory

ΔW_f is the difference in areas to the left of the magnetization curves.

ΔW_m is the shaded area which is bounded by the magnetization curves and the trajectory.

It is interesting to note that when the coil current is kept constant, for linear magnetization curves only, the total increase in electric energy delivered to the coil is divided in half. Half is stored in the magnetic field and half is delivered to the mechanical system. By referring to Fig. 4.13 (b),

$$\Delta W_e = \Delta\lambda\, I$$

$$W_{f2} = \frac{1}{2}(\lambda + \Delta\lambda)\, I$$

$$W_{f1} = \frac{1}{2}\lambda\, I$$

then,
$$f\,\Delta x = \Delta W_f = W_{f2} - W_{f1} = \frac{1}{2}\Delta\lambda\, I = \frac{1}{2}\Delta W_e \quad \text{(J)}$$

In summary,

$$f = -\frac{\partial W_f(\lambda,x)}{\partial x} = +\frac{\partial W_f(i,x)}{\partial x} \quad \text{(N)} \qquad (4.41)$$

The reluctance force that acts on any moving iron is equal to the space rate of change of the energy stored in the field of the device.

For rotational moving-iron devices, the reluctance force acts on the armature at a radius, r meters. For circular motion, the displacement, x, becomes,

$$x = r\,\theta \qquad \text{(m)}$$

$$dx = r d\theta$$

The reluctance torque is, then,

$$T = f\,r = \frac{\partial W_f}{\partial \theta}\frac{d\theta}{dx}\,r$$

or,

$$T = -\frac{\partial W_f(\lambda,\theta)}{\partial \theta} = +\frac{\partial W_f(i,\theta)}{\partial \theta} \qquad \text{(N-m)} \qquad (4.42)$$

The reluctance <u>torque</u> that acts on any moving iron is equal to the <u>angular</u> rate of change of the energy stored in the magnetic field of the device.

Example 4.7

What is the force acting on the armature for the electromagnet in Fig. 4.11 as a function of current or flux, and displacement?

Before the reluctance force acting on the armature can be calculated, the energy stored in the magnetic field of the device must be known as a function of the armature displacement, x. This analysis can be accomplished in several procedural steps:

1. The analog circuit is drawn for the singly-excited device.

2. Using Maxwell's MMF law, the flux, $\phi(x)$, is calculated, where the displacement of the movable iron, x, is measured from an <u>arbitrary</u> point in space.The reluctance force, acting on the movable iron, as in Fig. 4.11, will then be <u>positive</u> in the direction of assumed x.

$$\phi(x) = \frac{\mathcal{F}}{\mathcal{R}_{ckt}(x)} \qquad \text{(Wb)}$$

3. The flux linkage of the coil, , can then be calculated,

$$\lambda(x) = N\,\phi(x) \qquad \text{(Wb-t)}$$

4. The energy stored in the magnetic field of the device, as it varies with the displacement of the movable iron, is determined,

$$W_f(x) = \frac{1}{2}\lambda(x)\, i = \frac{1}{2} L(x)\, i^2 = \frac{1}{2}\mathcal{R}(x)\, \phi^2 \qquad \text{(J)}$$

5. The reluctance force acting on the movable iron is, then,

$$f = \frac{\partial W_f(x)}{\partial x} = \frac{1}{2}\, i^2 \frac{dL(x)}{dx} = -\frac{1}{2}\, \phi^2 \frac{d\mathcal{R}(x)}{dx} \qquad \text{(N)}$$

This analysis for any singly-excited device is now applied to the electromagnet in Fig. 4.11.

Maxwell's mmf law yields,

$$\mathcal{F} = F_c + 2F_g$$

$$Ni = H_c \ell_c + 2\, H_g\, (g_o - x)$$

$$= \frac{B_c \ell_c}{\mu_c} + \frac{2B_g\, (g_o - x)}{\mu_o}$$

$$Ni = \frac{\ell_c}{\mu_c\, A_c}\, \phi + \frac{2(g_o - x)}{\mu_o\, A_g}\, \phi \qquad \text{(A-t)} \qquad (4.43)$$

For a well-designed core, $F_c << F_g$, and Eqn. (4.43) becomes,

$$\phi = \frac{N\, \mu_o\, A_g}{2(g_o - x)}\, i \qquad \text{(Wb)} \qquad (4.44)$$

The flux linkage of the coil, $\lambda = N\,\phi$, is found from Eqn. (4.44),

$$\lambda = \frac{N^2\, \mu_o\, A_g}{2(g_o - x)}\, i \qquad \text{(Wb-t)} \qquad (4.45)$$

The mmf dropped along the core, in Eqn. (4.43) need not be neglected, in which case, the magnetization curve for a given displacement, x, could be nonlinear. In either case, the energy stored in the magnetic field, for any displacement, is the area to the left of the magnetization curve. This energy is,

$$W_f = \int_0^{\lambda} i\; d\lambda \qquad \text{(J)}$$

For a <u>linear</u> magnetization curve,

$$W_f = \frac{1}{2}\lambda i = \frac{1}{2}L i^2 = \frac{1}{2}\mathcal{R}\phi^2 \qquad \text{(J)} \qquad (4.46)$$

where,

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} \qquad \text{and} \qquad \mathcal{R} = \frac{Ni}{\phi}$$

The energy stored in the field can now be calculated by substituting Eqn (4.45), in Eqn (4.46),

$$W_f(i,x) = \frac{1}{2}\lambda i = \frac{N^2 \mu_o A_g}{4(g_o - x)} i^2 \qquad \text{(J)} \qquad (4.47)$$

Observe from Fig. (4.11) and Eqn. (4.47), that as the armature displacement, x, increases, the reluctance of the device decreases and the energy stored in the field increases for a given mmf driving this device. It is clear, then, from Eqn. (4.47), for a given mmf driving the device, the energy stored in the field is a function of the armature displacement, x.

then, $$f = \frac{\partial W_f(i,x)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{N^2\mu_o A_g i^2}{4}\right)(g_o - x)^{-1}$$

$$= \frac{N^2\mu_o A_g i^2}{4}(-1)(g_o - x)^{-2}(-1)$$

$$f = \frac{N^2\mu_o A_g i^2}{4(g_o - x)^2} \qquad \text{(N)} \qquad (4.48)$$

Alternatively, from Eqn. (4.45),

$$i = \frac{2(g_o - x)\lambda}{N^2\mu_o A_g}$$

$$W_f(\lambda,x) = \frac{1}{2}\lambda i = \frac{(g_o - x)}{N^2\mu_o A_g}\lambda^2$$

$$f = -\frac{\partial W_f(\lambda,x)}{\partial x} = \frac{\lambda^2}{N^2\mu_o A_g} \qquad \text{(N)} \qquad (4.49)$$

Equations (4.48) and (4.49) are identical if $\lambda(i)$ is substituted in Eqn. (4.49).

It is interesting to note that in steady state, if the armature is held at any value of x, the emf generated across the coil is zero and the current becomes,

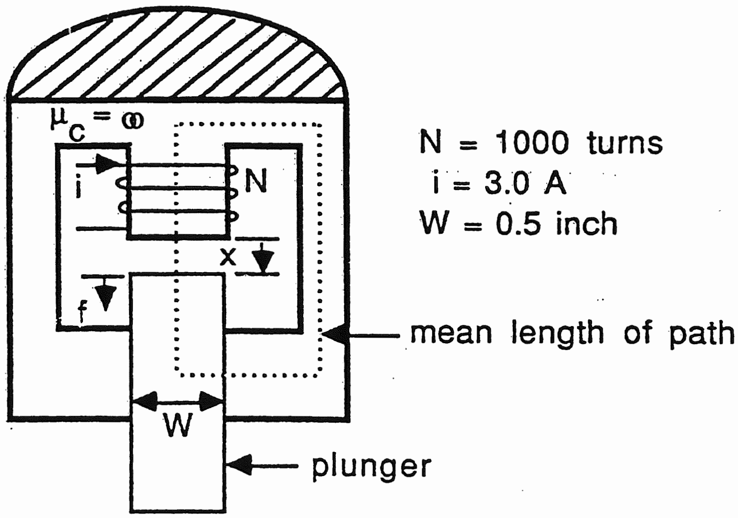
$$i = \frac{V}{R}$$

The force equation (4.48) is, then,

$$f = \frac{N^2\mu_o A_g V^2}{4R^2(g_o - x)^2} \qquad (N) \qquad (4.50)$$

It would appear, from Eqn. (4.50) that the force becomes infinite as the displacement approaches g_o, but it must be remembered that the mmf dropped along the core was neglected and the flux, in actuality, is limited by the core reluctance when the gap is zero.

Consider, next, a cylindrical solenoid,

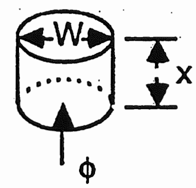


Calculate the force acting on the plunger, at x = 0.25 inch, and its direction.

Using Maxwell's mmf law –

$$Ni = \frac{x}{\mu_o A_x}\phi \qquad \text{where, } A_x = \frac{\pi W^2}{4}$$

$$= \frac{4x}{\mu_o \pi W^2}\phi \quad ; \qquad \phi = \frac{\mu_o \pi W^2 N}{4x} i$$



$$\lambda = N\phi = \frac{\mu_o \pi W^2 N^2}{4x} i \qquad (Wb\text{-}t)$$

$$W_f(i,x) = \frac{1}{2}\lambda i = \frac{\mu_o \pi W^2 N^2 i^2}{8x} \qquad (J)$$

$$f = \frac{\partial W_f(i,x)}{\partial x} = \frac{\mu_o \pi W^2 N^2 i^2}{8}(-1)(x)^{-2}$$

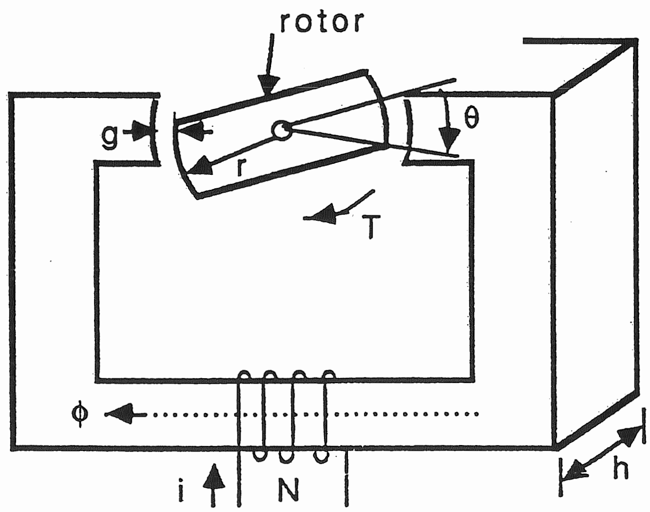
$$f = -\frac{\mu_o \pi W^2 N^2 i^2}{8x^2} \qquad (N)$$

$$f = -\frac{(4\pi x10^{-7})(\pi)(0.5 \times 0.0254)^2(1000)^2(3)^2}{(8)(0.25 \times 0.0254)^2}$$

$$= -\ 17.8\ \text{N} \quad x \quad 0.2248\frac{\text{lb.}}{\text{N}} \quad = \ -\ 4\ \text{lb. (force)}$$

The force is directed upward.

Example 4.8 Air-gap fringing negligible –



r = 2.54 cm
h = 2.54 cm
g = 0.254 cm
N = 1000 turns
i = 8.0 A

The magnetic circuit shown consists of a cylindrical rotor shaped so that it has more iron along its direct axis than it has along its quadrature axis. If air-gap fringing is neglected, what is the torque that will move this rotor to a position of least circuit reluctance?

From the mmf law,

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}_c + 2\mathcal{R}_g}$$

Assuming a well-designed core,

$$\mathcal{R}_c << 2\mathcal{R}_g \quad \text{and} \quad \mathcal{R}_g = \frac{g}{\mu_o A_g} = \frac{g}{\mu_o\, h\, r\, \theta}$$

then,

$$\phi = \frac{Ni\,(\mu_o\, h\, r\, \theta)}{2g}$$

The flux linkage of the coil is,

$$\lambda = \frac{N^2 \mu_o\, h\, r\, \theta}{2g}\, i$$

The energy stored in the magnetic field is,

$$W_f(i,\theta) = \frac{1}{2}\lambda\, i = \frac{N^2 \mu_o h r \theta}{4g} i^2$$

The torque is,

$$T = \frac{\partial W_f(i,\theta)}{\partial \theta} = \frac{N^2 \mu_o h r i^2}{4g} \qquad \text{(N-m)}$$

Observe that for this electromagnetic device, the torque is constant and independent of θ.

$$T = \frac{(1000)^2(4\pi)(10^{-7})(0.0254)(0.0254)(8)^2}{(4)(0.00254)} = 5.11 \text{ N-m} \quad \text{(CW)}$$

4-9 RELUCTANCE MOTOR

Air-gap fringing not negligible –

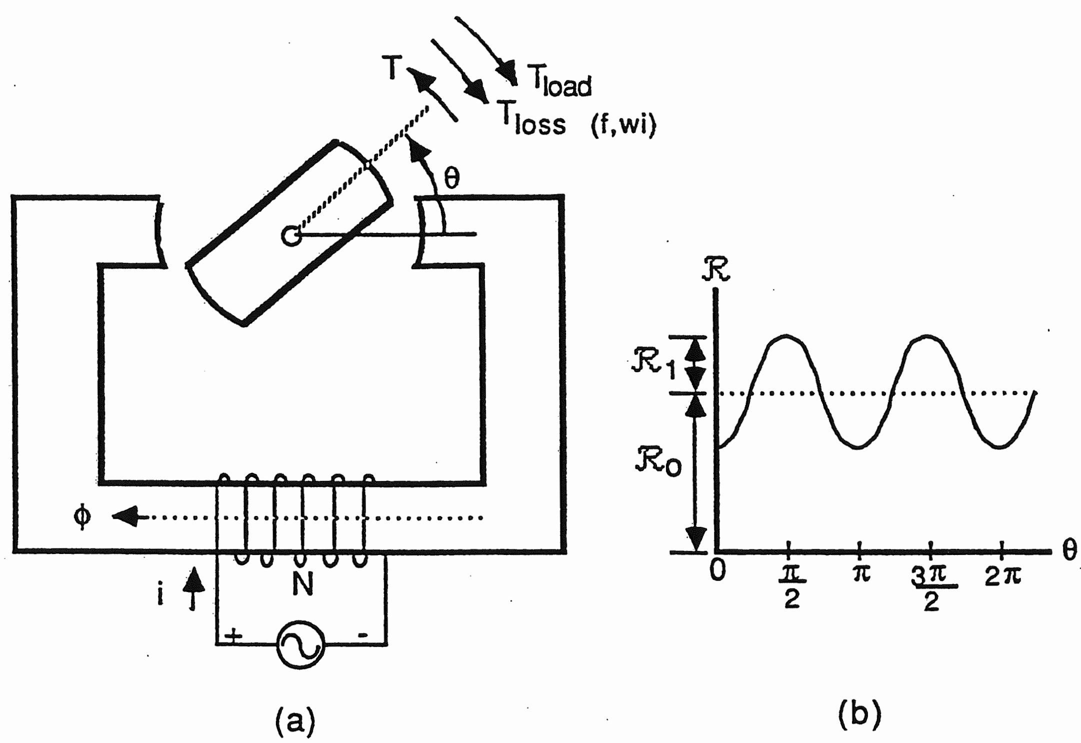


Figure 4.14 Reluctance Motor

As is seen in Ex. 4.8, a reluctance torque acts on the rotor to move it to a position of least circuit reluctance. Advantage will be taken of this phenomenon in Fig. 4.14, by connecting the coil on the stator (stationary iron of this device) toa single-phase source of frequency, f Hz, and we shall show that this device will operate as a motor at a speed synchronized to the source frequency. This device is used to drive clocks and other timing devices, since it rotates only at synchronous speed.

In Fig. 4.14, air-gap fringing is not neglected, so that when θ is zero, the circuit reluctance is minimum and when θ is 90°, the reluctance is maximum, and the complete cycle of variation repeats twice in one revolution of the rotor as shown in Fig. 4.14(b). Neglecting higher ordered variation, the circuit reluctance is assumed to vary sinusoidally with θ, as,

$$\mathcal{R} = \mathcal{R}_0 - \mathcal{R}_1 \cos 2\theta \qquad \text{(A-t/Wb)} \tag{4.51}$$

If the core is well designed, the core mmf is negligible and the flux is limited by the reluctance of the gaps. From the mmf law,

$$\phi = \frac{Ni}{\mathcal{R}} \quad ; \quad Ni = \phi\mathcal{R}$$

The energy stored in the magnetic field is,

$$W_f = \frac{1}{2}\lambda i = \frac{1}{2}\phi Ni = \frac{1}{2}\phi^2 \mathcal{R}(\theta)$$

The magnetic torque acting on the rotor is, then,

$$T = -\frac{\partial W_f(\phi,\theta)}{\partial \theta} = -\frac{1}{2}\phi^2 \frac{d\mathcal{R}(\theta)}{d\theta} \tag{4.52}$$

The derivative of Eqn. (4.51) is,

$$\frac{d\mathcal{R}}{d\theta} = 2\mathcal{R}_1 \sin 2\theta$$

and when substituted in Eqn. (4.52),

$$T = -\phi^2 \mathcal{R}_1 \sin 2\theta \qquad \text{(N-m)} \tag{4.53}$$

Equation (4.53) is very interesting in that we observe the magnetic torque acting on the rotor is a function of two variables, ϕ and θ.

As is commonly done with a function of two variables, we keep one variable constant and allow the other one to vary. For the purposes of this discussion we will keep the coil mmf and therefore the flux constant. When $\theta = 0$, the torque is zero and if we move the rotor to 90°, the torque is again zero. In between, the torque is negative, and the rotor is driven to $\theta = 0°$, If we continue this observation we note that when,

$\theta = 0$	$T = 0$
$0 < \theta < \frac{\pi}{2}$	T neg.
$\theta = \frac{\pi}{2}$	$T = 0$
$\frac{\pi}{2} < \theta < \pi$	T pos
$\theta = \pi$	$T = 0$
$\pi < \theta < \frac{3\pi}{2}$	T neg
$\theta = \frac{3\pi}{2}$	$T = 0$
$\frac{3\pi}{2} < \theta < 2\pi$	T pos
$\theta = 2\pi$	$T = 0$

In summary, for constant mmf or flux, the magnetic torque always returns the rotor to a position of least reluctance.

Now, let us allow the flux to vary. Since the coil is connected to an alternating source of frequency, f, the mmf and therefore the flux will vary sinusoidally with time. We shall start counting time when,

$$\phi = \phi_m \cos \omega t \qquad \text{(Wb)} \tag{4.54}$$

where,

$$\omega = 2\pi f \qquad \text{(rad/sec)}$$

Substituting Eqn. (4.54) into Eqn. (4.53),

$$T = -\phi_m{}^2 \mathcal{R}_1 \cos^2 \omega t \, \sin 2\theta \tag{4.55}$$

At this point, by external means, we shall bring the rotor up to a constant angular speed, ω_R rad/sec, in which case, the angle θ increases as,

$$\theta = \omega_R t - \delta \qquad \text{(rad)}$$

since,

$$\frac{d\theta}{dt} = \omega_R$$

where, δ is called the torque angle of the load, and will become meaningful as our discussion proceeds.

then, $$T = -\phi_m^2 \mathcal{R}_1 \cos^2 \omega t \; \sin(2\,\omega_R t - 2\,\delta) \tag{4.56}$$

Using the trignometric identities,

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\,\alpha)$$

$$\sin \alpha \cos \beta = \frac{1}{2}\sin(\alpha + \beta) + \frac{1}{2}\sin(\alpha - \beta)$$

Equation (4.56) becomes,

$$\begin{aligned} T = &-\frac{1}{2}\phi_m^2 \mathcal{R}_1 \sin(2\,\omega_R t - 2\,\delta) \\ &- \frac{1}{4}\phi_m^2 \mathcal{R}_1 \sin(2\,\omega_R t + 2\,\omega t - 2\,\delta) \\ &- \frac{1}{4}\phi_m^2 \mathcal{R}_1 \sin(2\,\omega_R t - 2\,\omega t - 2\,\delta) \qquad \text{(N-m)} \end{aligned} \tag{4.57}$$

In Eqn. (4.57), if $\omega_R \neq \omega$, all three terms are functions of time, and their average values over a cycle are zero. We conclude that we have no continuous magnetic torque if the rotor speed is not synchronous. However, if $\omega_R = \omega$, Eqn. (4.57) becomes,

$$\begin{aligned} T = &-\frac{1}{2}\phi_m^2 \mathcal{R}_1 \sin(2\,\omega t - 2\,\delta) \\ &-\frac{1}{4}\phi_m^2 \mathcal{R}_1 \sin(4\,\omega t - 2\,\delta) \\ &+\frac{1}{4}\phi_m^2 \mathcal{R}_1 \sin 2\,\delta \qquad \text{(N-m)} \end{aligned} \tag{4.58}$$

The average values of the first two terms of Eqn. (4.58) are zero over a cycle, but the last term is independent of time, so the average torque, when the motor is running at synchronous speed is,

$$T = \frac{1}{4}\phi_m^2 \mathcal{R}_1 \sin 2\delta \qquad \text{(N-m)} \qquad (4.59)$$

Equation (4.59) is plotted in Fig. 4.15.

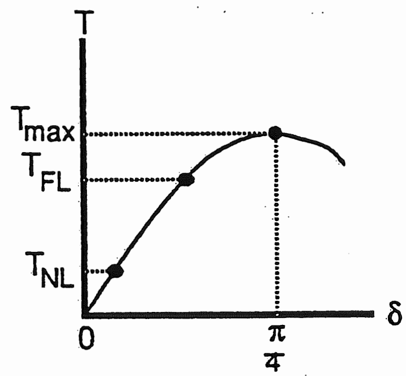


Figure 4.15 Average Reluctance Torque

We have shown the motor runs at constant, synchronous speed or it does not rotate at all. If the motor speed is constant, the average reluctance torque must at all instants of time, from Fig. 4.14, be equal to the sum of the back torques consisting of losses and load,

$$T = T_{loss} + T_{load}$$

If we were to illuminate the rotor with a strobe light flashing with source frequency, f Hz, the rotor would appear stopped at a torque angle, δ. At no-load, the generated torque would be equal to the losses torque and the torque angle would be quite small. As the motor is loaded, the generated torque increases and the torque angle opens to $\pi/4$ radians, where, if the load torque increases beyond this point the motor will stall. This motor has no starting torque as is evident from Eqn. (4.57) but must be externally brought up to synchronous speed and then loaded.

Example 4.9

The inductance of the motor in Fig. 4.14 was measured at the 1000-turn coil as 1.0 H, when $\theta = 0°$, and 0.5 H when $\theta = 90°$. If the coil current is constant at 0.65A, what is the torque acting on the rotor at $\theta = \frac{\pi}{4}$ rad?

now,

$$\mathcal{R} = \frac{Ni}{\phi} = \frac{N^2 i}{N\phi} = \frac{N^2}{L}$$

$$\mathcal{R}_{max} = \frac{(1000)^2}{0.5} = 2.0 \times 10^6 \quad \text{At/Wb}$$

$$\mathcal{R}_{min} = \frac{(1000)^2}{1.0} = 1.0 \times 10^6 \quad \text{At/Wb}$$

$$\mathcal{R}_1 = \frac{\mathcal{R}_{max} - \mathcal{R}_{min}}{2} = \frac{(2.0 - 1.0)\ 10^6}{2} = 0.5 \times 10^6 \quad \text{At/Wb}$$

therefore, $\mathcal{R}(\theta) = \mathcal{R}_o - \mathcal{R}_1 \cos 2\theta = (1.5 - 0.5 \cos 2\theta)\ 10^6 \quad \text{At/Wb}$

$$\mathcal{R}(\frac{\pi}{4}) = 1.5 \times 10^6 \quad \text{At/Wb}$$

$$\phi = \frac{Ni}{\mathcal{R}} = \frac{(1000)(0.65)}{(1.5)(10^6)} = 433 \quad \mu\text{Wb}$$

From Eqn. (4.53),

$$T = -\phi^2 \mathcal{R}_1 \sin 2\theta = -(433 \times 10^{-6})^2 (0.5 \times 10^6)\ (1)$$

$$T = -0.0937 \quad \text{N-m}$$

4-10 DOUBLY-EXCITED ELECTROMAGNETIC DEVICES

Many electromagnetic devices have more than one coil wound on their magnetic circuits, and it is desirable to find a general expression for the generated, magnetic force or torque acting on these devices. The foregoing sections revealed the existence of a Lorentz force acting on current-carrying conductors, and a reluctance force acting on the movable iron of an electromagnetic device. We shall now develop an expression for the force or torque that will predict the Lorentz as well as the Reluctance force acting on multiply-excited electromagnetic devices.

From the previous section consider the machine of Fig. 4.14 with a coil wound on the rotor.

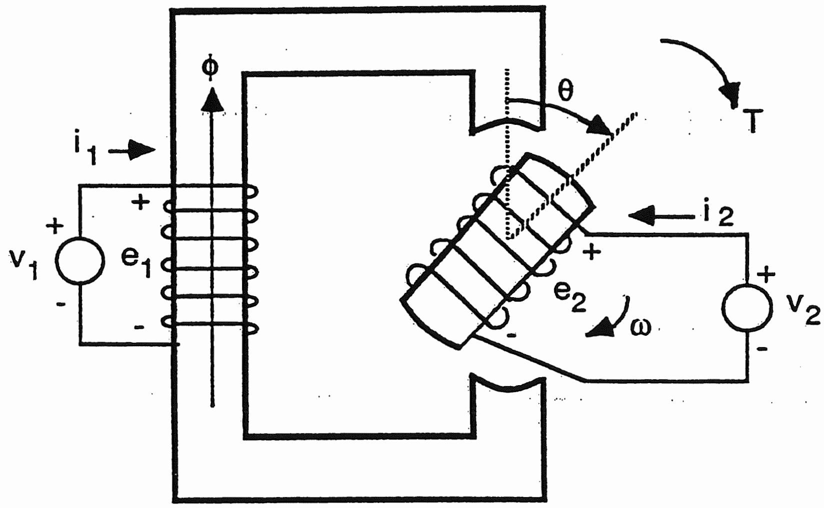


Figure 4.16 Doubly - Excited Device

In Fig. 4.16, there is the movable, unsymmetric iron of the rotor, as well as mutual coupling between the coils. As the rotor turns, Faraday emfs will be generated across each coil and the inductance of each coil will vary with θ because of the unsymmetric, rotor iron. For any value of θ, the flux linkage of each coil is,

$$\lambda_1 = L_{11}(\theta)\, i_1 + L_{12}(\theta)\, i_2$$

$$\lambda_2 = L_{21}(\theta)\, i_1 + L_{22}(\theta)\, i_2 \qquad \text{(Wb-t)} \qquad (4.60)$$

where, $L_{12} = L_{21}$

The emf generated across each coil is,

$$e_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} + i_1\frac{dL_{11}}{d\theta}\omega + i_2\frac{dL_{12}}{d\theta}\omega$$

$$e_2 = \frac{d\lambda_2}{dt} = L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} + i_1\frac{dL_{21}}{d\theta}\omega + i_2\frac{dL_{22}}{d\theta}\omega \qquad \text{(V)} \qquad (4.61)$$

The first two terms of each emf in Eqn. (4.61) are called "transformer emfs", since they vary with the time-rate of current. The second two terms are called "speed emfs", since they vary with the rotor angular velocity, $\omega = d\theta/dt$.

We shall now find a general expression for the torque acting on the rotor by realizing that a small change in electric energy delivered to both coils is partly stored in the magnetic field, and the remainder is delivered to the mechanical system.

$$dW_e = dW_f + T\,d\theta \qquad (J) \qquad (4.62)$$

The first term in Eqn. (4.62), $\underline{dW_e}$, is evaluated from the total electric power delivered to both coils,

$$\frac{dW_e}{dt} = e_1 i_1 + e_2 i_2 \qquad (W)$$

and using Eqn. (4.61),

$$\begin{aligned} dW_e &= L_{11}\, i_1\, di_1 + L_{12}\, i_1\, di_2 + i_1^2\, dL_{11} + i_1\, i_2\, dL_{12} \\ &\quad + L_{21}\, i_2\, di_1 + L_{22}\, i_2\, di_2 + i_1\, i_2\, dL_{21} + i_2^2\, dL_{22} \end{aligned}$$

or,

$$\begin{aligned} dWe &= L_{11}\, i_1\, di_1 + L_{12}\, i_2\, di_1 + L_{22}\, i_2\, di_2 + L_{12}\, i_1\, di_2 \\ &\quad + i_1^2\, dL_{11} + i_2^2\, dL_{22} + 2\, i_1 i_2\, dL_{12} \qquad (J) \qquad (4.63) \end{aligned}$$

The second term in Eqn. (4.62), $\underline{dW_f}$, is evaluated from the energy stored in the field,

$$W_f\,(i_1, i_2, \theta) = \frac{1}{2} L_{11}(\theta)\, i_1^2 + \frac{1}{2} L_{22}\,(\theta)\, i_2^2 + L_{12}(\theta)\, i_1 i_2 \qquad (J) \qquad (4.64)$$

Since the energy stored in the field is a function of three variables, the derivative of Eqn. (4.64) is taken first with respect to i_1, then i_2, and then θ,

$$\begin{aligned} dW_f &= L_{11}\, i_1\, di_1 + L_{12}\, i_2\, di_1 + L_{22}\, i_2\, di_2 + L_{12}\, i_1\, di_2 \\ &\quad + \frac{1}{2} i_1^2\; dL_{11} + \frac{1}{2} i_2^2\, dL_{22} + i_1 i_2\, dL_{12} \qquad (J) \qquad (4.65) \end{aligned}$$

When Eqns. (4.63) and (4.65) are substituted in Eqn. (4.62), the differential currents cancel on both sides of the equation and the magnetic torque acting on the rotor, for a linear system, becomes,

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta} \qquad (N\text{-}m) \quad (4.66)$$

The first two terms of Eqn. (4.66) are the reluctance torques that tend to minimize the reluctance of the unsymmetrical iron of the rotor and would be zero if the rotor were cylindrical. The third term always exists in a doubly excited device, and is the Lorentz torque that tends to align the magnetic fields of both coils.

The torque can then be expressed as,

$$T = \frac{\partial W_f(i_1, i_2, \theta)}{\partial \theta} \qquad \text{(N-m)} \qquad (4.67)$$

where,

$$W_f\,(i_1, i_2, \theta) = \frac{1}{2} L_{11}(\theta)\, i_1^2 + \frac{1}{2} L_{22}\,(\theta)\, i_2^2 + L_{12}(\theta)\, i_1 i_2 \qquad \text{(J)} \qquad (4.68)$$

Observe in Eqn. (4.66), the torque acting on this system is independent of differential changes in current, but depends entirely on the currents and the configuration of the system.

Also, for translational, doubly-excited devices, Eqn. (4.66) is valid if the variable, θ, is replaced by the linear displacement, x, and torque, T, is replaced by force, f.

The foregoing analysis can be extended to multiply-excited systems, for example, a triply-excited device, where the torque is,

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + \frac{1}{2} i_3^2 \frac{dL_{33}}{d\theta}$$

$$+ i_1\, i_2 \frac{dL_{12}}{d\theta} + i_2 i_3 \frac{dL_{23}}{d\theta} + i_3 i_1 \frac{dL_{31}}{d\theta} \qquad \text{(N-m)} \quad (4.69)$$

Example 4.10

1. In Fig. 4.16, the inductance of one coil is measured with the other coil open-circuited as,

(a) $L_1 = 1.0$ H at $\theta = 0°$; $L_1 = 0.5$ H at $\theta = 90°$ (coil 2 open)

(b) $L_2 = 10.0$ H at $\theta = 0°$; $L_2 = 5.0$ H at $\theta = 90°$ (coil 1 open)

2. The coils are now connected in series and the inductance is measured as,

$L = 12.5$ H at $\theta = 0°$; $L = 5.5$ H at $\theta = 90°$; $L = 9.5$ H at $\theta = 180°$

What is the magnetic torque acting on the rotor as a function of the coil currents?

Solution–

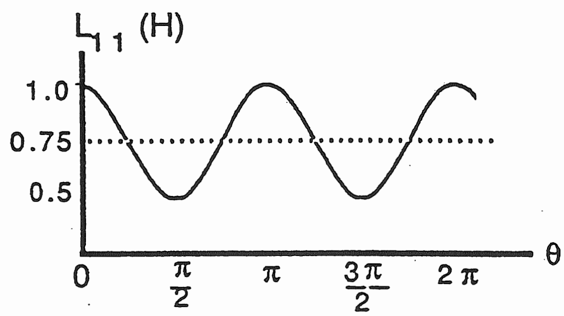
Realizing that for a sinusoidal drive on both coils,

$$V_1 = j\omega L_{11} I_1 + j\omega L_{12} I_2$$
$$V_2 = j\omega L_{21} I_1 + j\omega L_{22} I_2 \qquad (4.70)$$

From 1(a) and Eqn. (4.70),

$L_1 = L_{11} = 1.0$ H at $\theta = 0°$ and $L_{11} = 0.5$ H at $\theta = 90°$

Neglecting higher-ordered terms, L_{11} is assumed to vary sinusoidally with θ.

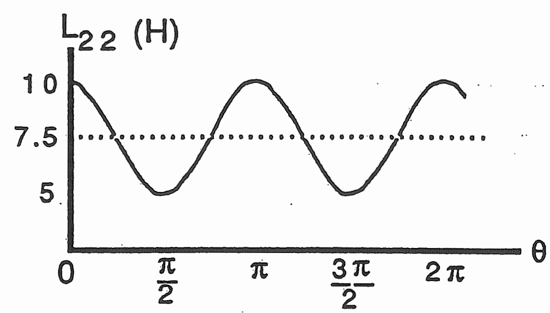


$L_{11} = 0.75 + 0.25 \cos 2\theta$ (H)

From 1(b) and Eqn. (4.70),

$L_2 = L_{22} = 10.0$ H at $\theta = 0°$ and $L_{22} = 5.0$ H at $\theta = 90°$

Neglecting higher-ordered terms, L_{22} is assumed to vary sinusoidally with θ.



$L_{22} = 7.5 + 2.5 \cos 2\theta$ (H)

From 2. and Eqn. (4.70),

$$L = L_{11} + L_{22} + 2L_{12}$$

$$\text{At } \theta = 0°, \qquad 12.5 = 1.0 + 10.0 + 2L_{12}$$

$$L_{12} = 0.75 \text{ H}$$

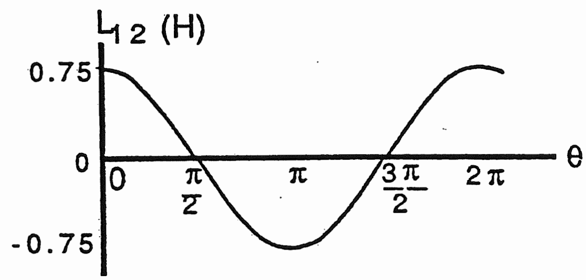
$$\text{At } \theta = 90°, \qquad 5.5 = 0.5 + 5 + 2L_{12}$$

$$L_{12} = 0\text{H}$$

$$\text{At } \theta = 180°, \qquad 9.5 = 1.0 + 10 + 2L_{12}$$

$$L_{12} = -0.75 \text{ H}$$

Neglecting higher-ordered terms, L_{12} is assumed to vary sinusoidally with θ.



$$L_{12} = 0.75 \cos \theta \quad \text{(H)}$$

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta}$$

$$= -\frac{1}{2} i_1^2 (0.25)(2) \sin 2\theta - \frac{1}{2} i_2^2 (2.5)(2) \sin 2\theta - i_1 i_2 (0.75) \sin \theta$$

$$T = -0.25\, i_1^2 \sin 2\theta - 2.5\, i_2^2 \sin 2\theta - 0.75\, i_1 i_2 \sin \theta \qquad \text{(N-m)}$$

The first two terms of the torque expression are the reluctance torques that tend to minimize the reluctance of the magnetic circuit in Fig. 4.16, and the third term is the Lorentz torque that tends to align the stator and rotor magnetic fields, thus maximizing the energy stored in the field.

4-11 SUMMARY

Any electromechanical, energy-conversion device can be represented by three blocks,

1. Electrical system
2. Coupling field
3. Mechanical system.

For a generator, the mechanical system is the source of energy and the electrical system is the sink of energy.

For a motor, the electrical system is the source of energy and the mechanical system is the sink of energy.

The two requisites for Lorentz-force, energy conversion are,

1. A magnetic field.
2. Current-carrying conductors capable of moving through this field.

Concomitant with these two requisites, energy conversion takes place only with the simultaneous generation of a Faraday emf and a Lorentz force.

When the conductor, field, and motion vectors are mutually perpendicular, a Faraday emf will be generated because of conductor motion,

$$E = B \ell u \qquad \text{(V)}$$

and a Lorentz force will be generated because of conductor current ,

$$f = B \ell I \qquad \text{(N)}$$

Generators are characterized by the fact that the driving force, or torque, determines the direction of motion; the Faraday emf is a forward emf and the Lorentz force is a back force.

Motors are characterized by the fact that the Lorentz force or torque determines the direction of motion; the Faraday emf is a back emf and the Lorentz force is a forward force.

The two requisites for Reluctance-force energy conversion are,

1. A magnetic field.
2. Movable iron in the path of the field.

Reluctance-force energy conversion is characterized by the fact that a magnetic force will act on all movable portions of the iron path so as to reduce path-reluctance to a minimum and increase the energy stored in the magnetic field to a maximum. Analysis of a Reluctance-force device is guided by the fact that a change in electric energy delivered to the device is both stored in the magnetic field and converted to mechanical energy,

$$\Delta W_e = \Delta W_f + \Delta W_m \qquad \text{(J)}$$

where,

$$\Delta W_e = e\, i\, \Delta t$$

$$\Delta W_f = i\, \Delta \lambda$$

$$\Delta W_m = f\, \Delta x = T\, \Delta \theta$$

From this guiding equation, for <u>singly-excited</u>, linear or nonlinear systems, the Reluctance-force or torque is,

$$f = -\frac{\partial W_f(\lambda,x)}{\partial x} = +\frac{\partial W_f(i,x)}{\partial x} \qquad \text{(N)}$$

$$T = -\frac{\partial W_f(\lambda,\theta)}{\partial \theta} = +\frac{\partial W_f(i,\theta)}{\partial \theta} \qquad \text{(N-m)}$$

where, for linear systems only,

$$W_f = \frac{1}{2}\lambda i = \frac{1}{2}L(x)\, i^2 = \frac{1}{2}\mathcal{R}(x)\, \phi^2 \qquad \text{(J)}$$

or,

$$W_f = \frac{1}{2}\lambda i = \frac{1}{2}L(\theta)\, i^2 = \frac{1}{2}\mathcal{R}(\theta)\, \phi^2 \qquad \text{(J)}$$

For <u>doubly- excited</u> , linear or nonlinear systems, the magnetic force or torque is,

$$f = \frac{\partial W_f(i_1,i_2,x)}{\partial x} \qquad \text{(N)}$$

$$T = \frac{\partial W_f(i_1,i_2,\theta)}{\partial \theta} \qquad \text{(N-m)}$$

where, for linear systems only,

$$W_f = \frac{1}{2}L_{11}(x)\, i_1^2 + \frac{1}{2}L_{22}(x)\, i_2^2 + L_{12}(x)\, i_1 i_2 \qquad \text{(J)}$$

or,

$$W_f = \frac{1}{2}L_{11}(\theta)\, i_1^2 + \frac{1}{2}L_{22}(\theta)\, i_2^2 + L_{12}(\theta)\, i_1 i_2 \qquad \text{(J)}$$

PROBLEMS

4.1 The translational generator in Fig. 4.2 has the following measured values:

$f_{drive} = 0.889$ (N)

$P_{f,wi} = 0.125\ P_i$

$u = 90$ (m/sec)

$R = 0.1$ (Ω)

$B = 1.0$ (T)

$\ell = 0.1$ (m)

Draw and label completely, with numerical values, the power flow diagram for this device.

4.2 The translational motor in Fig. 4.4 has the following measured values:

$I = 8.0$ (A)

$u = 90$ (m/sec)

$R = 0.1$ (Ω)

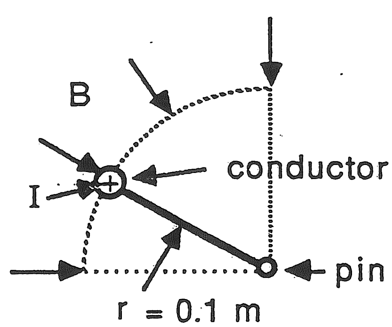
$B = 1.0$ (T)

$\ell = 0.1$ (m)

$P_{f,wi} = 0.125\ P_o$

Draw and label completely, with numerical values, the power flow diagram for this device.

4.3



A conductor, 20 cm is length, and resistance, 0.2 Ω is placed perpendicular to a uniform radial magnetic field of B = 1.0 T. The conductor carries a current, I = 5.0 A, into the page and is observed to move clockwise at constant tangential velocity, u = 50 m/sec. A voltmeter across the conductor measures 9.0 V. The conductor back force of friction and windage is 0.1 N.

a) What is the magnitude and direction of the generated Lorentz force?

b) What is the magnitude and direction of the generated Faraday emf?

c) Is this device a motor or generator? Why?

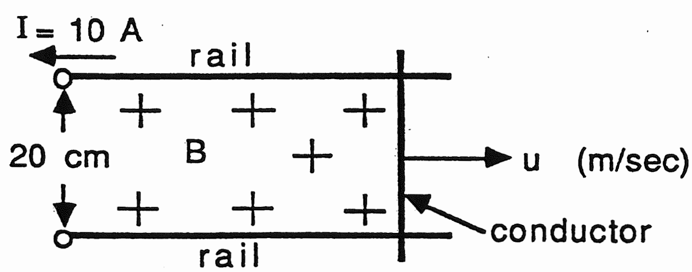
d) What is the input power, P_i, delivered by the source to this device?

e) What is the output power, P_o, delivered by the device to the sink?

f) What are the losses - P_{cu}, $P_{h,ec}$, $P_{f,wi}$?

4.4 Calculate Problem 4.3 for a reversed radial field of 1.0 T and a conductor carrying a current, I = 5.0 A, into the page, moving clockwise at constant tangential velocity = 40 m/sec. A voltmeter across the conductor measures 9.0 V, and the conductor back force of friction and windage is 0.1N.

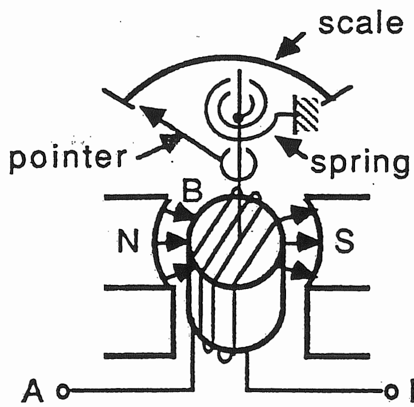
4.5



A conductor of resistance 0.3 Ω is sliding 25 m/sec along two conductive rails, each with negligible resistance, spaced 20 cm apart in a uniform magnetic field of 1.0 T. oriented into this page. The conductor back force of friction and windage is 0.1 N. Draw and completely label, with numerical values, the power-flow diagram for this device.

4.6 Do Problem 4.5 if the current is reversed with the same magnitude, all else remains the same.

4.7



A D'Arsonval meter movement is illustrated in the figure. The flux density, B = 1.0 T, is radial and uniform around the airgap. A 10-turn coil is wound on the cylindrical rotor whose diameter is 1.0 cm and length is 1.0 cm. The spiral spring exerts a back torque with a spring constant, $K_s = 1.6 \times 10^{-8}$ N-m/deg.

a) How many degrees of deflection will a current of 0.5 ma produce when flowing through the coil?

b) For upscale deflection, which terminal, A or B, is marked positive? Why?

4.8

a) In Problem 4.7, what coil current will produce a full-scale deflection of 100°?

b) What must be the direction of this current?

4.9 The rotational generator in Fig. 4.6 has a 100-turn coil, 0.01 Ω/turn, placed in a uniform, radial magnetic field of 1.0 T. The device constants are, r = 4 cm and ℓ = 10 cm. The mechanical losses are 2.0 % of the power input to this device.

a) If the machine is driven at 500 rad/sec, what current is drawn by a 39 Ω load placed across the brushes?

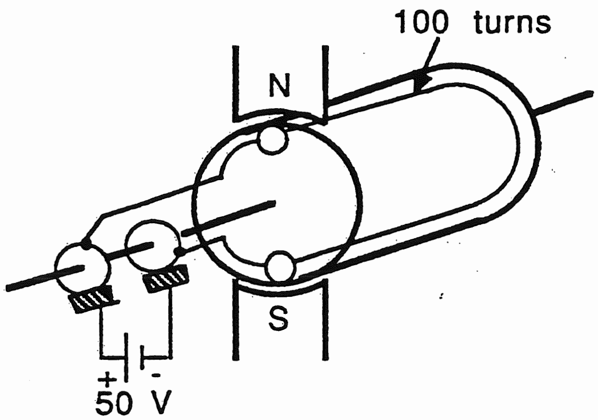
b) Draw and <u>completely</u> label, with numerical values, the power-flow diagram for this machine.

4.10 The rotational motor in Fig. 4.9 has the same description as the machine in Problem 4.9, except that the mechanical losses are 2.0% of the power output.

a) At what speed (rad/sec) will this machine rotate if a voltage source, V_t = 400 V, is placed across the brushes, and the motor is loaded with a load torque, T_{load} = 7.84 N-m.

b) Draw and <u>completely</u> label, with numerical values, the power-flow diagram for this machine.

4.11

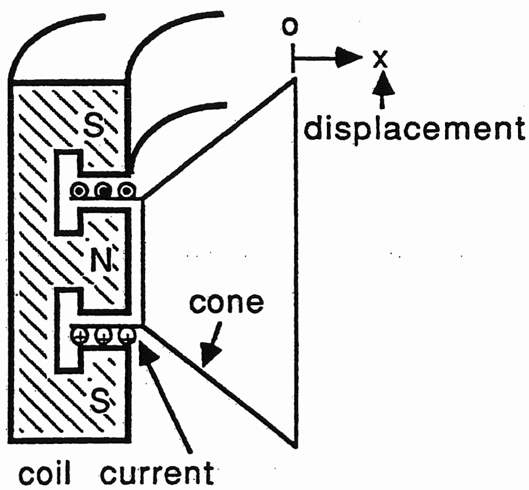


A permanent magnet motor is shown in the figure that establishes a uniform, radial flux density of 1.0 T in the air gap. A 100-turn coil with 0.5 Ω is placed on the rotor of length 12.7 cm, a diameter 12.7 cm, and is connected to slip rings as indicated.

a) What is the magnitude and direction of the torque acting on this coil?

b) Describe the motion of the rotor as pictured in the figure.

4.12



The cross-section of a cylindrical sound transducer is shown with a 20-turn coil, length per turn is 2.0 cm, wound on a cone placed in a uniform, radial magnetic field, B = 0.2 T. The positive directions of the coil current, i, and the cone displacement, x, are indicated on the figure. By measurement, the coil current is observed to be, at 1000 Hz,

$$i = 0.313 \cos \omega t \quad (A)$$

and the displacement is observed to be, at 1000 Hz,

$$x = 2 \times 10^{-5} \sin \omega t \quad (m)$$

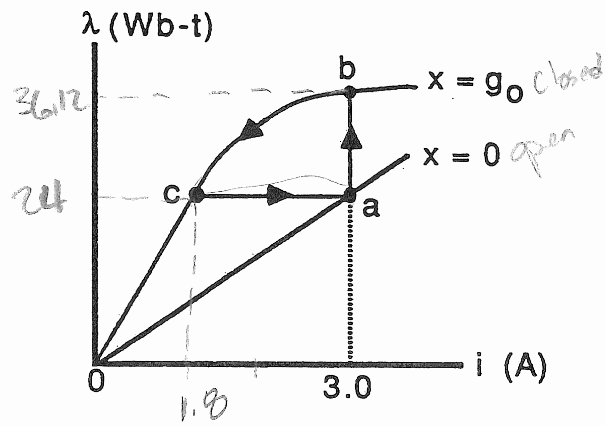
a) What is the Faraday emf generated in this coil?

b) What is the Lorentz force acting on the cone?

c) Is the transducer a loudspeaker or a microphone? Why?

d) Sketch the instantaneous power delivered to the coupling field for the first cycle of the displacement, x, as a function of time. Label all important magnitudes on both axes with numerical values.

4.13



In Fig. 4.11, the magnetization curves were measured, as shown above, at

armature open - $x = 0$: $\lambda = 8.0\, i$ (Wb-t)

armature closed - $x = g_0$: $\lambda = 15\, i^{0.8}$ (Wb-t)

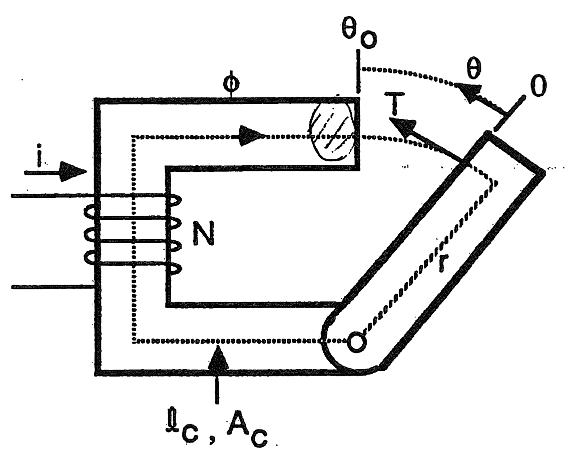
What is,

	ΔW_e	ΔW_f	ΔW_m	(J)?
a to b --------------				
b to c --------------				
c to a --------------				
b to c to a --------				

4.14 In Prob. 4.13, what is,

	ΔW_e	ΔW_f	ΔW_m	(J)?
a to c --------------				
c to b --------------				
b to a --------------				
b to a to c --------				

4.15



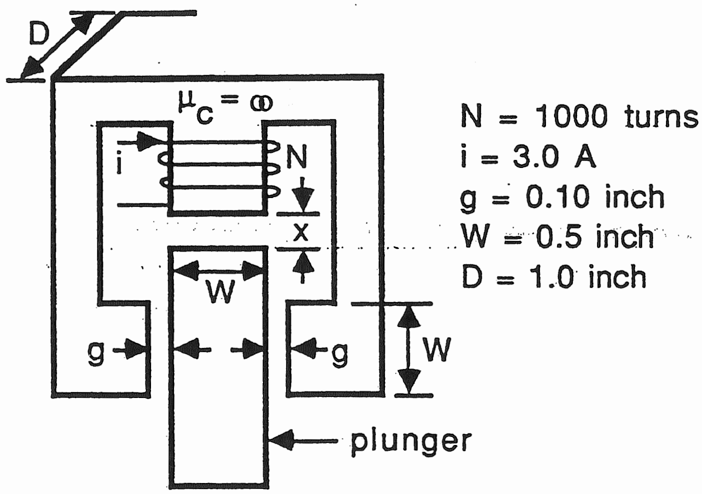
For the device shown, $\mu_c = \infty$,

a) What is the flux linkage of the coil, $\lambda(i,\theta)$?

b) What is the energy stored in the field, W_f (i, θ)?

c) What is the torque acting on the armature, $T(i,\theta)$?

4.16



The solenoid shown has rectangular cross-section with depth, D, and all gap fringing is negligible, $\mu_C = \infty$,

a) Plot, to scale, λ vs i, for plunger gaps, x = 0.10, 0.20, and 0.50 inches.

b) Calculate the energy, (J), stored in the field for each of the displacements in (a).

c) What is the expression for the coil inductance, L(x)? ; for W_f (i,x) ?

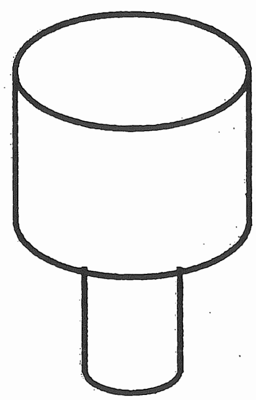
d) What is the expression for the energy stored in the field, $W_f(\lambda,x)$?

e) What is the force acting on the plunger, f(x), using part c)?

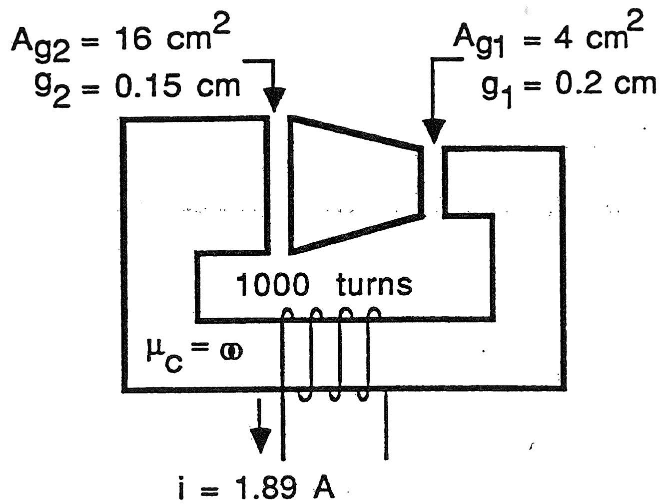
f) What is the force acting on the plunger, f(x), using part d)?

g) What is the force, (N), and its direction for x = 0.10, 0.20, 0.50 inches using parts e) and f)?

4.17 Do Problem 4.16, if the solenoid is cylindrical, as shown below, with cross-section shown in the figure of Problem 4.16.

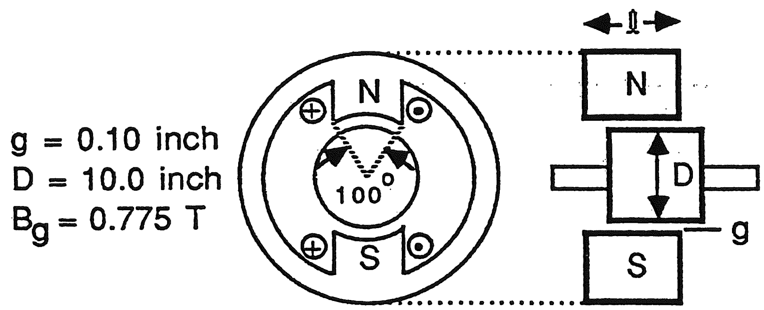


4.18



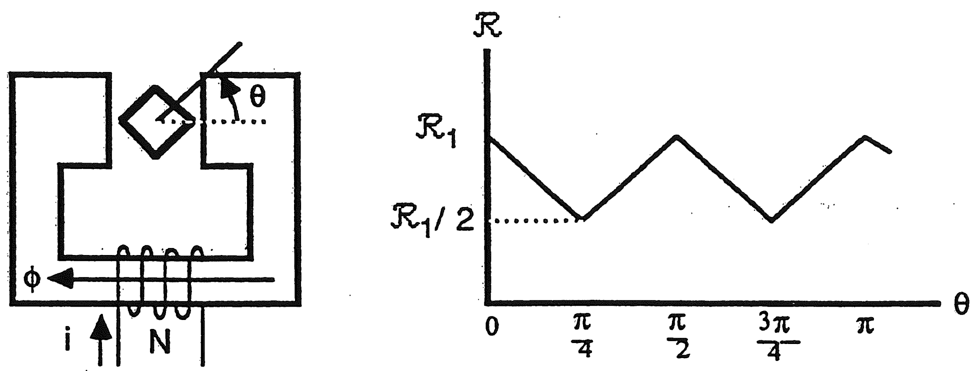
What is the force (N) acting on the movable slug in the figure? In what direction?

4.19



Because of thrust bearing wear, the armature of the two-pole machine in the figure is offset, axially, by 0.5". What is the force, (N), that will tend to align the armature with the pole pieces?

4.20

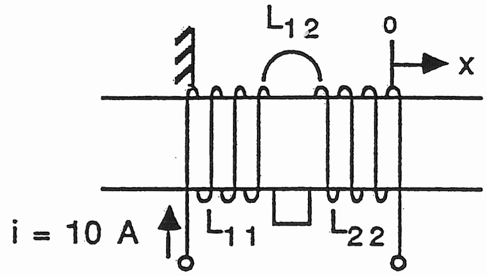


A reluctance motor has a square rotor and its path-reluctance, neglecting higher-ordered variation, is the triangular wave shown in the figure.

a) For a constant value of flux, ϕ, or mmf, write the expression for the torque acting on the rotor.

b) In a manner similar to section 4-9, tabulate the torque for critical ranges of θ, and summarize what you think happens to the rotor as it is moved.

4.21



$L_{11} = 1.0$ H

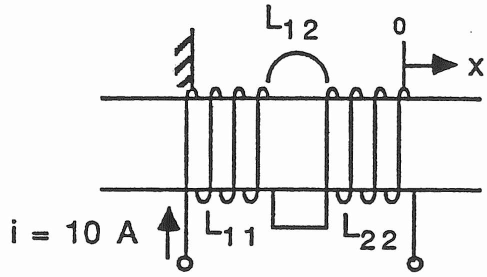
$L_{22} = 2.0$ H

$L_{12} = 1\text{-}x$ H

a) What is the force, (N), generated between these coils?

b) Will the coils move apart or together?

4.22



$L_{11} = 1.0$ H

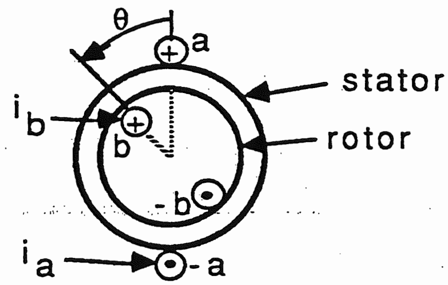
$L_{22} = 2.0$ H

$L_{12} = 1\text{-}x$ H

a) What is the force, (N), generated between these coils?

b) Will the coils move apart or together?

4.23



A machine has a coil, a-a, wound on the stator and a coil, b-b, wound on the rotor. The inductances of these windings are,

$$L_{aa} = 2.2 \quad (H)$$

$$L_{bb} = 1.0 \quad (H)$$

$$L_{ab} = \sqrt{2} \cos \theta \quad (H)$$

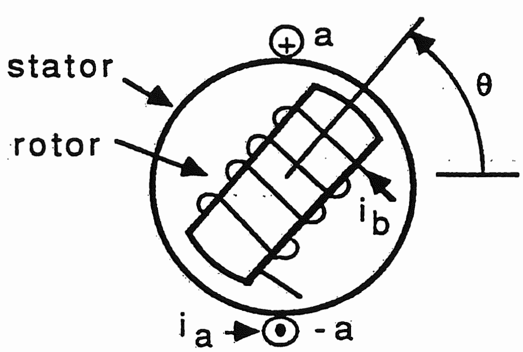
a) If i_a is constant at, 10 A, and i_b is constant at, −14.14 A, what is the expression for the torque acting on this rotor, $T(\theta)$? What happens to this rotor from its position indicated in the figure?

b) If $i_a = 14.14 \cos \omega t$ (A), and coil b is short-circuited (<u>induction motor</u>), what is the instantaneous torque expression, $T(\theta)$? Is this torque a reluctance or Lorentz torque?

c) In (b), what is the average torque at $\theta = 45°$?

d) In (b), what happens to this rotor from its position indicated in the figure?

4.24



This machine has a coil, a-a, wound on the stator and a coil, b-b, wound on a shaped rotor. The inductances of these windings are,

$$L_{aa} = 2.0 + 0.2 \cos 2\theta \quad (H)$$

$$L_{bb} = 1.0 \quad (H)$$

$$L_{ab} = \sqrt{2} \cos \theta \quad (H)$$

a) If i_a is constant at, 14.14 A, and i_b is constant at, 10 A, what is the expression for the torque acting on this rotor, $T(\theta)$? What happens to this rotor from its position indicated in the figure?

b) If $i_a = 14.14 \cos \omega t$ (A), and i_b is a direct current, $i_b = 10$A, (synchronous motor), what is the instantaneous torque expression $T(\theta)$? Which portion is the reluctance torque and which is the Lorentz torque?

c) In (b), what is the average torque at $\theta = 45°$?

d) In (b), what happens to this rotor from its position indicated in the figure?

e) Under what conditions will this rotor rotate continuously?

4.25.

The translational motor in Fig. 4.4 can be considered as a doubly excited device with a coil of N_1, turns wound on the magnetic circuit (not shown), and a movable coil, $N_2 = 1$ turn in the airgap. Realize that the magnetic field created by the movable coil is quite small, and that gap fringing is neglected.

a) What is the expression for the energy stored in the magnetic field, $W_f(i, i_2, x)$?

b) Show that the general force expression will predict the Lorentz force acting on the movable conductor,

$$f = \frac{\partial W_f(i_1, i_2, x)}{\partial x} = B \ell I \qquad (N)$$

Hint: Divide the total flux, ϕ, in the magnetic circuit into two parts, (Chapter 2, Fig. 2.17), one part that links both coils (mutual flux), and the other part that does not link each coil, (leakage flux).

CHAPTER 5

SYNCHRONOUS GENERATOR

To meet the enormous demand for electric power in our nation, an extensive, interconnected power grid has evolved. This grid consists of thousands of the components shown in the elementary power system of Fig. 5.1.

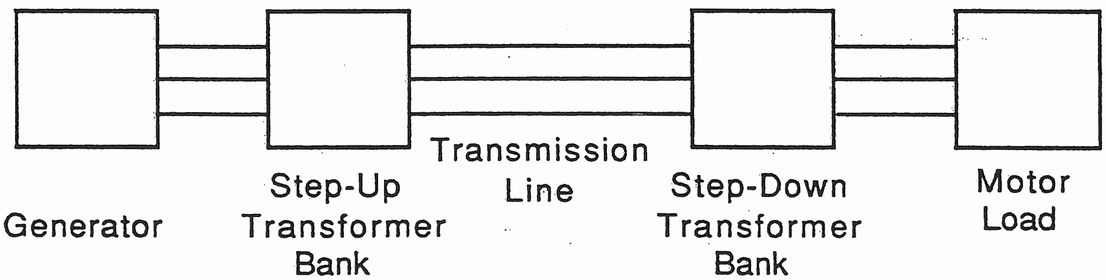


Figure 5.1 Elementary Power System

For a typical power system, one of many in the interconnected power grid, the load consists primarily of electric motors, which are energy converters, as is the synchronous generator in Fig. 5.1. Since electric power cannot be stored in existing power systems, the generator must supply, on demand, the power required by the system components as the demand varies each day. A watt-var balance, then, must be achieved at each instant of time, and the system must remain reliable and stable during this energy exchange. It is the purpose of this chapter to describe and model the synchronous generator and its adjustment, at each instant of time, to achieve this watt-var balance.

Synchronous machines are used in present-day power systems as generators of large blocks of power, or as motors in constant-speed load applications. The synchronous generator, often called an alternator, is the primary source of electrical energy, since it is capable of delivering large blocks of power, and will continue to be the primary source in the foreseeable future.

It is available with widely different ratings from a small, portable, gasoline-engine driven, single-phase machine of 500 watts to a large, steam-turbine driven, three-phase machine of 1000 or more megawatts.

The synchronous generator converts mechanical energy to electrical energy where the mechanical energy is obtained from its prime mover. Prime movers consist of gasoline or diesel engines, or gas, or water, or steam turbines. The prime movers, in turn, convert energy in raw form to mechanical energy. Energy in raw form consists of the fossil fuels - gas, oil and coal or nuclear fuel, or the energy stored in water held behind a river dam. While this scheme of energy conversion may seem unnecessarily extensive, literally billions of dollars have been invested in it, since it is practical, and satisfies the enormous demand for electrical power of our society.

While all of the above-described synchronous machines have the same model and analysis, attention will be focused on the large, steam-turbine driven generators of large blocks of power. A simplified diagram of one of these generators is given in Fig. 5.2.

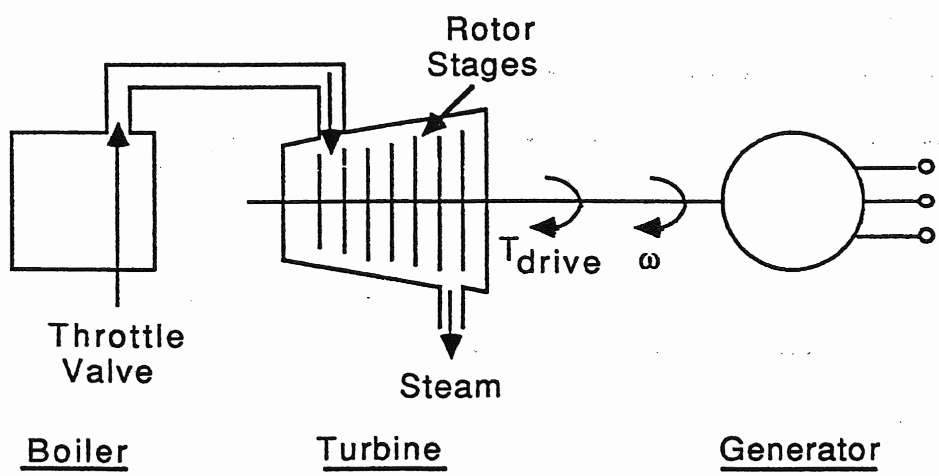


Figure 5.2 Steam-Turbine Driven Synchronous Generator

Figure 5.2 becomes important in the later analysis of this machine. The steam generated in the boiler is controlled by a throttle valve and is fed to nozzles on each stage of the turbine rotor. Each turbine-rotor stage consists of concave blades or buckets, on which the steam impinges, generating a drive torque on the shaft. The steam is then condensed and fed back to the boiler. The machine then rotates at a steady-state speed, synchronous to the frequency of the line to which it is connected - hence the name, synchronous machine.

5-1 SYNCHRONOUS MACHINE CONSTRUCTION

The synchronous machine is a cylindrical device as shown in Fig. 5.3.

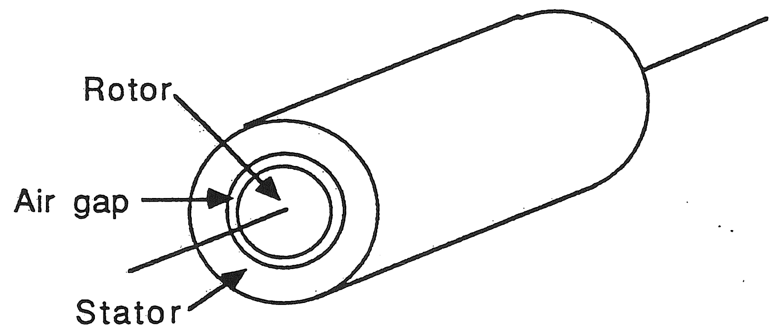


Figure 5.3 Synchronous Machine Construction

The two requisites for energy conversion are available in the machine of Fig. 5.3. An air-gap field, called the rotor field, originates on the rotor and rotates past stationary conductors on the stator, generating a Faraday emf in the stator conductors. For a given load, currents will flow in the stator conductors that produce a rotating stator field which interacts with the rotor field producing a Lorentz torque on the rotor. This, very briefly, describes the energy-conversion process of the machine in Fig. 5.3, and will be considered in detail in subsequent sections. Since the analysis of this machine and its mathematical model are better understood from its physical construction, the windings on the rotor and stator will be closely examined.

5-2 ROTOR FIELD

Two types of rotors are used in synchronous machines - the cylindrical rotor and the salient-pole rotor. For the purposes of this introductory text, only the cylindrical rotor will be considered, and the machine will be assumed well-designed, i.e., very little of the winding mmfs are dropped along the iron portions of the flux paths and are, therefore, dropped across the air gaps, thus creating the air-gap fields to be described. The cylindrical rotor and its excitation are shown in Fig. 5.4.

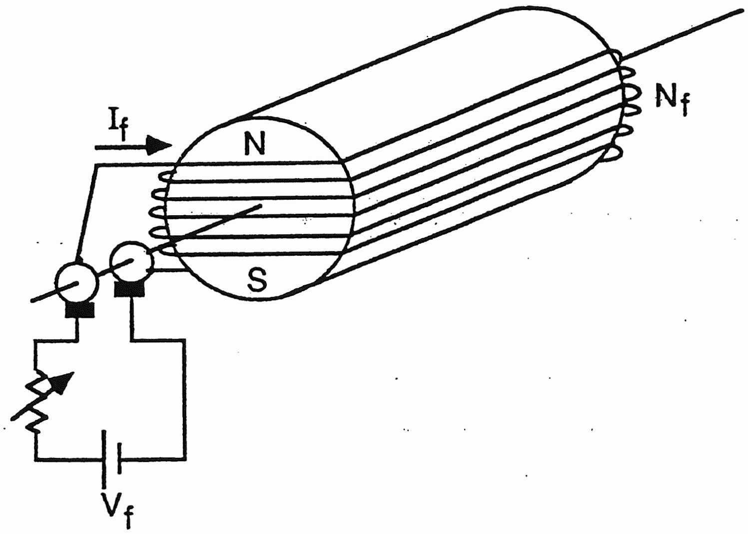


Figure 5.4 Synchronous Machine Cylindrical Rotor

A rotor winding of N_f - turns is placed in slots cut axially on the surface of a high-permeability, laminated, iron cylinder. The coil terminals are connected to slip rings, allowing rotary motion and continuous current flow. Connected across the brushes is a rheostat and dc voltage source. The dc source is a disadvantage for this machine, since this source is not generally available in ac power systems, however, large machines include a dc generator, called an exciter, on the same shaft with the rotor. The dc field-current in Fig. 5.4 produces an mmf creating north and south poles, external to the winding, as indicated, and this air-gap field is called the rotor field which rotates with the rotor. Facing the slip rings in Fig. 5.4, a cross-section of a two-pole rotor is shown in Fig. 5.5, together with the distribution of flux density that exists in the airgap for each value of θ.

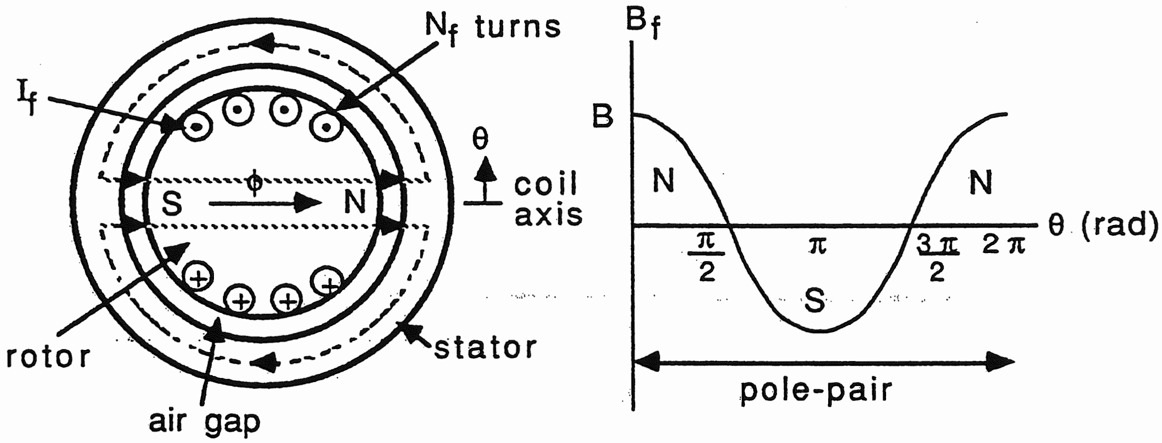


Figure 5.5 Two-Pole Rotor Magnetic Field

The cross-section of the rotor is placed in a polar coordinate system and the variable, θ, is measured, around the air-gap, from the coil axis which lies halfway between the coil sides. The magnitude of the flux density is assumed positive over half of the airgap where flux exits from the rotor (North pole), and negative over the other half of the air-gap where the flux enters the rotor (South pole), as is indicated in the figure. Observe that one pole-pair per revolution exists for a two-pole machine. There would be two pole-pairs per revolution for a four-pole machine, three pole-pairs per revolution for a six-pole machine, etc. It is clear, then, that there are,

$$\frac{P}{2}\ \frac{\text{pole-pairs}}{\text{revolution}} \tag{5.1}$$

for a machine with P-poles, where P is an even integer.

The rationale for the cosinusoidal distribution of flux density in Fig. 5.5 is given in Fig. 5.6, first for a rotor with N_f-turns concentrated in one pair of slots, and then for the rotor in Fig. 5.5.

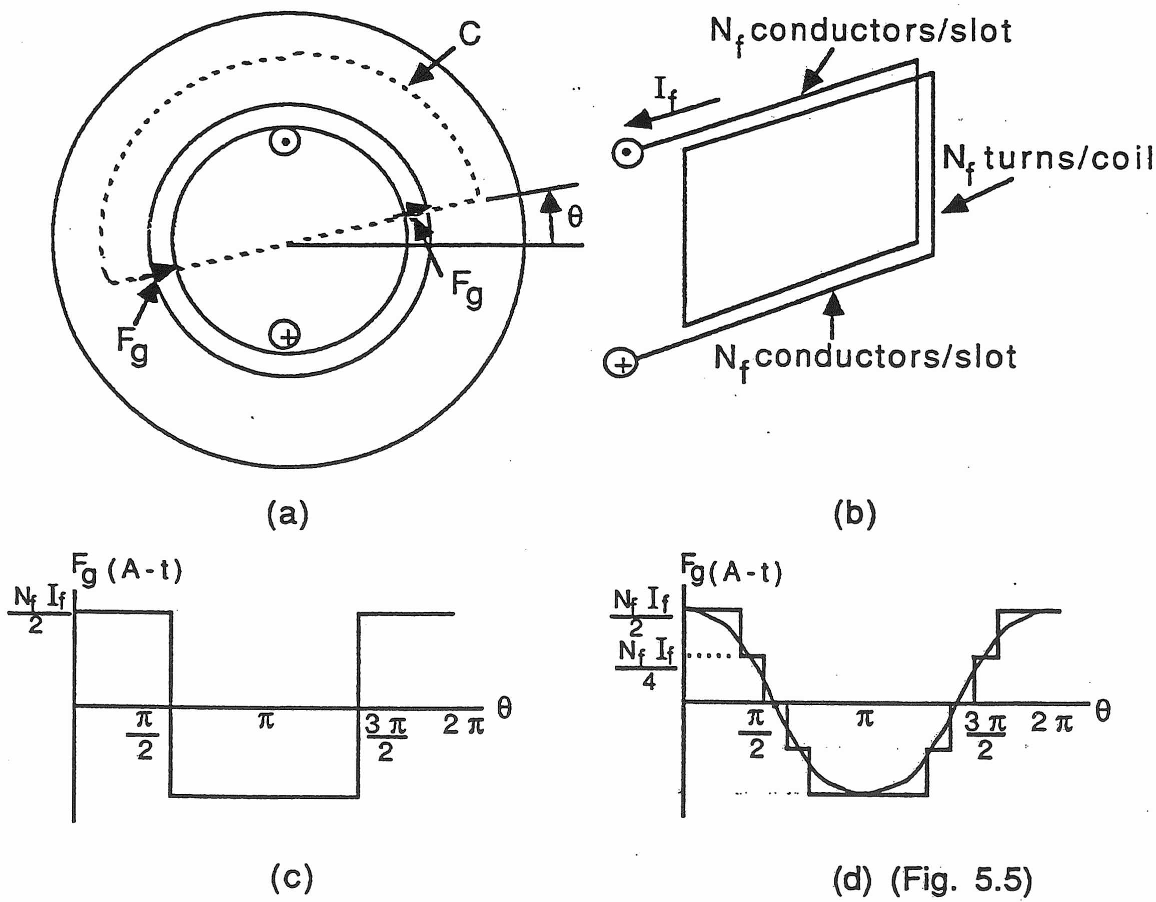


Figure 5.6 Rationale for Figure 5.5

The mmf dropped across the air-gap, F_g, for each value of θ, will now be determined. In Fig. 5.6 (b), an N_f-turn coil is concentrated in the slot-pair of Fig. 5.6 (a). It follows, therefore, there are N_f-conductors, <u>each</u> carrying I_f (A), in <u>each</u> slot. Maxwell's mmf law is true for any contour, C, bounding a surface, S. Taking advantage of symmetry in Fig. 5.6 (a), a semicircular contour, C, is chosen. The current enclosed, as a rise in mmf, is then dropped around the contour as,

$$N_f\, I_f = \Sigma\, \cancelto{0}{H_c l_c} + 2\, F_g \qquad \text{(A–t)} \qquad (5.2)$$

Since the machine is well-designed, the mmf dropped along the iron portions of the contour shown in the Figure, is negligible, so that the mmf dropped across <u>each</u> gap is,

$$F_g = \frac{N_f\, I_f}{2} \qquad \text{(A-t)} \qquad (5.3)$$

Different contours are chosen as the semicircle is rotated through an angle, θ; the mmf enclosed for each contour is evaluated, and the mmf dropped across each gap is plotted in Fig. 5.6 (c).

For the rotor in <u>Fig. 5.5</u>, the gap mmf, for each value of θ, is plotted in Fig. 5.6 (d) and is a staircase distribution. Observe, that for many rotor slots, as in Fig. 5.5, the staircase distribution approaches a cosine wave, so that,

$$F_g\,(\theta) = \frac{N_f\, I_f}{2} \cos\theta \qquad \text{(A-t)} \qquad (5.4)$$

The air-gap, magnetic intensity for each value of θ, is, then,

$$H_g\,(\theta) = \frac{F_g(\theta)}{g} = \frac{N_f\, I_f}{2g} \cos\theta \qquad \text{(A–t/m)} \qquad (5.5)$$

and the air-gap flux density for each value of θ, is,

$$B_g(\theta) = B_f = \mu_o\, H_g\,(\theta) = \frac{\mu_o N_f\, I_f}{2\,g} \cos\theta = B\cos\theta \quad \text{(T)} \quad (5.6)$$

where, Eqn. (5.6) is plotted in Fig. 5.5.

Observe, in Eqn. (5.6), the magnitude of the flux density distribution is,

$$B = \frac{\mu_o N_f}{2g} I_f \qquad (T) \qquad (5.7)$$

The magnitude of the flux density distribution is a function of the field current which can be varied by the rheostat in Fig. 5.4. Related to this magnitude, and of considerable importance in later analysis, is the total flux that passes through each pole. In Fig. 5.5, consider the total flux that passes through the cylindrical air-gap area of the North pole.

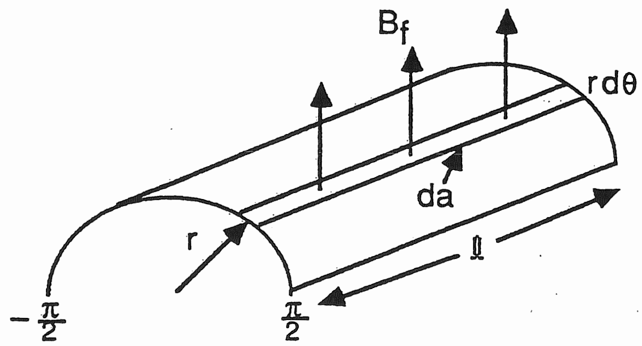


Figure 5.7 Rotor Surface of Integration

Since the width of the air-gap, g, is small, the total flux per pole, Φ_f ,is found by integrating the flux density over the surface of the rotor, of radius, r- meters and axial length, ℓ- meters, from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ radians, where B_f is constant along axial length, ℓ, but varies consinusoidally with θ.

$$\Phi_f = \int B_f \, da = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} B \cos\theta \, \ell \, r \, d\theta = 2B \, \ell \, r \qquad \text{(Wb/pole)} \qquad (5.8)$$

Substituting Eqn. (5.7) in Eqn. (5.8),

$$\Phi_f = \frac{\mu_o N_f \ell \, r}{g} I_f \qquad \text{(Wb/pole)} \qquad (5.9)$$

The flux per pole, as well as the magnitude of the flux-density distribution, is a function of the field current, and is plotted in Fig. 5.8.

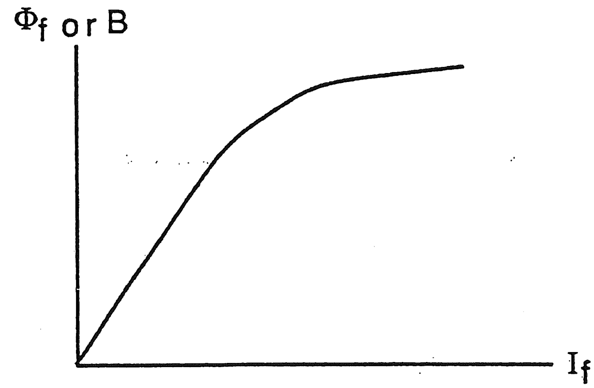


Figure 5.8 Synchronous Machine Magnetization Curve

In Fig. 5.8, as long as the machine iron is not saturated, Eqn. (5.9) holds, and the flux increases linearly with I_f. Beyond this region the iron quickly saturates with a resultant small increase in flux. In summary, the rheostat in Fig. 5.4 varies the field current, which, in turn, varies the magnitude of the flux-density distribution in Fig. 5.5, or, Φ_f, from zero to saturation, beyond which there is little change. Since I_f is one of the two variables for adjusting this machine, this is an important physical observation, useful in later analysis.

The rotor field, then, is a sinusoidally distributed waveform of air-gap, flux density that rotates with the rotor, at rotor speed, around the air-gap, and its magnitude can be varied from zero to saturation by varying the field current with the rheostat in the excitation circuit.

5-3 STATOR WINDINGS

The current-carrying conductors, called the armature, are placed, stationary, on the stator of this machine, because, for large machines, they could carry hundred or thousands of amperes which would be impossible to carry by brushes and slip rings on the rotor.

Three, N-turn windings are placed in slots cut axially along the inside surface of the stator annular- cylinder, as shown in Fig. 5.9.

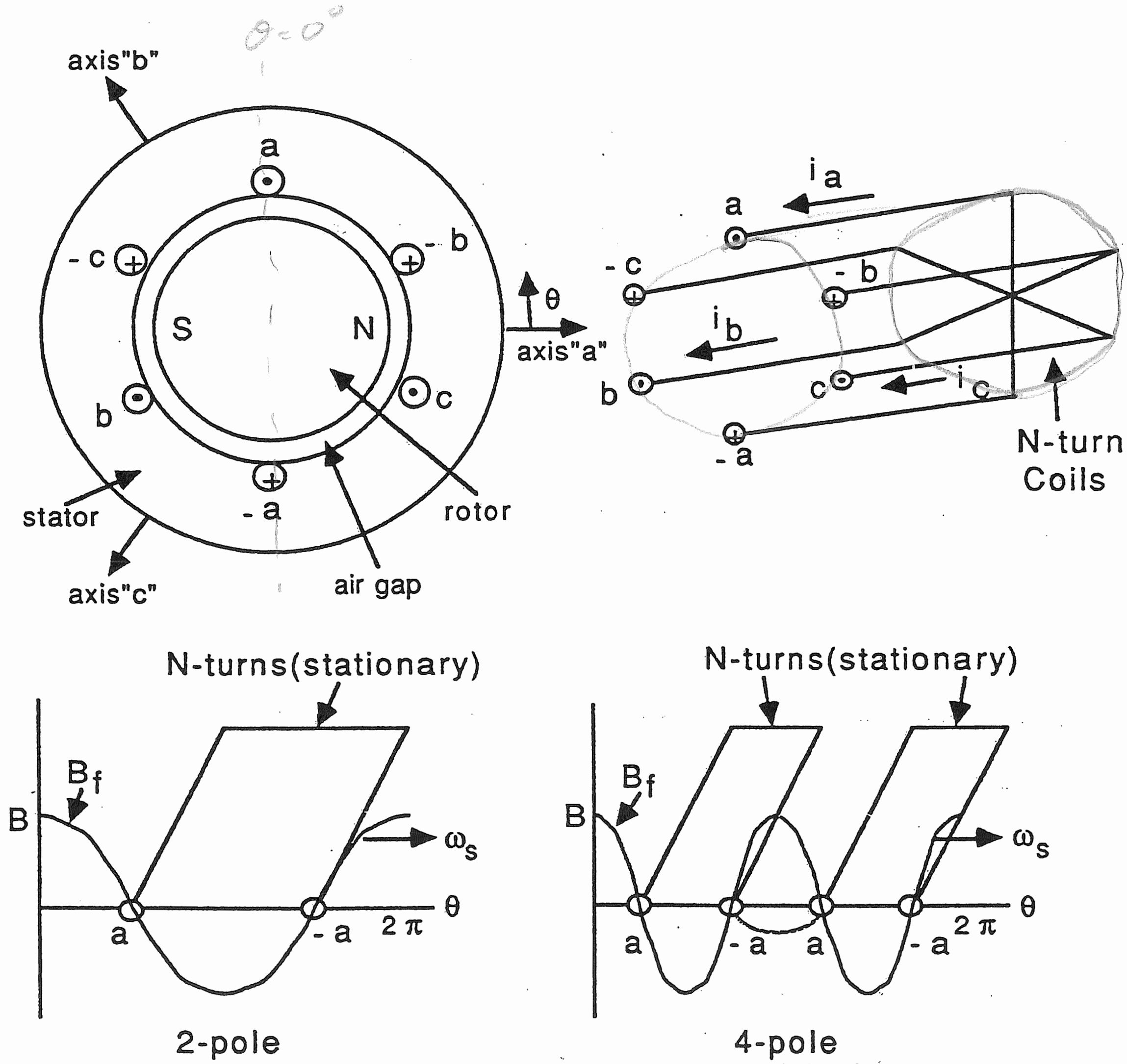


Figure 5.9 Stator Windings of a Synchronous Machine

The three windings are concentrated in three pairs of slots as shown in the Figure, for the two-pole machine, and constitute the three phases a, b and c of this machine. With the assumed positive directions of currents indicated, using the right-hand rule, the axes of these windings are shown 120° apart. For the remainder of this chapter, the variable, θ, will always be measured, around the air gap, from the axis of the a winding. In the lower portion of the Figure, the revolving rotor field distribution is shown relative to the stationary stator windings for a two and four pole machine at the instant of time indicated. Observe, for the four-pole machine, the two, stationary, N-turn windings for phase a are linked simultaneously by the same rotor field, so the emfs generated in each of these windings are in time phase with the result that both windings are connected in series and act as one winding.

The symbolic diagram for this machine is shown in Fig. 5.10 with the windings connected in wye, on their respective axes, thus providing a physical neutral that can be grounded for safety and other reasons.

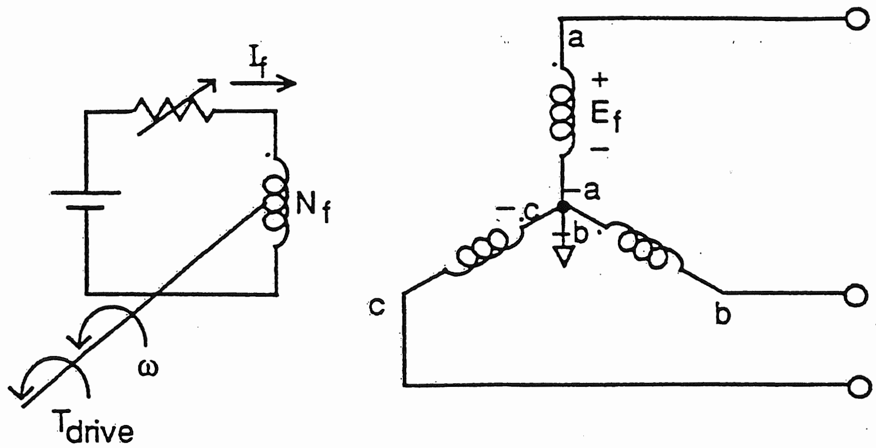


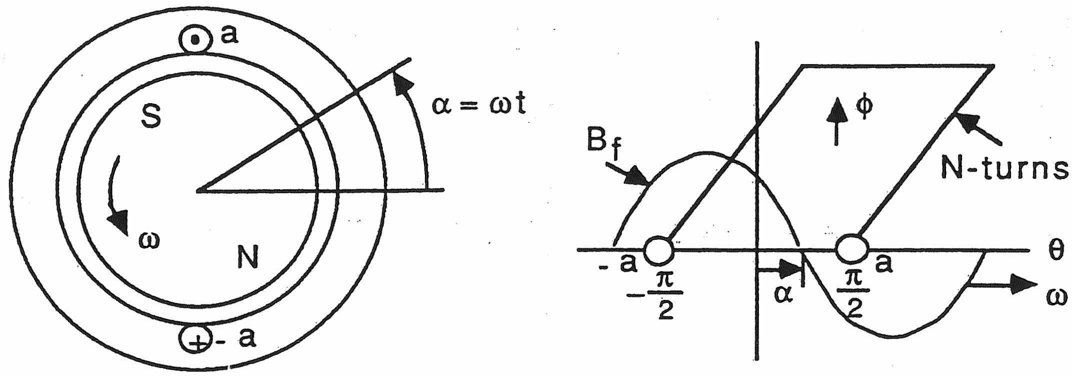
Figure 5.10 Symbolic Diagram of Synchronous Generator

5-4 STATOR GENERATED EMF

In Fig. 5.10, the rotor is driven, with constant velocity, ω, rad/sec, with a prime mover whose torque is T_{drive} (N-m). As the rotor turns, its sinusoidally-distributed field moves past the stationary, stator windings a, b, and c. The flux linkages with each winding then change with time and an emf will be generated in each winding, according to Faraday's law,

$$e = \frac{d\lambda}{dt} \qquad \text{(V)} \qquad (5.10)$$

The flux linkages with winding a will now be examined, keeping in mind that the analysis for windings b and c is identical, except that the generated emfs will occur, successively, 120° later in time phase. Time is arbitrarily chosen at the instant when the rotor field links winding a as in Fig. 5.11.



$$B_f = - B \sin(\theta - \alpha)$$

Figure 5.11 Rotor Flux Linkages with Stator Winding "a"

The angle, α, in Fig. 5.11, is measured to the zero-crossover of the <u>moving</u> rotor field and increases as, ωt (rad). When $\alpha = \pi/2$ rad, or 90°, the flux piercing the cylindrical area of winding <u>a</u>, or the linkage with winding <u>a</u>, is a maximum. When $\alpha = 0$ rad, or 0°, the linkage with winding <u>a</u> is zero. The variation of flux linkage with winding <u>a</u> is then found by referring to Fig. 5.7, and integrating the flux density over the cylindrical area of winding <u>a</u> for any value of α, and then multiplying by N, the number of times the flux pierces the coil surface,

$$\phi = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} - B \sin(\theta - \alpha)\, \ell\, r\, d\theta = B\, \ell\, r \Big[\cos(\theta - \alpha)\Big]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

$$\phi = 2 B \ell r \sin\alpha \quad \text{(Wb)} \qquad \text{where, } \alpha = \omega t$$

using Eqn (5.8), $\lambda = N\phi = N2B\ell r \sin\alpha = N\Phi_f \sin\omega t$ (Wb-t) (5.11)

and, Eqns. (5.10), (5.11) are plotted in Fig. 5.12.

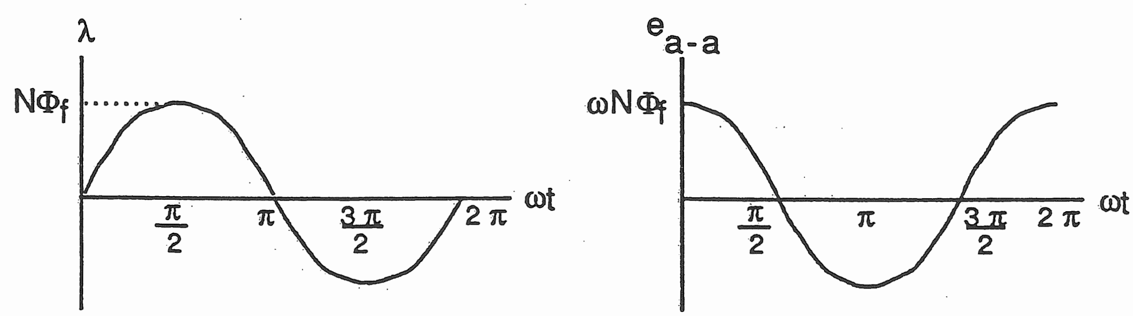


Figure 5.12 Flux Linkage and Generated Emf of Winding "a"

As the rotor turns through one revolution from $\alpha = 0°$, the flux linkages with winding a increase from zero to a positive maximum and then through zero to a negative maximum, returning to zero at $\alpha = 360°$. The generated emf, which is the time derivative of the λ - variation, is,

$$
\begin{aligned}
e_{a\text{-}a} &= \frac{d\lambda}{dt} = \omega N \Phi_f \cos \omega t \quad (V) \\
e_{b\text{-}b} &= \qquad = \omega N \Phi_f \cos (\omega t - 120°) \quad (V) \qquad (5.12) \\
e_{c\text{-}c} &= \qquad = \omega N \Phi_f \cos (\omega t + 120°) \quad (V)
\end{aligned}
$$

The windings b and c are also linked, simultaneously, by the rotor field and since their axes are displaced in space by 120°, the rotor field links them later in time phase by 120°, as indicated in Eqns. (5.12). The rms value of Eqns. (5.12) is,

$$E_f = \frac{\omega N \Phi_f}{\sqrt{2}} = \frac{2\pi f N \Phi_f}{\sqrt{2}} = 4.44 f N \Phi_f \quad (V) \qquad (5.13)$$

The rotor field, then, induces a balanced set of emfs in winding a, b, and c, called excitation emfs, whose rms values can be varied with the rotor field current, I_f, as shown in Eqn. (5.9) , and Fig. 5.8. The frequency, f (Hz), of the generated emfs in Eqn. (5.13), is established by realizing that whenever a pole-pair sweeps by a winding in space, a cycle of emf is generated in the winding, i.e.,

$$\underset{\frac{rad}{sec}}{\omega} = \underset{\frac{rev}{min}}{n} \times \underset{\frac{pole\text{-}pair}{rev}}{\frac{P}{2}} \times \underset{\frac{rad}{pole\text{-}pair}}{2\pi} \times \underset{\frac{min}{sec}}{\frac{1}{60}} = 2\pi f \qquad (rad/sec)$$

or,

$$n_s = \frac{120 f}{P} \qquad (rpm) \qquad (5.14)$$

The rotor speed (rpm) is tied irrevocably to the frequency of the generated emfs in the stator windings and, therefore, this speed is called synchronous speed for the synchronous machine. If the generated frequency is 60 Hz, the rotor speed must be,

3600 rpm – 2 -pole machine

1800 rpm – 4-pole machine

etc.

5-5 STATOR ROTATING FIELD

The symbolic diagram for a loaded synchronous generator, driven by a prime mover, at synchronous speed corresponding to voltage-frequency, f, is now given in Fig. 5.13.

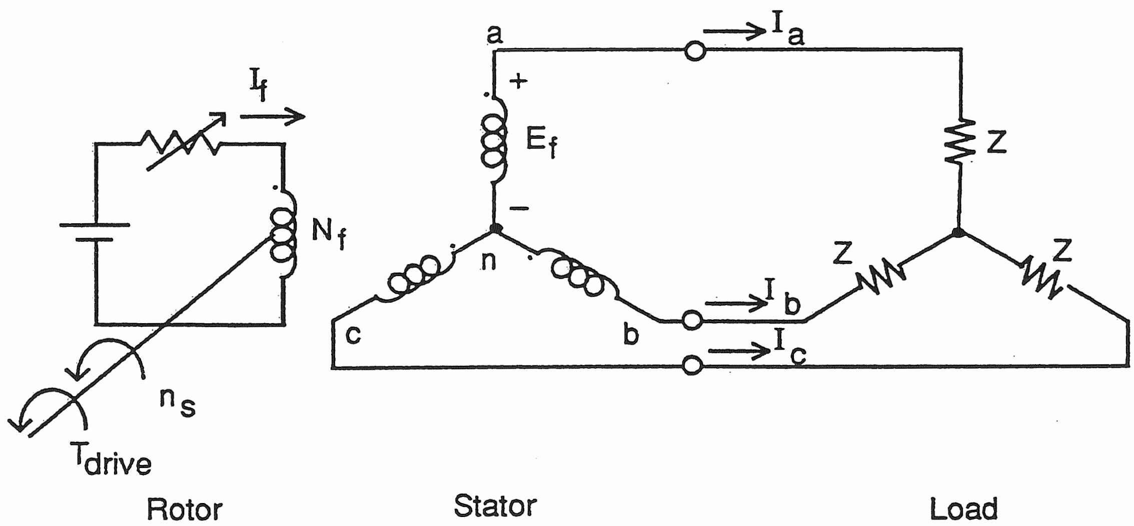


Figure 5.13 Symbolic Diagram of Loaded Synchronous Generator

To this point, the only rotating field in the air-gap is the rotor field. As a consequence of this rotating field, a balanced, excitation emf, E_f, is generated in each phase winding as indicated in Fig. 5.13, whether the machine is loaded or unloaded (open-circuit).

When the machine is loaded with a balanced wye or delta-connected load, three balanced line currents, I_a, I_b, and I_c will flow, because they are driven by the three balanced excitation emfs given in Eqn. 5.12. We now have another set of distribution of currents in space on the stator, similar to the rotor distribution, as indicated in Fig. 5.14.

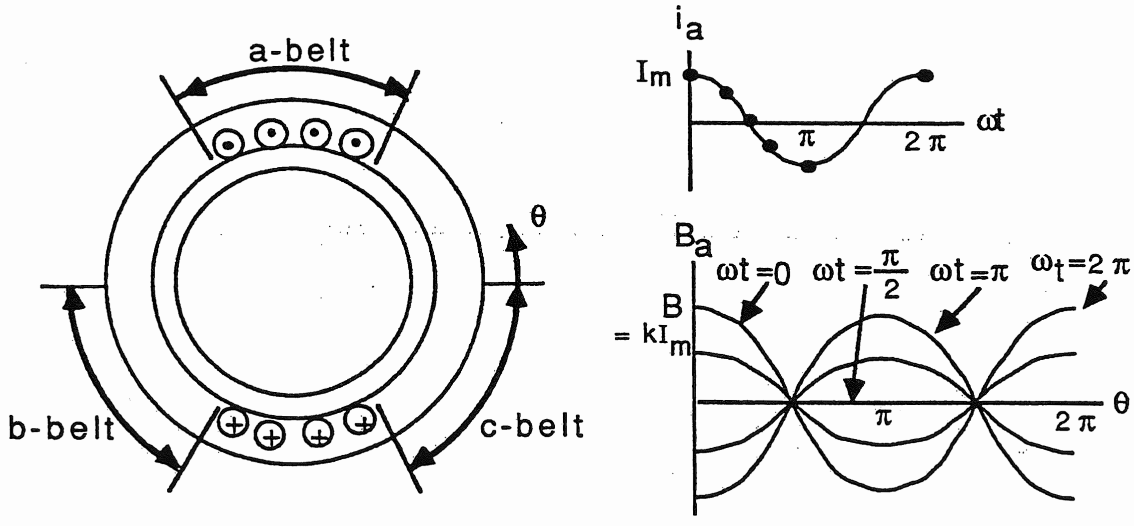


Figure 5.14 Stationary, Pulsating Stator Field of Winding "a"

In a practical synchronous machine, the stator windings a, b and c are not concentrated in one set of slots, rather, they are distributed over the slots in the a, b and c belts. In Fig. 5.14, only the a winding, with its assumed positive direction of current flow, is shown for clarity. The independent variable, θ, is always measured from the axis of the a stator winding. The independent variable, t, is measured from the instant current in winding a is a maximum as indicated in the upper right diagram in Fig. 5.14. The three stator load currents are, then,

$$i_a = I_m \cos \omega t \quad \text{(A)}$$

$$i_b = I_m \cos (\omega t - 120°) \quad \text{(A)} \qquad (5.15)$$

$$i_c = I_m \cos (\omega t + 120°) \quad \text{(A)}$$

Once again, the stator field, produced by the load current, i_a, flowing through the stator winding of N turns, is predicted by Maxwell's mmf law and a semicircle contour in a manner similar to Fig. 5.6. The sinusoidally distributed field of stator winding a is similar to Eqn (5.6) and is,

$$B_a = (k\, i_a) \cos \theta \quad \text{(T)}$$

$$B_b = (k\, i_b) \cos (\theta - 120°) \quad \text{(T)} \qquad (5.16)$$

$$B_c = (k\, i_c) \cos (\theta + 120°) \quad \text{(T)}$$

Equations (5.16) follow since the windings b and c are delayed in space by ± 120°.

The magnetic field produced by the load current, i_a, in stator winding a is shown in the lower right diagram of Fig. 5.14. Observe that the fields of all three windings are always distributed cosinusoidally around the air-gap, but their magnitudes (in parentheses) depend on their currents at given instants of time. A point of constant phase (angle) remains stationary, as indicated in Fig. 5.14, and therefore these fields are stationary pulsating waveforms around the air-gap. If Eqns. (5.15) are substituted in Eqns. (5.16),

$$B_a = kI_m \cos\omega t \cos\theta \qquad = B \cos\omega t \cos\theta \qquad (T)$$

$$B_b = kI_m \cos(\omega t - 120°)\cos(\theta - 120°) = B\cos(\omega t - 120°)\cos(\theta - 120°) \qquad (T) \qquad (5.17)$$

$$B_c = kI_m \cos(\omega t + 120°)\cos(\theta + 120°) = B\cos(\omega t + 120°)\cos(\theta + 120°) \qquad (T)$$

The armature or load currents of Eqns. (515) flowing through the stator windings a, b and c produce the three stationary, pulsating fields of Eqns. (5.17), the sum of which, for an unsaturated machine, is the stator field. Using trigonometric identity,

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha + \beta) + \frac{1}{2}\cos(\alpha - \beta)$$

the stator field is,

$$B_s = B_a + B_b + B_c = \frac{3}{2} B \cos(\theta - \omega t) \qquad (T) \qquad (5.18)$$

The stator field in Eqn. (5.18) is a forward rotating field, whose magnitude is 3/2 the peak magnitude of any one winding, rotating at synchronous speed around the airgap. The coefficient of the time term, ω, is determined by the frequency of the currents that produced this waveform, which, in turn was determined by the frequency of the rotor emfs, E_f, which, in turn, was determined by rotor speed.

In summary, when three armature currents, 120° apart in time, flow through three windings 120° apart in space, the result is a rotating field of constant magnitude rotating at synchronous speed around the airgap. This extraordinary fact can be extended, in general, to n-windings on the stator. The reality of this field can be demonstrated by placing a compass needle within a three-phase stator and it will attempt to track the rotating stator field at synchronous speed.

We now have two fields – the rotor field and the stator field – both rotating in a forward direction at synchronous speed and their sum is called the resultant air-gap field, displaced from the rotor field by the torque angle, δ, as indicated in Fig. 5.15.

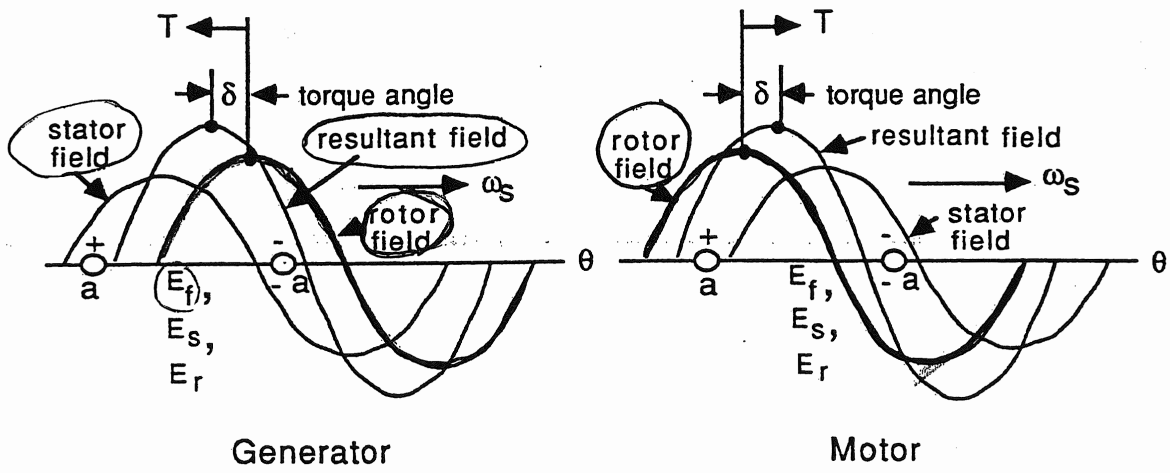


Figure 5.15 Synchronous Machine Air-Gap Fields

In Fig. 5.15 when the machine is a generator, the prime mover drive torque drives the rotor field ahead of the resultant field by the torque angle, δ, and a generated back Lorentz torque acts on the rotor.

When the machine is a motor, the load torque drags the rotor field behind the resultant field by the torque angle, δ, and a generated forward Lorentz torque acts on the rotor. The calculation of the magnitude of the Lorentz torque will be deferred until the machine equivalent-circuit is determined.

It must be emphasized in Fig. 5.15, that the only field that exists in the air gap is the resultant field which is the sum of its component fields – the rotor field produced by the rotor current, I_f, and the stator field produced by the load or armature currents, i_a, i_b, and i_c. It, therefore, follows that at no-load, the resultant field is the rotor field.

Shown in Fig. 5.15 is the stationary a winding (b and c windings are omitted for clarity) on the stator. As the fields sweep by the windings, Faraday emfs will be induced in the windings of rms value predicted by Eqn. (5.13). If, N_ϕ = total turns per phase,

$$\begin{aligned} &\text{Rotor field:} && E_f = 4.44\, f\, N_\phi\, \Phi_f \leftarrow I_f \quad (V) \\ &\text{Stator field:} && E_s = 4.44\, f\, N_\phi\, \Phi_s \leftarrow i_a, i_b, i_c \quad (V) \qquad (5.19) \\ &\text{Resultant field:} && E_r = 4.44\, f\, N_\phi\, \Phi_r = E_f + E_s \quad (V) \end{aligned}$$

The only emf that exists across each winding is, of course, the resultant emf, which is the phasor sum of its components, which will become evident after the machine equivalent circuit is determined.

5-6 SYNCHRONOUS MACHINE,PER-PHASE,EQUIVALENT CIRCUIT

Keeping in mind the foregoing physical basis for this machine, analysis will continue by realizing the machine is a balanced, three-phase device, if one looks into the three stator terminals, as in Fig. 5.16,

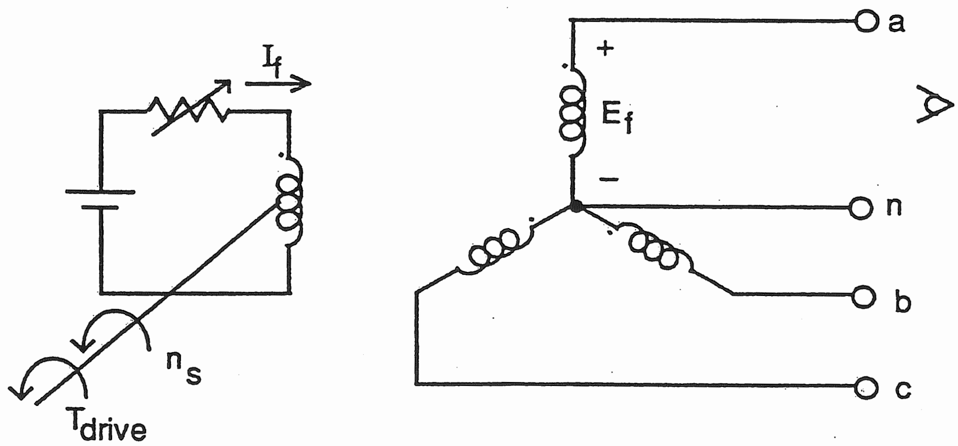


Figure 5.16 Three Ports of the Synchronous Machine

The three stator ports, each defined from line to neutral, are shown in Fig. 5.16. Phase a is chosen as the port of analysis, since ports b and c have identical equivalent circuits. The machine is linearized by finding the Thevenin equivalent circuit looking into port a, which consists of a Thevenin voltage in series with a Thevenin impedance.

The Thevenin voltage is the open-circuit voltage across terminals a-n, and the magnitude of the Thevenin impedance is the open-circuit terminal voltage, divided by the short-circuit terminal current.

Clearly, the open-circuit terminal voltage is the excitation emf, E_f, since, on open-circuit, the only air-gap field is the rotating, rotor field, which can be varied by adjusting the rheostat and therefore I_f. The Thevenin parameters can be found from the open-circuit, short-circuit tests of the machine, driven as a generator, at synchronous speed. The results of these tests are shown in Fig. 5.17.

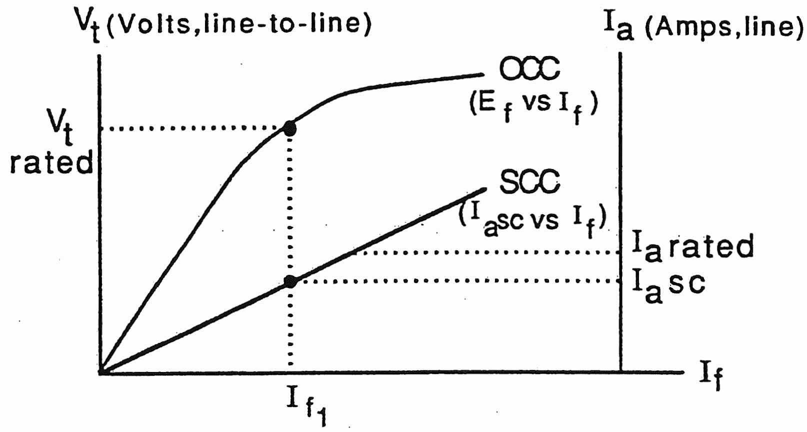


Figure 5.17 Synchronous Machine Open-Circuit,Short-Circuit Characteristics

The open-circuit characteristic, OCC, is the line to line voltage, measured on open-circuit,as the field current is varied from zero to maximum. This curve is a measure of the excitation emf as a function of the field current, and is nonlinear.

The short-circuit characteristic, SCC, is found by measuring the line currents, on short-circuit, by varying the field current and therefore, the driving excitation emf, E_f, until the short-circuit currents reach rated value. On short-circuit, the machine is not saturated, so this curve is linear.

There are an infinite number of ratios of open-circuit voltage to short-circuit current, constant in the linear region, and decreasing in the saturation region. From experience, an excellent operating value for the magnitude of the Thevenin impedance is found by entering the open-circuit characteristic at rated voltage, and dividing by the corresponding value of the short-circuit current, which is not necessarily rated current. The magnitude of the Thevenin impedance is, then,

$$|Z_{Th}| = |Z_s| = \frac{V_t\,OC\ (rated)}{\sqrt{3}\,I_a SC} = \sqrt{R_a^2 + X_s^2} \quad (\Omega/\phi) \quad (5.20)$$

where,

Z_s = synchronous impedance (Ω/ϕ)

R_a = winding resistance (Ω/ϕ)

X_s = synchronous reactance (Ω/ϕ)

The per-phase, equivalent circut is given in Fig. 5.18.

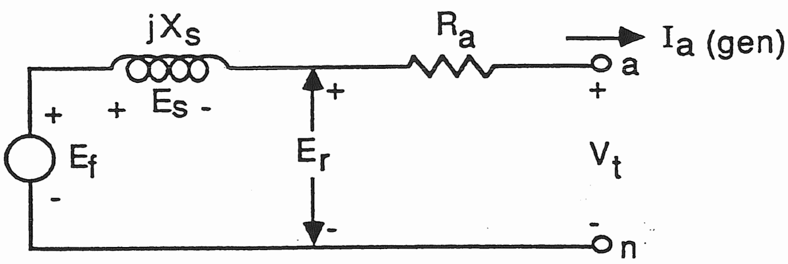
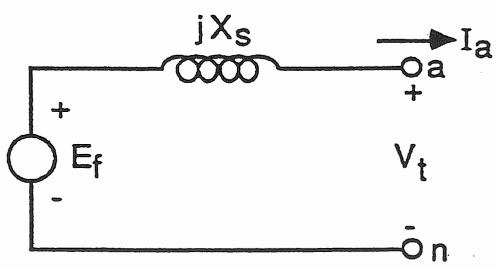


Figure 5.18 Synchronous Generator Equivalent Circuit

The machine model in Fig. 5.18 consists of a constant-voltage source, whose magnitude, E_f, does not vary with the load or torque, but varies <u>only</u> with the rotor field current, I_f. The emf, E_r, generated by the revolving, resultant field in winding <u>a</u> is different from the terminal voltage, V_t, only by the R_aI_a drop, and is the phasor sum of the excitation and stator emfs as given in Eqns. (5.19). For large machines, with megawatt ratings, the armature resistance is more than ten times smaller than the synchronous reactance and is usually neglected, as shown in Fig. 5.19.



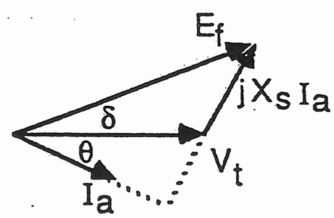


Figure 5.19 Power-System Model

For large machines, the resultant emf, E_r, is approximately equal to the terminal voltage, V_t. Therefore, the angle between the excitation and terminal voltage phasors is the torque angle, δ, defined in Fig. 5.15, and is shown in the phasor diagram of Fig. 5.19, for a lagging, machine power-factor angle, θ.

Example 5.1

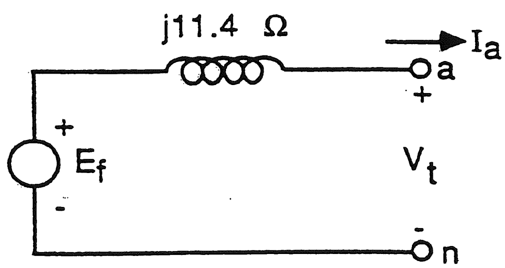
A 60 Hz, three-phase, 20,000 kVA, 13.8 kV, Y-connected, two-pole synchronous machine is driven as a generator at synchronous speed. At no-load, the field current is adjusted to 185 A, for rated line voltage across the machine terminals. On short circuit, with no change in the field current, the line current is 699 A. With the short-circuit removed, the machine is loaded and the field current is readjusted so that rated load, 0.85 pf lagging, at rated voltage, is delivered by the machine.

a) At what speed, rpm, must the machine be driven?

$$n_s = \frac{120\,f}{P} = \frac{(120)(60)}{2} = 3600 \text{ rpm}$$

b) What is the synchronous reactance of this machine in Ω/ϕ and draw the machine model.

$$|Z_s| \approx X_s = \frac{V_t OC\ (\text{rated})}{\sqrt{3}\, I_a SC} = \frac{13{,}800}{\sqrt{3}\,(699)} = 11.4\ \Omega/\phi$$



c) When the machine is delivering rated load, 0.85 pf lagging, at rated terminal voltage, is the field current larger, smaller or equal to 185A?

$$V_t = \frac{13{,}800}{\sqrt{3}} \angle 0^\circ = 7{,}967 \angle 0^\circ \text{ V} \ ; \ I_a \text{ rated} = \frac{20{,}000}{\sqrt{3}\,(13.8)} \angle -31.8^\circ = 837 \angle -31.8^\circ \text{ A}$$

From the above diagram,

$$E_f = j\,X_s\,I_a + V_t = (11.4 \angle 90^\circ)(837 \angle -31.8^\circ) + 7{,}967 \angle 0^\circ = 15.3 \angle 32^\circ \text{ kV } (\phi)$$

$$= 26.5 \text{ kV (line)}$$

From the open-circuit characteristic in Fig. 5.17, at E_f = 26.5 kV (line), the operating field current is greater than 185 A.

d) With the machine under rated load, draw the phasor diagram and label all machine voltages and currents, in per-unit, using the machine rating as a base.

$V_{base} = 13.8 \text{ kV}$

$I_{base} = \dfrac{20{,}000}{\sqrt{3}\,(13.8)} = 837 \text{ A}$

$VA_{base} = 20 \text{ MVA}$

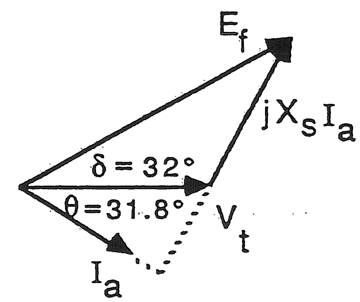
$Z_{base} = \dfrac{(13.8)^2}{20} = 9.52\ \Omega$

$$E_f = \frac{15.3\angle 32^\circ}{13.8/\sqrt{3}} = 1.92\angle 32^\circ \text{ pu}$$

$$V_t = \frac{7{,}967\angle 0^\circ}{13.8/\sqrt{3}} = 1.0\angle 0^\circ \text{ pu}$$

$$j X_s I_a = \frac{9.54\angle 58.2^\circ}{13.8/\sqrt{3}} = 1.2\angle 58.2^\circ \text{ pu}$$

$$I_a = \frac{837\angle -31.8^\circ}{837} = 1.0\angle -31.8^\circ \text{ pu}$$



Observe in the phasor diagram, for the machine as a generator as in Fig. 5.15, the rotor field or its excitation emf always leads the resultant field or its terminal voltage by the torque angle, δ.

5-7 SYNCHRONOUS MACHINE POWER FLOW

In steady state, the machine power flow is represented by its mechanical system, coupling field and electrical system as shown in Figure 5.20. All power flow is shown total, or three-phase, for the machine, for an inductive load. The factor, 3, accounts for the contribution to total power of the three phases.

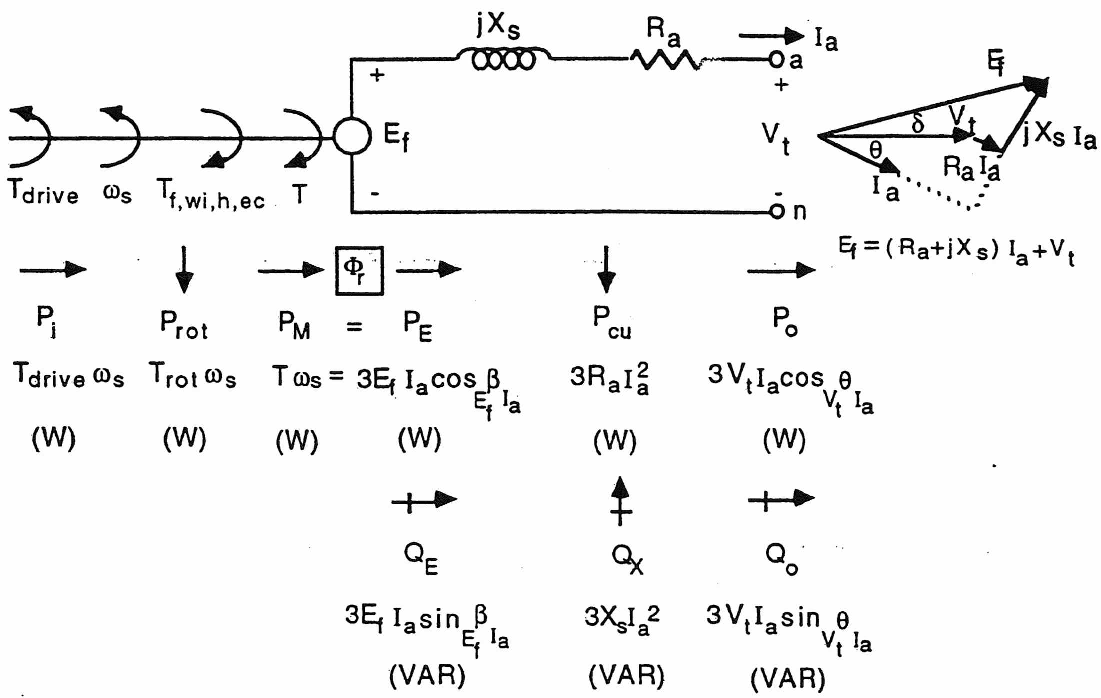


Figure 5.20 Synchronous Generator Power Flow Diagram

The mechanical source is the prime mover whose drive-torque times ω_s is the total mechanical power delivered to this machine. Since the torques acting on the shaft are independent of the machine pole-pairs,

$$\omega_s = \underset{\frac{rev}{min}}{n_s} \times \underset{\frac{rad}{rev}}{2\pi} \times \underset{\frac{min}{sec}}{\frac{1}{60}} \qquad \text{(rad/sec)} \qquad (5.21)$$

The mechanical power input, then, supplies, continuously, the real power required by the losses and load, thus achieving a watt balance,

$$T_{drive}\,\omega_s = T_{rot}\,\omega_s + T\,\omega_s$$

$$T\,\omega_s = 3\,E_f\,I_a \cos \beta_{E_f I_a} \qquad \text{(W)} \qquad (5.22)$$

$$3\,E_f\,I_a \cos \beta_{E_f I_a} = 3\,R_a\,I_a^2 + 3\,V_t\,I_a \cos \theta_{V_t I_a}$$

In steady state, the coupling field delivers the vars required by the synchronous reactance and the inductive load (the quadrature component of I^* is positive). Reactive power surges back and forth, each quarter cycle, between the coupling field and load, with an average value of zero, requiring no input from the drive torque in steadystate.

The var balance is,

$$3\,E_f I_a \sin \beta_{E_f I_a} \quad = \quad 3\,X_s I_a^2 + 3\,V_t I_a \sin \theta_{V_t I_a} \qquad \text{(VAR)} \qquad (5.23)$$

The watt-var balance in Eqns (5.22), (5.23) must be well understood to appreciate the role the synchronous generator plays when placed in an electric power system.

Example 5.2

The 13.8 kV, 20,000 kVA synchronous generator of Example 5.1 has a synchronous reactance of 11.4 Ω/phase and is delivering rated load, 0.85 pf lagging, at rated voltage. The excitation emf is calculated at $E_f = 15.3\underline{/32^\circ}$ kV(ϕ), and the phasor diagram is drawn in part d) of Example 5.1. Compute the power flow through the machine if the rotational and electrical losses are negligible.

$$P_M = T_{drive}\,\omega_s = T\omega_s = 3\,E_f I_a \cos(\delta + \theta) = (3)\,(15{,}300)(837)\cos(32^\circ + 31.8^\circ)$$

$$= 17 \text{ MW}$$

$$|\,T_{drive}\,| = |\,T\,| = \frac{3\,E_f I_a}{\omega_s}\cos(\delta+\theta) = \frac{17 \times 10^6}{\underbrace{(3600)}_{\frac{rev}{min}}\underbrace{(2\pi)}_{\frac{rad}{rev}}\underbrace{\frac{1}{60}}_{\frac{min}{sec}}} = \underbrace{45{,}000 \text{ N-m},}_{\text{(approx. 5 tons force at a meter !)}}$$

$$P_E = 3\,E_f I_a \cos(\delta+\theta) = 3\,V_t I_a \cos\theta = (3)\,(7{,}967)(837)\cos 31.8^\circ = 17 \text{ MW}$$

$$Q_E = 3\,E_f I_a \sin(\delta+\theta) = (3)(15{,}300)(837)\sin 63.8^\circ = 34.5 \text{ MVAR}$$

$$Q_X = 3\,X_s I_a^2 = (3)\,(11.4)(837)^2 = 24 \text{ MVAR}$$

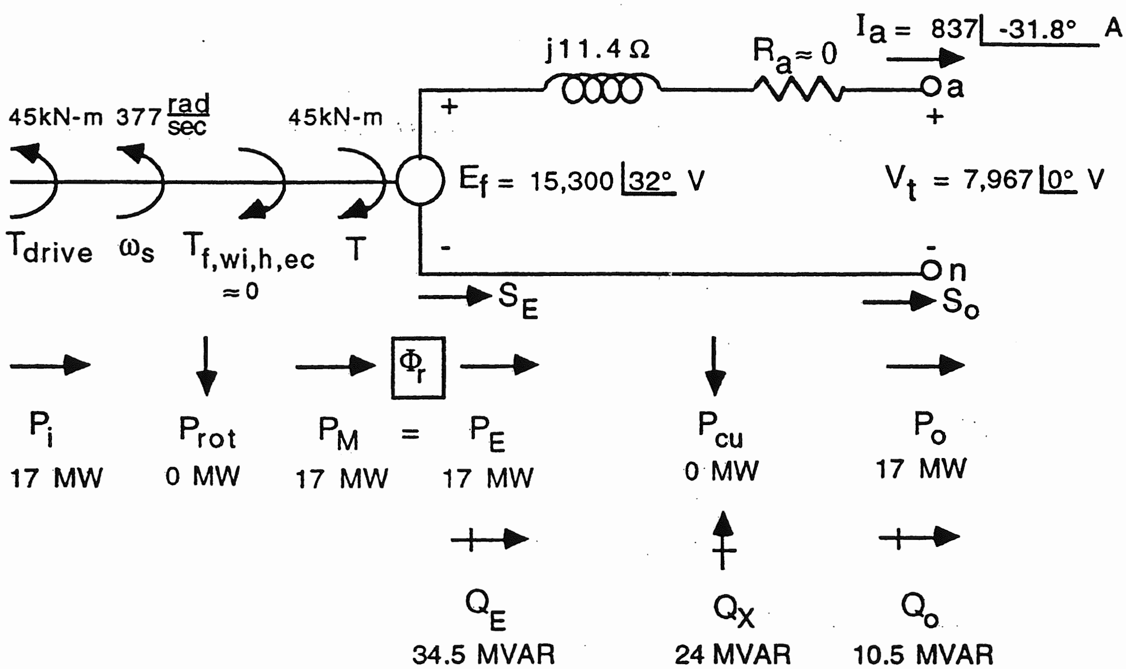
$$Q_o = 3\,V_t I_a \sin\theta = (3)\,(7{,}967)\,(837)\sin 31.8^\circ = 10.5 \text{ MVAR}$$

Check:

$$S_E = 3\,E_f I_a^* = (3)(15{,}300\,\underline{/32^\circ})(837\,\underline{/31.8^\circ}) = 38.4\,\underline{/63.8^\circ} = 17 + j\,34.5 \text{ MVA}$$

$$S_o = 3\,V_t I_a^* = (3)(7{,}967\,\underline{/0^\circ})(837\,\underline{/31.8^\circ}) = 20\,\underline{/31.8} = 17 + j\,10.5 \text{ MVA}$$

The power-flow diagram is,



5-8 MACHINE STEADYSTATE CHARACTERISTICS

To this point, the synchronous generator, with its load, has been considered as an entity. In practice, the large machine, with megawatt rating, is connected to the power grid to help supply, together with other large generators, hundreds of thousands of passive and active loads connected to three-phase lines.

To better understand how the synchronous generator helps achieve a watt-var balance in a large electric power system, the generator will, for the purposes of this discussion, be connected to an <u>infinite</u> bus, as in Fig. 5.21.

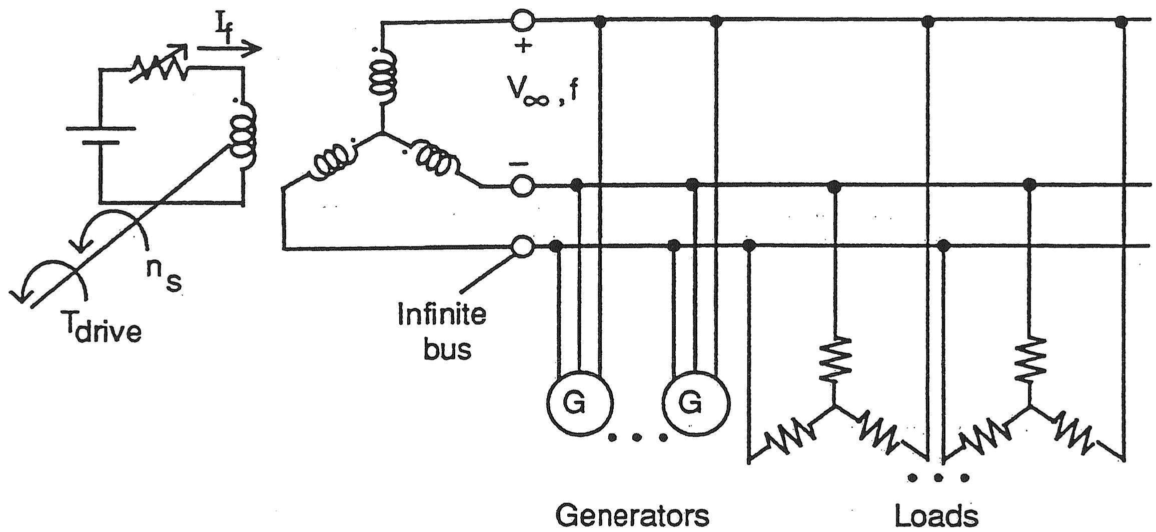


Figure 5.21 Synchronous Generator Connected to an Infinite Bus

A bus in a power system is a junction point of one or more sets of three-phase currents and a bus is often called a power node. An infinite bus has the special characteristics that the line voltage and its frequency remain absolutely constant, regardless of the line currents drawn by this bus. In practice, power systems approach this ideal definition, since hundreds of large generators, properly adjusted in the power grid, assure constant voltage and frequency (rated for the machine) at the infinite bus. Further simplification is assumed by considering the large generator in Fig. 5.21 as lossless, i.e., the friction, windage, hysteresis and eddy-current mechanical losses and the I^2R electrical losses are negligible, compared to the power output of this machine. In practice, this is essentially true for these large machines.

With the above assumptions, several very important observations can be made concerning the synchronous generator in Fig. 5.21.

1. When the machine is on line, the only two, physical parameters of the machine that can be varied are the field current, I_f, and the prime-mover torque, T_{drive}.

2. The field current varies the excitation emf, E_f, independent of machine torque and load.

3. The drive torque, at constant, synchronous speed, determines the mechanical power (watts) delivered to the machine and, therefore, the electrical watts delivered to the infinite bus.

These observations will become important in the subsequent discussion.

5-9 MACHINE FLOATING ON THE LINE

When the machine is brought up to speed by its prime mover (illustrated in Fig. 5.2), and synchronized to an infinite bus, whose voltage and frequency are rated for the machine, the field current can be adjusted so that the magnitude of the excitation emf, E_f, is equal to, smaller or greater than, the magnitude of the bus voltage, and the prime-mover throttle is closed. Because the prime-mover throttle is closed, and the watt-flow is zero, the machine is said to be floating on the line as shown in Fig. 5.22.

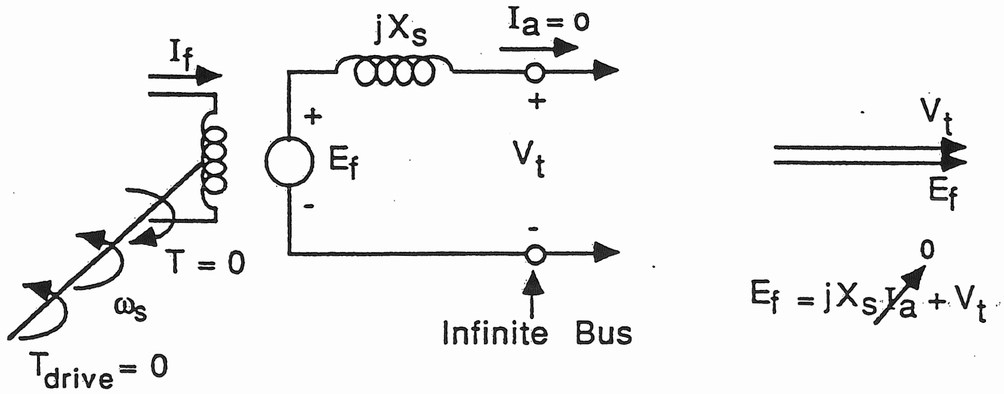


Figure 5.22 Synchronous Machine Floating on the Line

Because the prime-mover throttle is closed, the drive torque is zero, but the machine continues to rotate at synchronous speed, since the rotor is energized and the frequency of the consequent, excitation emf cannot be different than the frequency of the infinite bus. Should the machine decrease speed, restoring torques will bring it back to synchronous speed. Since the drive torque is zero, and the machine is lossless, no real power will be delivered to the machine or the infinite bus. Since the magnitudes of the excitation voltage and bus voltage were adjusted to be equal, no line currents flow, as shown in the phasor diagram of Fig. 5.22. The machine delivers no watts, and is said to be floating on the line.

5-10 GENERATOR STEADYSTATE CHARACTERISTICS

The machine can be made a generator by opening the throttle of the prime mover, thus increasing the drive torque as illustrated in Fig. 5.23.

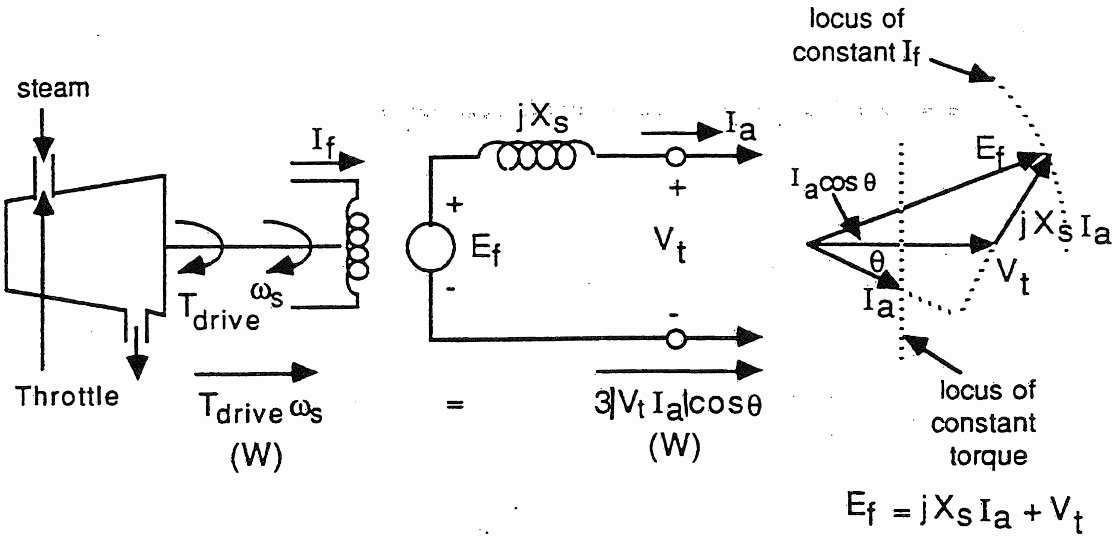


Figure 5.23 Generator Connected to an Infinite Bus

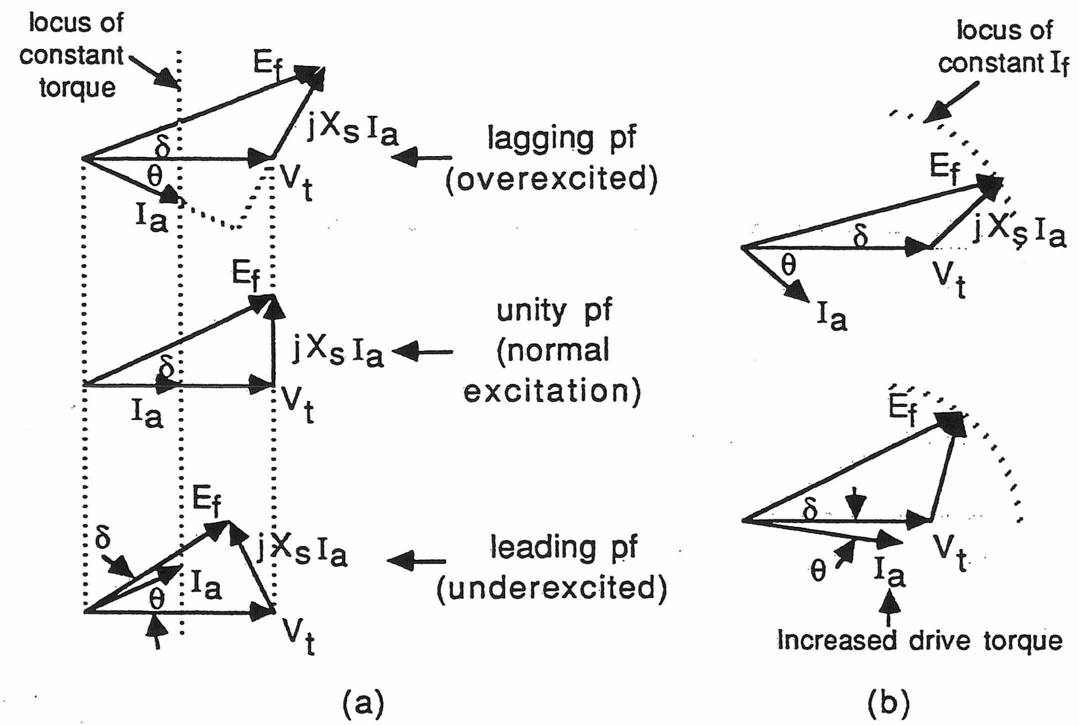
Since the machine is rotating at synchronous speed, mechanical power, $T_{drive}\,\omega_s$, watts, are delivered to the machine. Since the machine is lossless, they are seen in electrical form as $3|V_t I_a| \cos\theta$, watts, delivered to the line. Since the terminal voltage, V_t, is kept constant by the infinite bus, the drive torque is then proportional to the projection of I_a on V_t.

$$T_{drive} = K|I_a| \cos\theta$$

Because the drive torque is proportional to the above projection, at constant throttle or drive torque, the locus of the armature current must be the vertical, dashed line on the phasor diagram of Fig. 5.23.

Also, at constant field current, since the magnitude of the excitation emf, E_f, is determined by I_f, the locus of constant I_f must be the dashed circle on the phasor diagram of Fig. 5.23.

The machine characteristics are, then, a function of two variables, T_{drive} and I_f. As is common with a function of two variables, one is kept constant and the other is allowed to vary, as is illustrated in Fig. 5.24.



Constant Torque
Variable Field Current

Variable Torque
Constant Field Current

Figure 5.24 Synchronous Generator Characteristics

In Fig. 5.24 (a), the drive torque is kept constant, so therefore,

$$T_{drive}\,\omega_s = 3|V_t I_a|\cos\theta \qquad \text{(W)}$$

and the watt-flow, to the loads beyond the infinite bus, remains the same for all three diagrams shown.

In descending order, the field current is first made large, so that,

$$|E_f|\cos\delta > V_t$$

and the machine is said to be overexcited, with a consequent lagging power factor, and it delivers watts and vars (the quadrature component of I_a^* is positive) to the infinite bus.

Next, the field current is decreased, so that,

$$|E_f|\cos\delta = V_t$$

and the machine is said to be normally excited, with a consequent unity power factor, and it delivers watts, but no vars (the quadrature component of I_a^* is zero) to the infinite bus.

Finally, the field current is further decreased, so that,

$$|E_f|\cos\delta < V_t$$

and the machine is said to be underexcited, with a consequent leading power factor, and it delivers watts to , but absorbs vars from (the quadrature component of I_a^* is negative) the infinite bus.

In Fig. 5.24 (b), the constant field current, variable torque characteristics are shown. The machine is shown, initially overexcited, so that it delivers real and reactive power to the infinite bus. In the lower phasor diagram, the prime-mover throttle is opened, with increased drive torque, so that $I_a \cos\theta$ must increase, with a corresponding increase in the magnitude of I_a. The synchronous reactance drop, $X_s I_a$, must then increase, and since the field current is constant, E_f must rotate counterclockwise to accommodate the increased drop. The net result is increased watt-flow to the infinite bus, with a change in var-flow dependent on the new power factor.

In summary, from Fig. 5.24 (a), the synchronous generator achieves a watt-var balance with an adjustment of the drive torque and the field current, by realizing that,

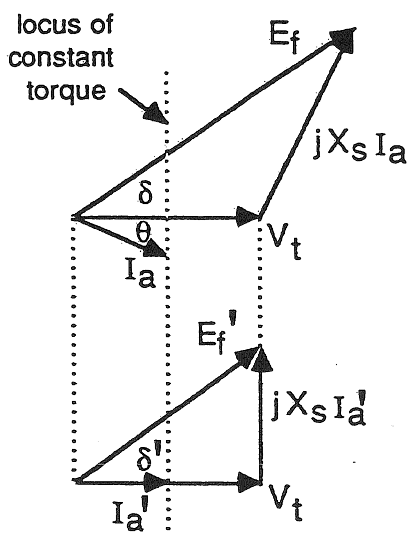
1. The prime-mover torque determines the watt-flow to the infinite bus.
2. The generator field current determines the vars delivered to, or absorbed from, the infinite bus.

Example 5.3

The 13.8 kV, 20,000 kVA, $X_s = 11.4\ \Omega/\phi$, synchronous generator of Example 5.1 is connected to a 13.8 kV, 60 Hz, infinite bus. The prime mover torque and the generator field current are adjusted so that the machine delivers rated kVA, 0.85 pf lagging, to the line.

a) With <u>constant</u>, prime-mover throttle or <u>drive torque</u>, by what percentage must the field current be reduced so that power is delivered to the infinite bus at unity pf? What is this power?

From Example 5.1, part c) and Example 5.2 –



<u>Overexcited</u>

$E_f = 15.3\angle 32°$ kV $\quad P_o = 17$ MW

$I_a = 837\angle{-31.8°}$ A $\quad Q_o = 10.5$ MVAR

$V_t = 7.967\angle 0°$ kV

<u>Normally excited</u>

$I_a' = I_a\cos\theta = 837\cos 31.8° = 711\angle 0°$ A

$E_f' = jX_sI_a' + V_t$

$= (11.4\angle 90°)(711\angle 0°) + 7{,}967\angle 0°$

$= 11.4\angle 45.5°$ kV

$$\frac{I_f'}{I_f} = \frac{E_f'}{E_f} = \frac{11.4}{15.3} = 0.75$$

The field current is reduced 25%

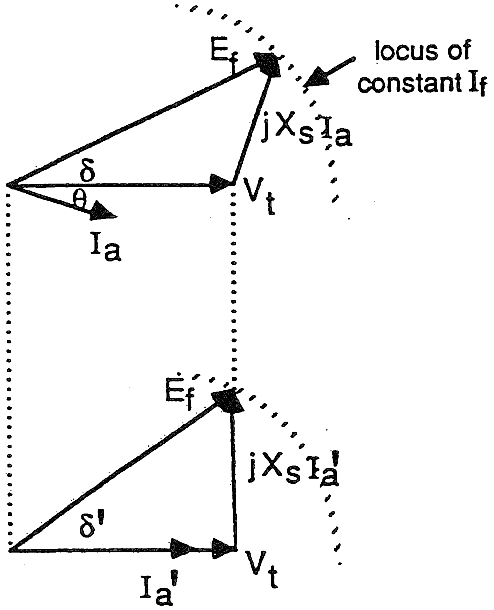
$$S_o = 3V_t(I_a')^* = (3)(7{,}967\angle 0°)(711\angle 0°) = 17\angle 0° = 17 + j0 \text{ MVA}$$

$$P_o = 17 \text{ MW}$$

$$Q_o = 0 \text{ MVAR}$$

b) Returning to the original adjustment of torque and field current, with constant field current, by what percentage must the drive torque be increased so that power is delivered to the infinite bus at unity pf? What is this power?

From Example 5.1, part c) and Example 5.2



<u>Overexcited</u>

$E_f = 15.3\angle 32^\circ$ kV $\quad P_o = 17$ MW

$I_a = 837\angle -31.8^\circ$ A $\quad Q_o = 10.5$ MVAR

$V_t = 7.967\angle 0^\circ$ kV

<u>Same Excitation</u>

$$\cos\delta' = \frac{V_t}{E_f} = \frac{7{,}967}{15{,}300}$$

$\delta' = 58.6°$

$E_f = jX_sI_a' + V_t$

$$I_a' = \frac{E_f\angle\delta' - V_t\angle 0^\circ}{jX_s}$$

$$= \frac{15{,}300\angle 58.6^\circ - 7{,}967\angle 0^\circ}{11.4\angle 90^\circ}$$

$$= 1{,}146\angle 0^\circ \text{ A}$$

$$\frac{T_{drive}'}{T_{drive}} = \frac{I_a'\cos\theta'}{I_a\cos\theta} = \frac{1{,}146}{711} = 1.61$$

The drive torque must be increased 61%.

$$S_o = 3V_t(I_a')^* = (3)(7{,}967\angle 0^\circ)(1{,}146\angle 0^\circ) = 27.4\angle 0^\circ = 27.4 + j0 \text{ MVA}$$

$$P_o = 27.4 \text{ MW}$$
$$Q_o = 0 \text{ MVAR}$$

5-11 SYNCHRONOUS GENERATOR MAXIMUM POWER

This section will describe a serious steadystate limitation to the synchronous generator when it is connected to an infinite bus. When the prime-mover torque is increased, more and more mechanical watts are delivered to the generator, resulting in more electrical watts delivered to the infinte bus, with a consequent increase in torque angle, δ, defined in Fig. 5.15. There is a limit, however, to the magnitude of the real power, (and the consequent torque angle), that can be transferred through the synchronous generator to the infinite bus. This limitation is defined by considering the complex power delivered to the infinite bus as in Fig. 5.25.

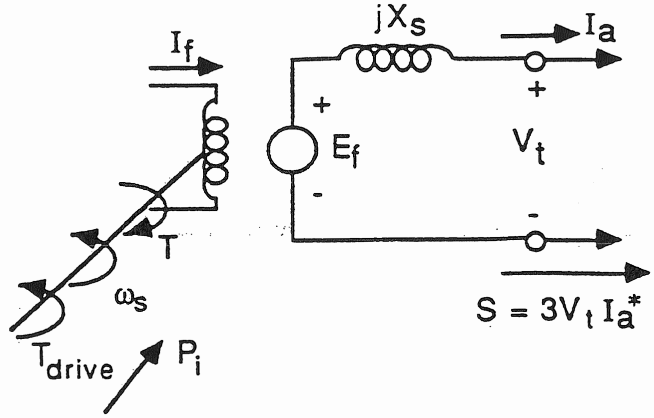


Figure 5.25 Synchronous Generator Connected to an Infinite Bus

$S = 3\,V_t\,I_a^*$, where, $\quad I_a = \dfrac{E_f \angle\delta - V_t \angle 0°}{j\,X_s}$

$$I_a^* = \frac{E_f \angle{-\delta} - V_t \angle 0°}{-j\,X_s}$$

then, $$S = 3\left[\frac{V_t \angle 0° \times E_f \angle{-\delta} - V_t \angle 0° \times V_t \angle 0°}{-j\,X_s}\right]$$

$$= 3\left[\frac{|V_t|\,|E_f|\,(\cos\delta - j\sin\delta) - |V_t|^2}{-j\,X_s}\right]$$

$$S = \underbrace{\frac{3\,|V_t|\,|E_f|}{|X_s|}}_{P_{max}} \sin\delta + j\underbrace{\left[\frac{(3)\,|V_t|\,|E_f|}{|X_s|}\cos\delta - \frac{(3)\,|V_t|^2}{|X_s|}\right]}_{Q} = P + jQ \quad (5.24)$$

Of interest in Eqn. (5.24), is the real part of S,

$$P = \frac{3|V_t\,E_f|}{X_s}\sin\delta \qquad \text{(W)} \qquad (5.25)$$

where, $$P_{max} = \frac{3|V_t\,E_f|}{X_s} \qquad \text{(W)}$$

The conclusion reached, from Eqn. (5.25), is , that if the generator is operated with variable drive torque and constant field current, the power delivered to the infinite bus varies as the sine of the torque angle. As the prime-mover torque is increased, the torque angle opens, as is shown in the plot of Eqn. (5.25) in Fig. (5.26),

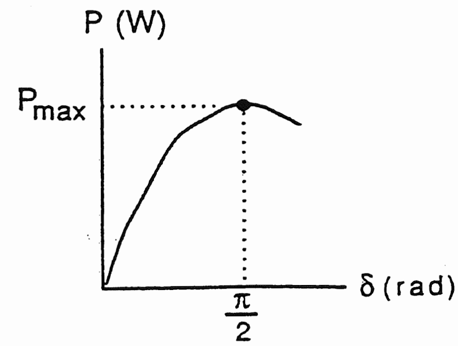


Figure 5.26 Generator Power Delivered to an Infinite Bus

As the mechanical watts delivered to the generator increase, the electrical watts delivered to the infinite bus increase until the torque angle opens to 90°. Maximum power is now transferred through the machine. If the mechanical watts are increased beyond this point, the machine snaps out of synchronism and runs away, since the excess mechanical watts are stored as kinetic energy in the rotor. The torque angle, in practice, is seldom allowed to exceed 30° - 40°, since the sin δ changes very rapidly beyond this point. Furthermore, for a synchronous generator with given V_t and X_s, the machine is seldom drastically underexcited, since the maximum-power capability of the machine is diminished, (Eqn. 5.25) and for a sudden load increase, the machine could lose synchronism.

Example 5.4

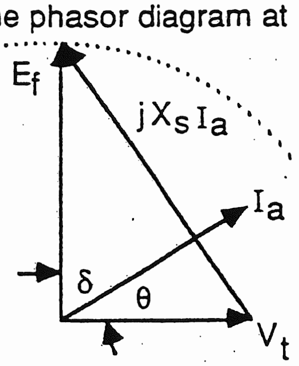
The 13.8 kV, 20,000 kVA, $X_s = 11.4\ \Omega/\phi$, synchronous generator of Example 5.1 is delivering rated kVA, 0. 85 lagging, to a 13.8 kV infinite bus. If the prime-mover, drive-torque is increased with constant field current,

a) What is the maximum power this machine can deliver?

$$P_{max} = \frac{3\,V_t\,E_f}{X_s} = \frac{(3)(7{,}967)(15{,}300)}{11.4} = 32\text{ MW} = T_{drive}\omega_s$$

$$T_{drive} = \frac{32 \times 10^6}{3{,}600 \times \frac{2\pi}{60}} = 84{,}883 \quad \text{(N-m)}$$

b) Draw the phasor diagram at maximum power



$E_f = 15{,}300\ \underline{/90^\circ}$ V

$V_t = 7{,}967\ \underline{/0^\circ}$ V

$X_sI_a = 17{,}250$ V

$I_a = 1{,}513\ \underline{/27.5^\circ}$ A

5-12 SUMMARY

The physical behavior of the synchronous machine is summarized in Fig. 5.15. For a generator, the rotor field is driven ahead of the resultant field by the torque-angle, δ. For a motor, the rotor field is torqued behind the resultant field by the torque-angle, δ.

When a direct-current, I_f, flows through the rotor winding, a sinusoidally-distributed rotor field is created, rotating around the airgap, with the rotor, at synchronous speed.

When three stator-load currents, 120° apart in time phase, flow through three stator windings, 120° apart in space, a sinusoidally distributed stator field is created, that rotates around the airgap at synchronous speed.

The sum of these two fields is the resultant field that lags the rotor field by the torque-angle δ, for a generator, and leads the rotor field by the torque-angle, δ, for a motor.

The interaction, then, of the resultant field with the rotor field produces a Lorentz torque on the rotor as shown in Fig. 5.15. When the torque is increased on either of these machines, the torque-angle, in steadystate, opens to a maximum of 90°, and then the machines lose synchronism.

The resultant field, with its component-rotor and stator fields, induce Faraday emfs in the stator windings, which form the basis for the machine, per-phase, equivalent circuit.

The open-circuit, short-circuit tests yield the equivalent-circuit parameters, R_a, X_s and E_f.

The excitation emf, E_f, is variable only with the field current, and the in-phase component of the armature current, $I_a \cos \theta$, is determined only by the drive or load torques

As a consequence of these two facts, at constant torque-variable field current, overexcited generators deliver vars to the line and underexcited generators absorb vars from the line. The drive torque determines the watt-flow, and the field current determines the var-flow. Important to this analysis is, the locus of constant torque is a vertical line and the locus of constant field current is a circle.

The maximum power that can be transferred through a synchronous generator, represented only by its synchronous reactance in its equivalent circuit is,

$$P_{max} = \frac{3 V_t E_f}{X_s} \qquad \text{(W)}$$

where the power delivered to the line, as a function of the torque angle is,

$$P = \frac{3 V_t E_f}{X_s} \sin \delta \qquad \text{(W)}$$

PROBLEMS

5.1 The two-pole, cylindrical rotor of Fig. 5.5 is changed to have five slots on the top and five slots on the bottom of the rotor. Wound in these slots is a field coil, $N_f = 100$ turns.

a) Sketch the staircase distribution of mmf around the airgap, $g = 0.16$ cm, as a function of θ, if the field current is 6.36 A. Label important magnitudes on both axes with numerical values.

b) On another set of axes, sketch the distribution of flux density as a function of θ, and label with numerical values.

c) What is the total flux per pole, Φ_f, for a rotor of diameter 10 ft and axial length, 30 ft, if $I_f = 6.36$ A?

d) What is the maximum field current for linear operation along the rotor magnetization curve, if the rotor is made of silicon sheet steel?

5.2 Do Problem 5.1 for $I_f = 10$ A.

5.3 A 3-phase, 4-pole synchronous generator, Y-connected, is driven, no-load, at 1800 rpm. Each stator coil (a, -a) has 4 turns. I_f is adjusted so that $\Phi_f = 0.25$ Wb/pole.

a) Determine the generator phase voltage, rms.

b) Determine the generator line voltage, rms.

c) For an abc sequence, t = 0, when the flux linkages with winding a are zero and positive increasing. Write a set of time equations for v_{an}, v_{bn}, v_{cn} and v_{ab}, v_{bc}, v_{ca}.

5.4 A 3-phase, 4-pole, 60 Hz, synchronous generator, Y-connected, is driven, no-load, at 1800 rpm. Each stator coil (a, -a) has 2 turns. I_f is adjusted so that $\Phi_f = 0.25$ Wb/pole.

a) Determine the generator phase voltage, rms.

b) Determine the generator line voltage, rms.

c) For an abc sequence, t = 0 when the flux linkages with winding a are negative maximum and increasing. Write a set of time equations for v_{an}, v_{bn}, v_{cn} and v_{ab}, v_{bc}, v_{ca}.

5.5 A 10 MVA, 13.8 kV, 2-pole, 60 Hz, Y-connected synchronous generator has the stator in Fig. 5.14 with 8 slots/per coilside in each belt, one conductor per slot as indicated. The machine is delivering rated MVA, unity power factor, at rated voltage, to a load. Write the equation for the rotating stator field, $B_s(\theta,t)$, produced around the airgap, g = 0.16 cm, resulting from these stator load currents. Calculate numerical values, for the constants in this equation.

5.6 For a 60 Hz, 2-pole, 2-phase synchronous machine, the two concentrated stator windings of 100 turns each are displaced 90° apart in space, and the winding currents, 4.5 A rms, are displaced 90° in time phase. If θ is measured from the axis of winding a and the current in winding a is positive-maximum at t = 0, write the equation for the stator field, $B_s(\theta,t)$, produced around the airgap, g = 0.16 cm, resulting from these stator currents. Calculate numerical values for the constants in this equation.

5.7 A 3-phase, 9,375 kVA, 13.8 kV, 2-pole, 60 Hz, Y-connected synchronous machine has a stator resistor, $R_a = 0.064\ \Omega/\phi$.
The open-circuit, short-circuit test measurements are,

V_t OC V(line to line)	I_a SC (A)	I_f (A)
13,000	392	169
13,800	446	192

a) What is the synchronous reactance?

b) Draw and label the per-phase, equivalent circuit.

c) What approximate I_f (A) is required for the machine to deliver rated kVA, 0.8 pf leading at rated voltage to the line? Sketch the phasor diagram and label completely with numerical values, in per-unit, using the machine rating as a base.

5.8 A 3-phase, 10 MVA, 13.8 kV, 4-pole, 60 Hz, synchronous machine has a stator resistance, $R_a = 0.07\ \Omega/\phi$.
The open-circuit, short-circuit test measurements are,

V_t OC V(line to line)	I_a SC (A)	I_f (A)
13,000	418	170
13,800	492	200

a) What is X_s, Ω/ϕ?

b) Draw and label the equivalent circuit.

c) The machine is delivering rated MVA, 0.8 pf lagging at rated voltage to the line. What is the drive torque, N-m, for negligible losses?

5.9 A 3-phase, 60 Hz, 20-pole, 25 kVA, 208 V, Y-connected synchronous generator, with a reactance, $X_s = 1.7\ \Omega/\phi$, is connected to a 208-volt, infinite bus.

a) What is the machine speed, rpm?

b) If the generator delivers rated kVA, 0.85 pf lagging, to the bus, what is the excitation emf, E_f, (polar form)?

c) With constant field current, what is the new torque angle, δ, and the reactive power delivered to the bus if the prime mover torque is adjusted so that the machine delivers 24,000 watts.

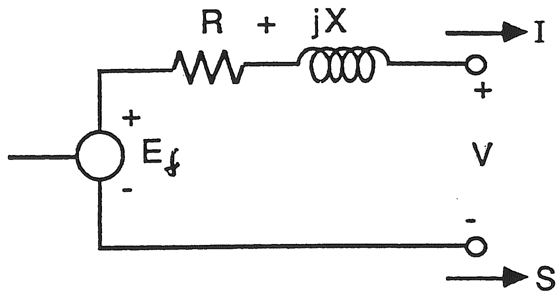
5.10 A three-phase, 9,375 kVA, 13.8 kV, 4-pole, 60 Hz, Y-connected synchronous generator has a synchronous reactance, $X_s = 17.9\ \Omega/\phi$, and is connected to an infinite bus at rated voltage. If the rotational and electrical losses are negligible, what is I_a, E_f, δ, θ, T_{drive}, phasor diagram, and power flow diagram (label completely with numerical values) for,

a) the machine as a generator ; the prime-mover throttle is opened and I_f is adjusted so that the machine delivers rated kVA, 0.8 pf lagging, to the line.

b) the machine floating on the line ; the prime-mover throttle is closed and I_f remains unchanged from part (a).

c) the machine as a generator ; the drive torque is adjusted so that maximum power is delivered to the line with I_f remaining unchanged from part (a).

5.11



A synchronous generator has the per-phase equivalent circuit shown, where the excitation emf, E, leads V by the torque angle, δ, degrees. Show that the per-phase real and reactive power delivered by the machine is,

$$P = \frac{VE}{Z}\cos(\theta - \delta) - \frac{V^2R}{Z^2} \qquad \text{(W)}$$

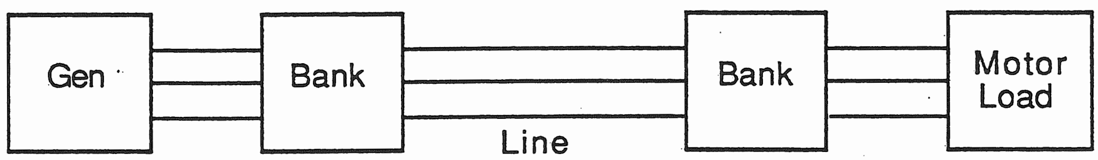
$$Q = \frac{VE}{Z}\sin(\theta - \delta) - \frac{V^2X}{Z^2} \qquad \text{(VAR)}$$

where, $Z^2 = R^2 + X^2$ and $\theta = \tan^{-1}\left[\frac{X}{R}\right]$

CHAPTER 6

MOTOR LOADS

The analysis of the synchronous generator in the preceding chapter emphasized the role of a generator in a power system, as achieving a watt-var balance at all instants of time as the load demand varies throughout each day. This chapter, then, will consider this load demand, as a motor load, which terminates an elementary power system.



A typical power system, one of many in the interconnected power grid of our nation, has a load that consists primarily of motors - as much as 80% - the remaining 20% consisting of the myriad communication, electronic, heating, lighting and other load applications. This enormous motor load consists primarily of three-phase and single-phase induction motors that power the lathes, punch-presses, vacuum cleaners, etc., that help make possible our high standard of living.

It is indeed significant that the transformer makes it possible to transmit large blocks of power over long distances, but it is the induction machine (invented by Nikola Tesla in 1887) that is the primary reason for this transmission of power. The remaining portion of the motor load is also very important in that it consists, mainly, of synchronous motors for constant-speed applications and dc motors for traction, hoisting and other variable-speed applications. These three motors, which are energy converters, will be modeled and analyzed in this chapter together with the demand they place on a power system.

6-1 SYNCHRONOUS MOTOR

Since the synchronous machine is a reversible energy converter, it can be operated as a motor by replacing its drive torque with a load torque as indicated in Fig. 6.1.

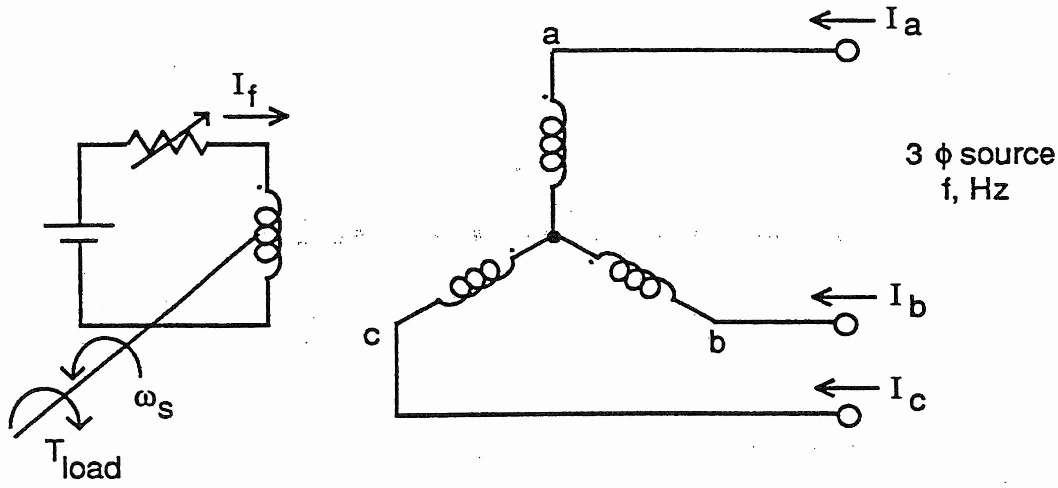


Figure 6.1 Synchronous Motor Symbolic Diagram

When the stator of the synchronous motor is connected to a three-phase source of rated voltage and frequency, a rotating, sinusoidally-distributed stator field is created, rotating around the airgap at synchronous speed, determined by the frequency of the balanced stator currents, i_a, i_b, and i_c. This stator field is given in Chapter 5-5 as,

$$B_s = \frac{3}{2} B \cos(\theta - \omega_s t) \qquad (T)$$

where,

$$\omega_s = \underset{\frac{rev}{min}}{n_s} \times \underset{\frac{rad}{rev}}{2\pi} \times \underset{\frac{min}{sec}}{\frac{1}{60}} \qquad (rad/sec)$$

and,

$$n_s = \frac{120f}{P} \qquad (rpm)$$

Since the rotor is excited with a direct current, I_f, a sinusoidally-distributed rotor field also exists in the air gap, and when the rotor is brought up to synchronous speed, as explained later in this section, the rotor field locks in behind the stator field and the sum of the two fields, called the resultant field, rotates around the air gap at synchronous speed as shown in Fig. 5.15.

Because the rotor is load-torqued, the rotor field falls behind the resultant field by a torque-angle, δ, and a driving Lorentz torque, on the rotor, is created that equals the back torques of losses and load in steadystate. The synchronous motor has no average starting torque because, at start, the inertia of the rotor prevents the rotor field from tracking or locking in with the rotating stator field and it must, by other means, be brought up to synchronous speed. The synchronous motor, therefore, runs at constant, synchronous speed or it does not run at all.

6-2 SYNCHRONOUS MOTOR EQUIVALENT CIRCUIT

When the motor is running, Faraday emfs are generated in the stator windings, as given in Eqn (5.19), and these emfs form the basis of the per-phase equivalent circuit in Fig. 6.2,

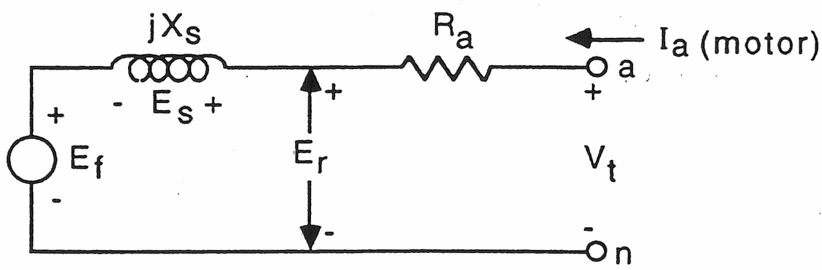


Figure 6.2 Synchronous Motor Equivalent Circuit

6-3 SYNCHRONOUS MOTOR POWER-FLOW DIAGRAM

The power-flow diagram for the motor consists of its mechanical system, coupling field, and electrical system as shown in Fig. 6.3, for a leading power factor drawn from the line.

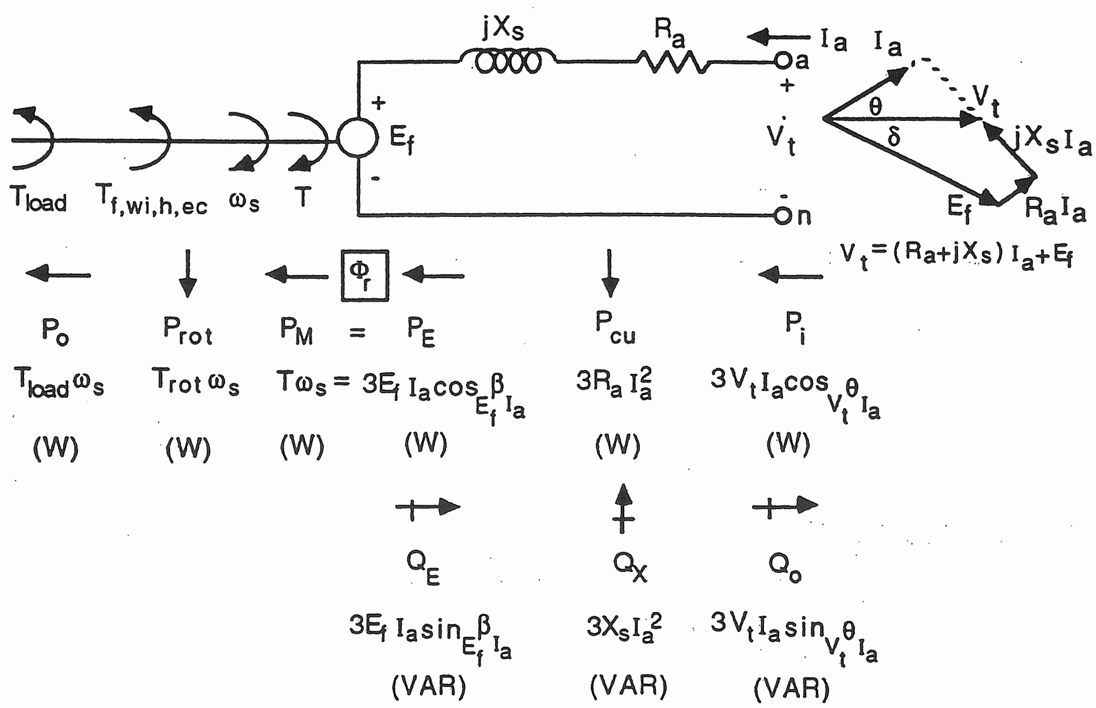


Figure 6.3 Synchronous Motor Power Flow Diagram

The real, input power is continuous, supplying the losses and load. The reactive power (slashed arrows) is delivered to the line, since the motor, for a leading power factor, looks like a large capacitive load.

6-4 SYNCHRONOUS MOTOR STEADYSTATE CHARACTERISTICS

The steadystate characteristics of the motor are interesting and important, and for simplicity, the motor will be considered large so that its mechanical and electrical losses are negligible compared to the real power (demand) it draws from the line. Furthermore, it will be connected to an infinite bus whose voltage and frequency are constant, at rated values, as shown in Fig. 6.4.

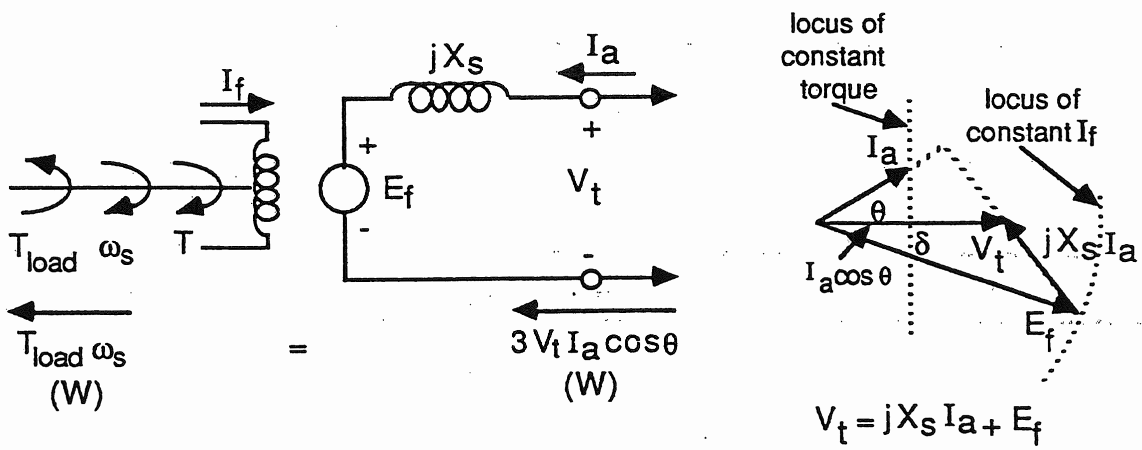


Figure 6.4 Synchronous Motor Connected to an Infinite Bus

The total electrical, real power drawn from the line is $3V_t I_a \cos\theta$, watts, and since the motor is lossless, this power is seen again, in mechanical form, as $T_{load}\,\omega_s$, watts output. Since the terminal voltage, V_t, is kept constant by the infinite bus and the rotor rotates at constant, synchronous speed, the projection of the armature current on V_t is then proportional to the load torque,

$$I_a \cos\theta = k\, T_{load} \qquad \text{(A)}$$

At <u>constant load torque</u>, then, the locus of the armature current must be the vertical dashed line on the phasor diagram of Fig. 6.4.

At <u>constant field current</u>, since the magnitude of the excitation emf, E_f, is determined by I_f, the locus of constant I_f must be the dashed circle on the phasor diagram of Fig. 6.4.

The motor characteristics are, then, a function of two variables, T_{load} and I_f. As is common with a function of two variables, one is kept constant and the other is allowed to vary, as is illustrated in Fig. 6.5.

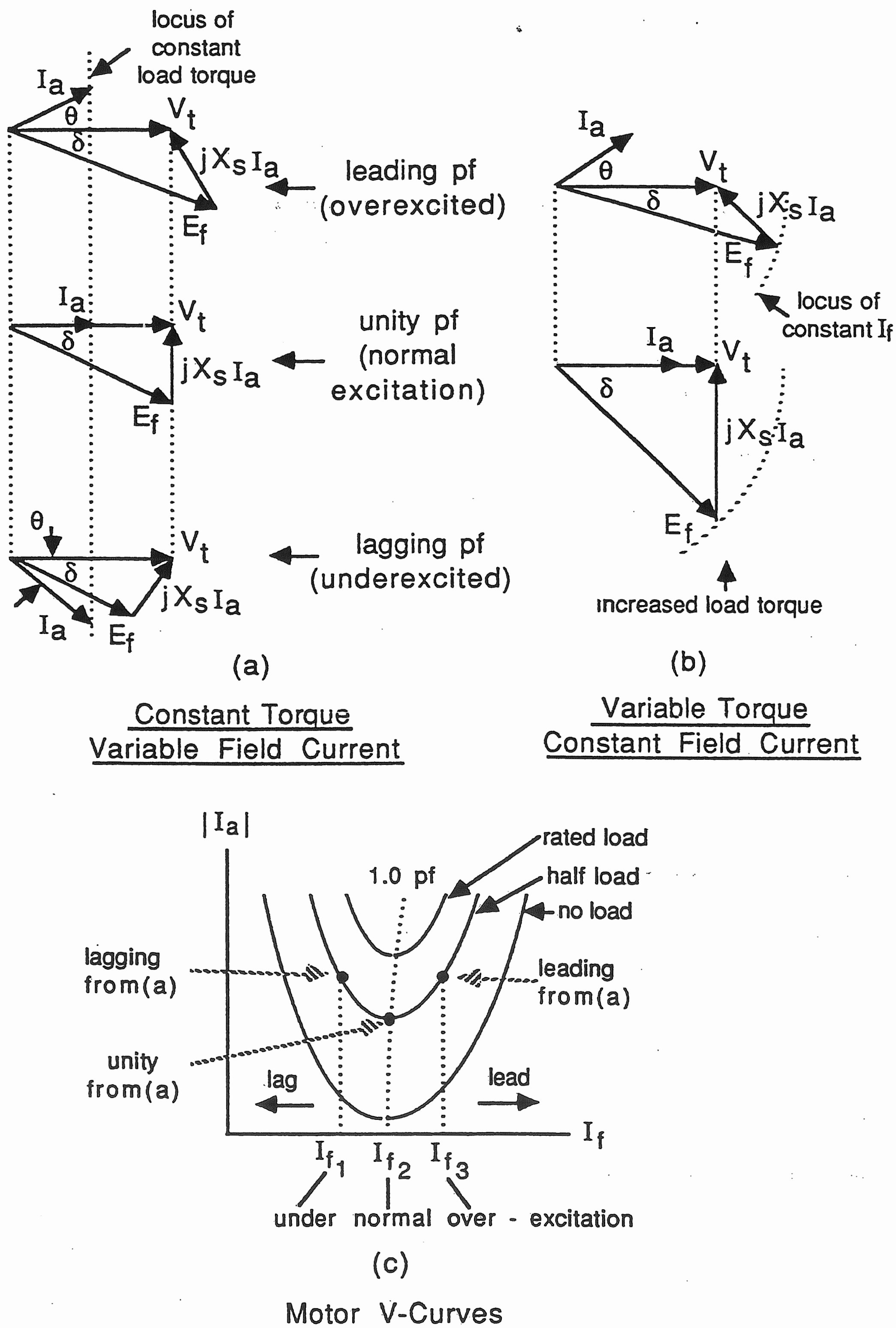


Figure 6.5 Synchronous Motor Characteristics

In Fig. 6.5 (a), the load torque is kept constant, and since

$$3\ V_t I_a \cos\theta \ = \ T_{load}\,\omega_s \qquad \text{(W)}$$

the watt-flow from the infinite bus remains the same for all three diagrams shown.

In descending order, the field current is first made <u>large</u>, so that,

$$E_f \cos\delta \ > V_t$$

and the motor is said to be <u>overexcited</u>, with a consequent leading power factor and it absorbs watts from and delivers vars to the infinite bus, (the quadrature component of I_a^* is negative). The motor, then, looks like a capacitive load on the infinite bus.

Next, the field current is <u>decreased</u>, so that,

$$E_f \cos\delta \ = \ V_t$$

and the motor is said to be <u>normally</u> excited, with a consequent unity power factor, and it absorbs watts but no vars from the infinite bus, (the quadrature component of I_a^* is zero). The motor, then, looks like a resistive load on the infinite bus.

Finally, the field current is <u>further</u> decreased, so that,

$$E_f \cos\delta \ < V_t$$

and the motor is said to be <u>underexcited</u>, with a consequent lagging power factor, and it absorbs watts and vars from the infinite bus, (the quadrature component of I_a^* is positive). The motor, then, looks like an inductive load on the infinite bus.

In summary, the synchronous motor is a load on the power system and it requires electrical watts from the line which are converted to useful mechanical watts determined by the load torque.

The motor V-curves in Fig. 6.5(c) are a plot of the magnitude of the armature current, I_a, for a <u>constant</u> load torque of zero (no-load), half load and full-load with <u>variable</u> excitation, I_f. Observe that the magnitude of the armature current follows the three phasor diagrams in Fig. 6.5 (a) for each value of constant load torque. Observe, also, that when the motor is overexcited it delivers vars to the infinite bus and when underexcited, it absorbs vars from the infinite bus. At unity power factor the magnitude of the armature current is

minimum for all three constant load torques. It can then be said, that an overexcited motor or generator delivers vars to the line while an underexcited motor or generator absorbs vars from the line.

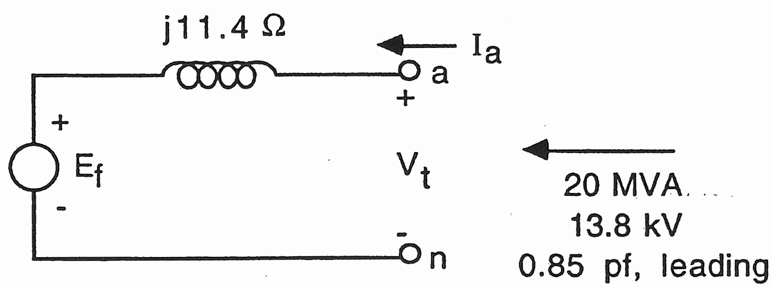
Example 6.1

A two-pole, 60 Hz, Y-connected, 13.8 kV, 20,000 kVA, $X_s = 11.4\ \Omega/\phi$, synchronous motor is connected to an infinite bus at rated voltage and frequency. The load torque and field current are adjusted so that the motor draws rated kVA, 0.85 pf leading, from the line.

(a) At what speed, rpm, will this motor run?

$$n_s = \frac{120\, f}{P} = \frac{(120)(60)}{2} = 3600 \text{ rpm}$$

(b) Draw the machine model, and determine its excitation when it is drawing rated kVA, 0.85 power factor leading from the line.



$$V_t = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} = 7{,}967\,\underline{/0^\circ}\ \text{V}(\phi)\ ;\quad I_a \text{ rated} = \frac{20{,}000}{\sqrt{3}\,(13.8)}\underline{/31.8^\circ} = 837\,\underline{/31.8^\circ}\ \text{(A)}$$

$$V_t = j X_s I_a + E_f$$

$$E_f\,\underline{/\delta} = V_t - j X_s I_a = 7{,}967\,\underline{/0^\circ} - (11.4\,\underline{/90^\circ})(837\,\underline{/31.8^\circ}) = 15.3\,\underline{/-32^\circ}\ \text{kV}\ (\phi)$$

The motor is overexcited since, $E_f \cos\delta > V_t$

(c) With the motor loaded as in part (b), draw the phasor diagram and label all voltages and currents, in per-unit, using the machine rating as a base.

$$V_{base} = 13.8 \text{ kV} \qquad I_{base} = \frac{20{,}000}{\sqrt{3}\,(13.8)} = 837 \text{ A}$$

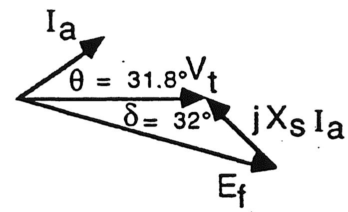
$$VA_{base} = 20 \text{ MVA} \qquad Z_{base} = \frac{(13.8)^2}{20} = 9.52\ \Omega$$

$$E_f = \frac{15.3\,\underline{/-32^\circ}}{13.8/\sqrt{3}} = 1.92\,\underline{/-32^\circ}\ \text{pu}$$

$$V_t = \frac{7.967\,\underline{/0^\circ}}{13.8/\sqrt{3}} = 1.0\,\underline{/0^\circ}\ \text{pu}$$

$$j X_s I_a = \frac{9.74 \angle 121.8^\circ}{13.8/\sqrt{3}} = 1.2 \angle 121.8^\circ \text{ pu}$$

$$I_a = \frac{837 \angle 31.8^\circ}{837} = 1.0 \angle 31.8^\circ \text{ pu}$$



Observe in this phasor diagram, for the machine as a motor, the rotor field (E_f) <u>always lags</u> the resultant field (V_t) by the torque angle, δ.

(d) Compute the power flow through this motor if the mechanical and electrical losses are negligible.

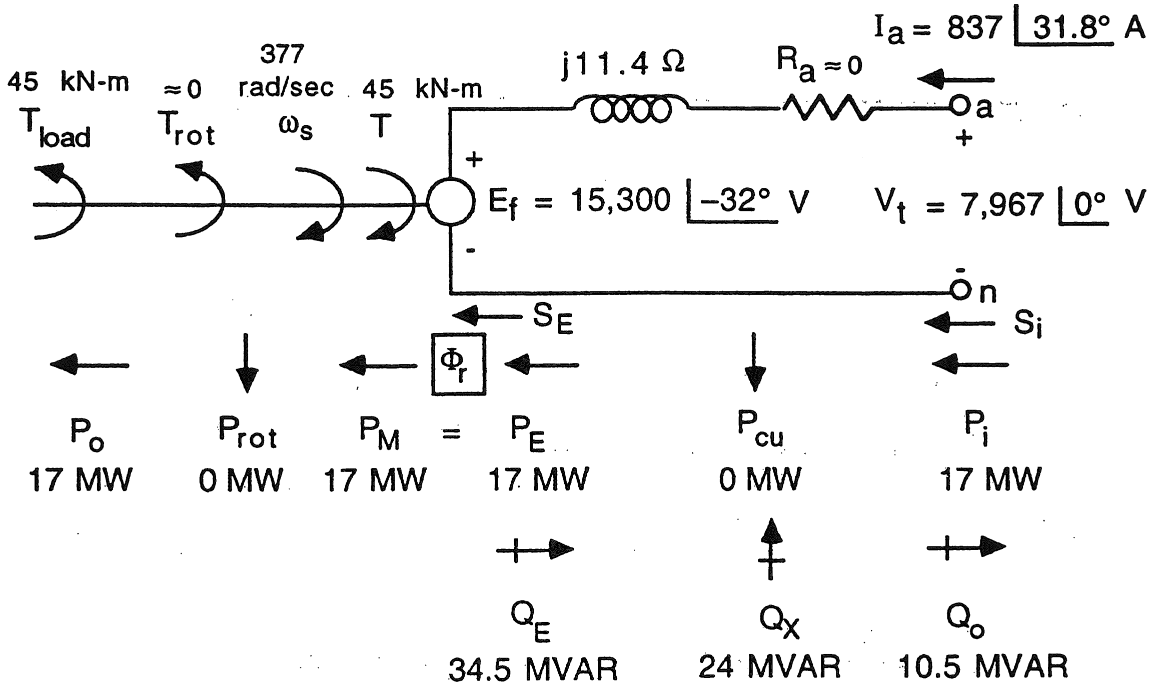
$$P_i = 3 V_t I_a \cos\theta = (3)(7{,}967)(837)\cos 31.8^\circ = 17 \text{ MW} = P_o$$

$$P_o = T_{load}\,\omega_s \ ; \ T_{load} = \frac{17 \times 10^6}{(3{,}600)(2\pi)\frac{1}{60}} = 45{,}000 \text{ N-m}$$

$$S_i = 3 V_t I_a^* = (3)(7{,}967 \angle 0^\circ)(837 \angle{-31.8^\circ}) = 17 - j\,10.5 \text{ MVA}$$

$$Q_x = 3 X_s I_a^2 = (3)(11.4)(837)^2 = 24 \text{ MVAR}$$

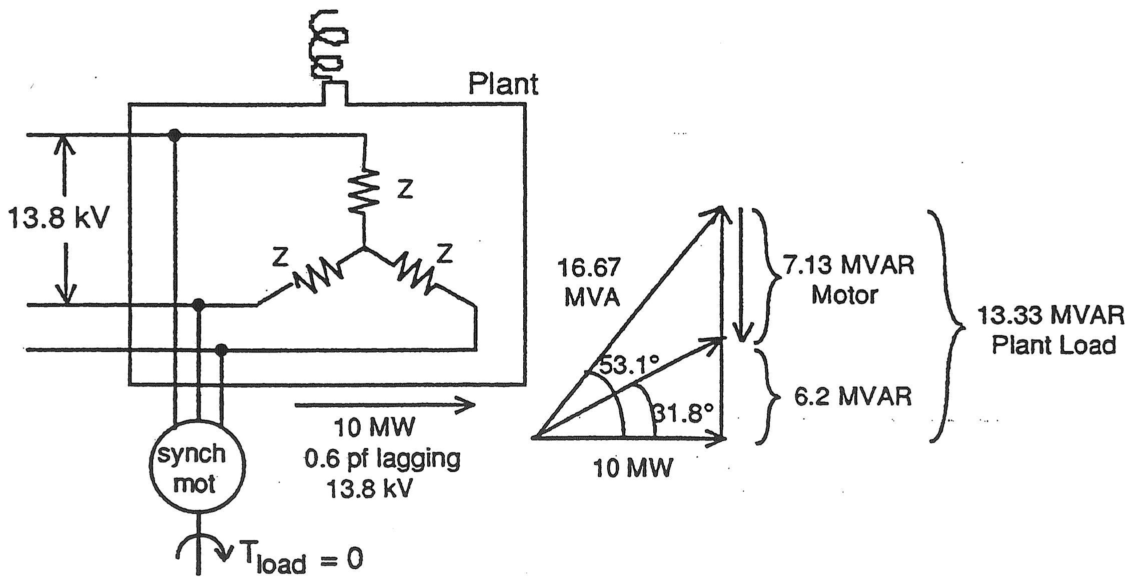
$$S_E = 3 E_f I_a^* = (3)(15{,}300 \angle{-32^\circ})(837 \angle{-31.8^\circ}) = 38.4 \angle{-63.8^\circ} = 17 - 34.5 \text{ MVA}$$



Example 6.2

A plant load consists of 10 MW, 0.6 pf lagging at 13.8 kV. Since the plant line current is higher than necessary at this low power factor, to avoid a penalty increase in rates, or even cutoff from the local utility, the synchronous motor, overexcited, of Example 6.1, will be placed across the line, running at no-load, (no watt-flow through the machine), to improve the plant power factor to 0.85 lagging, by supplying MVARs locally, thus reducing the input line current.

(a) How many MVARS must be supplied by this machine, if the input line voltage remains constant



The plant load, uncorrected, is,

$$10 + j\,13.33 = 16.67 \underline{/53.1^\circ} \quad \text{MVA}$$

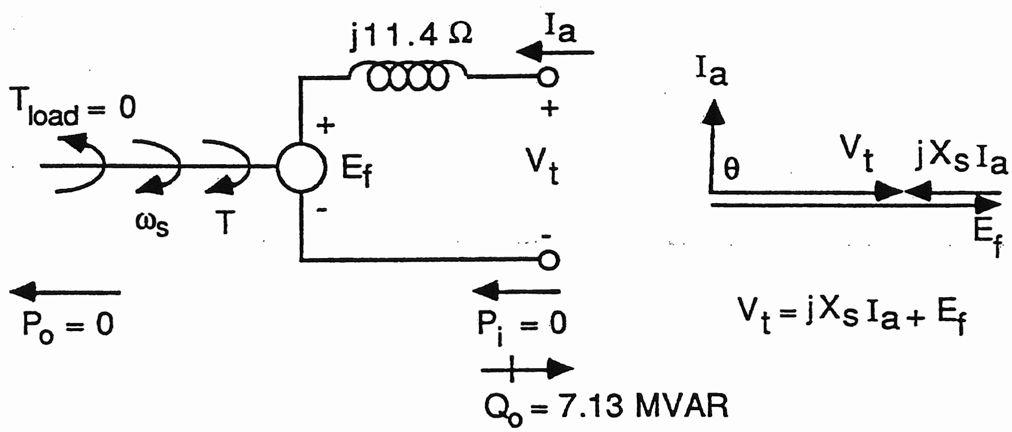
After correction, the plant-input MVAR is,

$$10 \tan 31.8^\circ = 6.2 \text{ MVAR}$$

Therefore, the motor must supply,

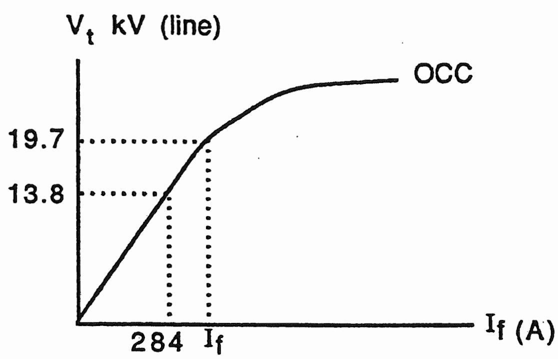
$$13.33 - 6.2 = \underline{7.13 \text{ MVAR}}$$

(b) If the open-circuit characteristic of this machine has a value, $V_t = 13.8$ kV, $I_f = 284$ A, what field current must the motor excitation be set to, so that it delivers 7.13 MVAR to the line?



$$\overset{0}{\uparrow}{T_{load}}\,\omega_s = 3V_tI_a\cos\theta = 0\;;\; Q_o = 3V_tI_a\sin\theta = (3)(7{,}967)I_a\sin 90^\circ = 7.13 \text{ MVAR}$$

$$\therefore\; I_a\cos\theta = 0$$



$$I_a = \frac{7.13 \times 10^6}{(3)(7{,}967)} = 298\underline{/90^\circ} \text{ A}$$

$$E_f = V_t - jX_sI_a$$

$$= 7{,}967\underline{/0^\circ} - (11.4\underline{/90^\circ})(298\underline{/90^\circ})$$

$$= 11{,}364\underline{/0^\circ} \text{ V}(\phi)$$

$$= 19.7 \text{ kV (line)}$$

$$I_f = \frac{19.7}{13.8} \times 284 = 405 \text{ A}$$

The motor is floating on the line, overexcited.

6-5 SYNCHRONOUS MOTOR MAXIMUM POWER

As was true with the synchronous generator is also true with the synchronous motor, a maximum real power can be drawn from the line by the motor in steady state. The power delivered to an infinite bus by a synchronous machine is given by Eqn. (5.25),

$$P = \frac{3 V_t E_f}{X_s} \sin \delta \qquad \text{(W)}$$

where,

$$P_{max} = \frac{3 V_t E_f}{X_s} \qquad \text{(W)}$$

This equation is plotted in Fig. 6.6, for a motor, where power drawn from the line is negative.

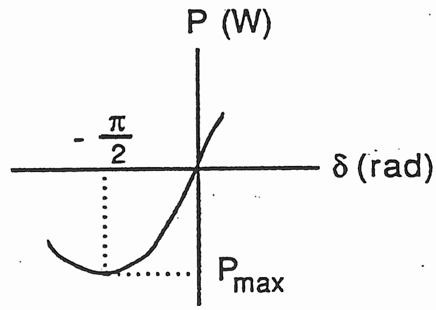


Figure 6.6 Motor Power Taken From An Infinite Bus

As the motor load-torque is increased from zero, more and more power is drawn from the line, and the rotor field continues to lag the resultant field by an increasing torque angle, to a maximum of 90°. If the load torque is increased beyond this point, more mechanical power is required than can be transferred through the motor, and the motor will then snap out of synchronism and will stall.

6-6 STARTING A SYNCHRONOUS MOTOR

When starting a synchronous motor, the stator is connected to a three-phase line, and a stator field, rotating at synchronous speed, is set up in the airgap. The rotor, with its field, is not moving at start and if the rotor possesses appreciable inertia, the rotor field cannot lock into synchronism with the stator field so the synchronous motor, inherently, has no average starting torque. A common method for starting a synchronous motor is to include an Amortisseur winding, in addition to the field winding, on the rotor. This is illustrated in Fig. 6.7.

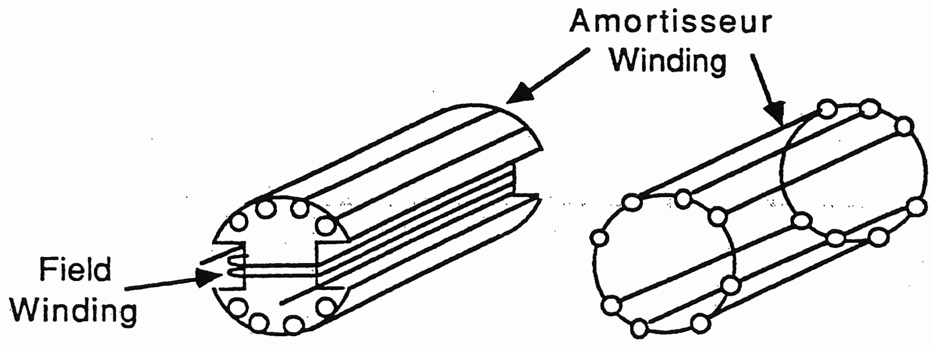


Figure 6.7 Synchronous Motor Amortisseur Winding

As indicated in Fig. 6.7, the Amortisseur winding is a "squirrel-cage" winding embedded in the rotor surface with shorted, end-rings that allow short-circuit, induced currents to flow. At start, the stator field is rotating at synchronous speed with respect to the Amortisseur winding on the stationary rotor. Voltages and currents are induced in this winding that result in a rotating rotor field that locks in with the stator field producing a starting torque on the rotor. The rotor then comes up to synchronous speed at which time the rotor field winding is energized locking the rotor at synchronous speed. Since the rotor is now turning at synchronous speed no voltages and currents are induced in the Amortisseur winding and it remains inoperative under normal load-running conditions. The details of the Amortisseur winding phenomenon will be covered in the next section on induction machines.

6-7 THREE-PHASE INDUCTION MOTOR

The induction machine is similar to the synchronous machine in that they have identical stator windings. The rotors of the machines, however, differ in their excitation. The rotor of a synchronous machine is excited with a direct current, resulting in a rotor field that is independent of loading, and the machine rotates at constant, synchronous speed. The rotor of an induction motor, however, cannot rotate at synchronous speed, and relies on its excitation by induction - hence its name. Since the induction motor cannot rotate at synchronous speed, it is an asynchronous machine, with an economically simple construction. For this reason, and the fact that it does not require dc excitation, it is by far the major load on a power system. While the induction machine is a reversible energy converter, it is far less suitable as a generator, when compared to the synchronous generator, and, therefore, its primary use is that of a motor and it will be discussed in that context. The induction machine will be considered first as a three-phase motor and later as a single-phase motor.

6-8 THREE-PHASE INDUCTION MOTOR CONSTRUCTION

The Induction machine is a cylindrical device with a stator and two types of rotors, as shown in Fig. 6.8.

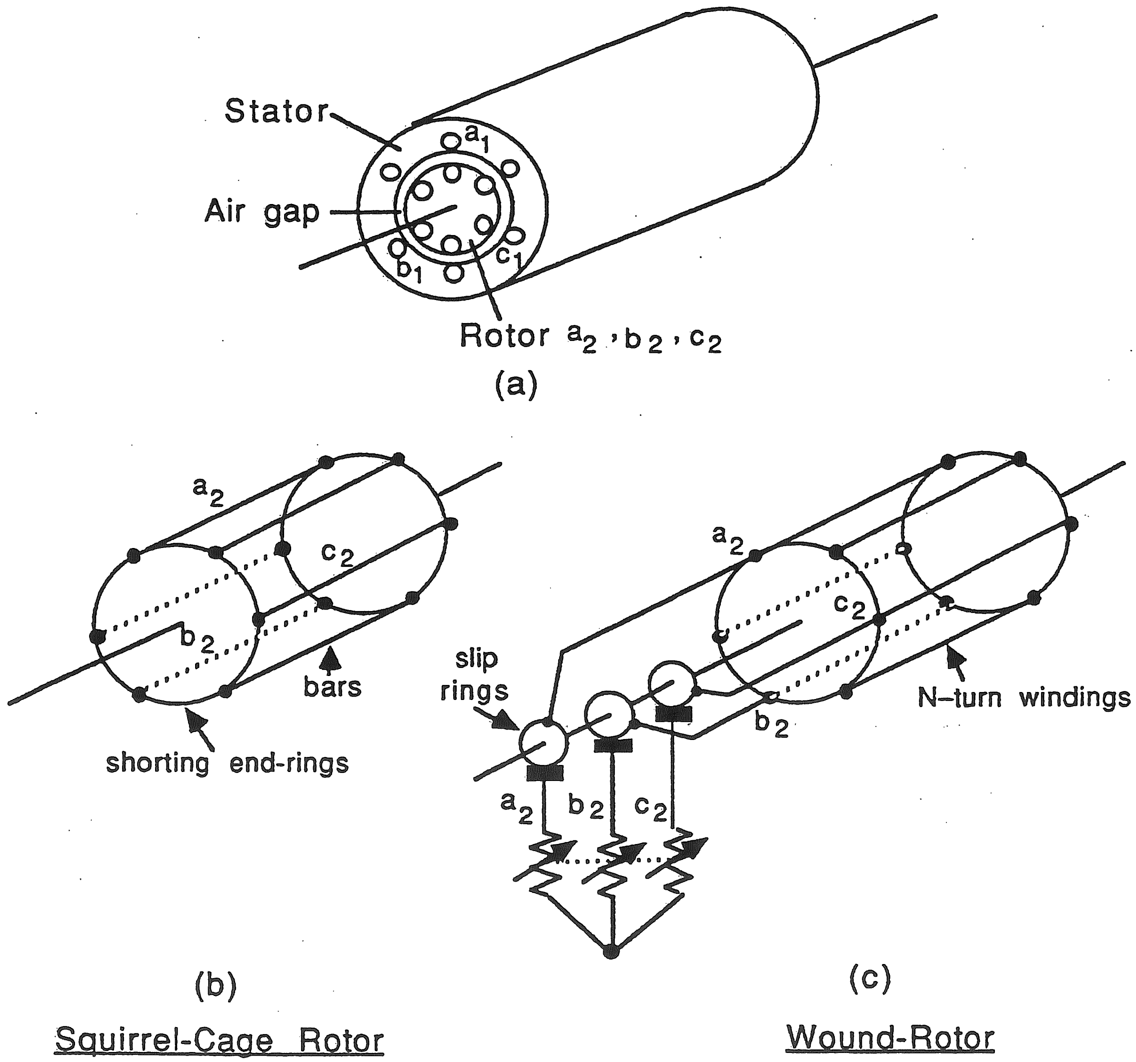


Figure 6.8 Three-Phase Induction Machine Construction

The stator of the induction machine in Fig. 6.8 (a) is identical to the stator of the synchronous machine and has three windings consisting of the phases, a, b, and c, placed on axes 120° apart, in slots cut axially along the inner surface of the stator.

The wound rotor of the induction machine in Fig. 6.8 (c) is different fro the single winding of the synchronous machine rotor, in that it has thre windings consisting of the phases a_2, b_2, and c_2, connected in wye, and placed on axes 120° apart in slots cut axially on the surface of the rotor. These winding phases are brought out to slip rings whose brushes are connected to a ganged, Y-connected, resistance bank. The resistance of the bank can be varied from zero (short-circuit) to a maximum design value, thus continuously varying the resistance of the rotor windings. Because the windings are wound in the rotor slots, this is called a wound-rotor, induction motor, which is very expensive to manufacture, and must be cost-justified in a motor load application.

The squirrel-cage rotor in Fig. 6.8 (b) is manufactured by punching sheets of hot-rolled silicon steel with the requisite number of slots and shaft opening, and then assembling them on a shaft with the slots in-line. The assembled laminated rotor is then placed in an injection molding machine where a molten aluminum alloy is injected into the slots forming, at the same time, the slot bars and the shorting end-rings. When the rotor cools, it possesses, permanently, the impedance characteristics predesigned by the molten aluminum alloy chosen for the injection. These squirrel-cage induction motors are manufactured, relatively cheaply, by the hundreds of millions, and constitute the major portion of the load on a power system. The symbolic diagram for either the squirrel-cage or the wound-rotor machine is shown in Fig. 6.9.

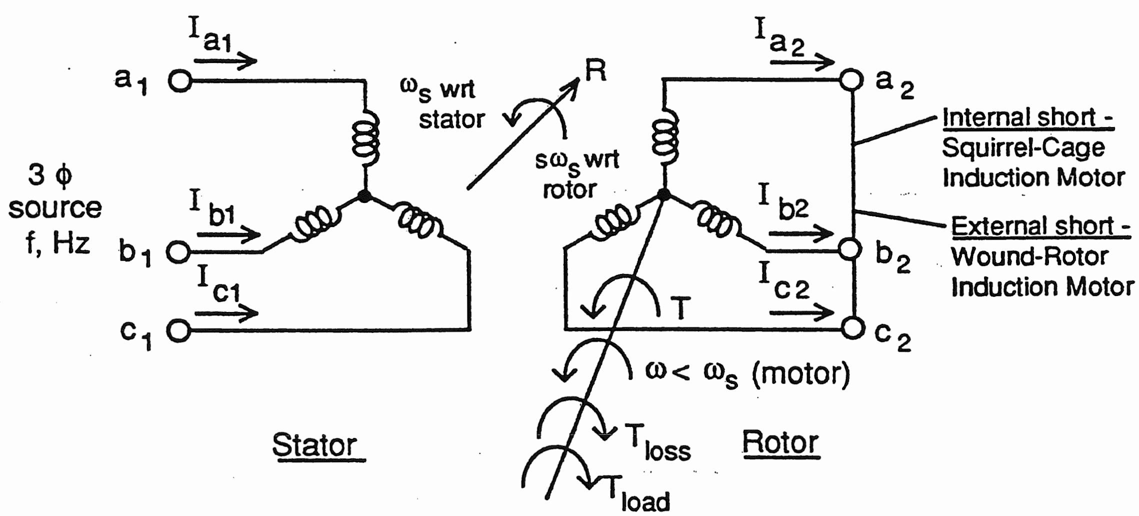


Figure 6.9 Induction Motor Symbolic Diagram

When the stator of the induction machine is connected to a three-phase source of frequency, f Hz, three balanced currents, 120° apart in time phase, limited by the balanced stator impedances, will flow through the three stator windings whose axes are 120° apart in space. A rotating stator field is produced whose

speed is synchronized to the frequency of the currents that produced it, (Eqn.5.18). If the rotor is turning at less than synchronous speed, there is relative motion between the rotor windings and the stator field, thus generating balanced emfs, 120° apart in time phase, in the short-circuited rotor windings. Balanced, short-circuit rotor currents, 120° apart in time phase, limited by the balanced rotor impedances, will then flow through the three rotor windings, generating a rotating rotor field which tracks the stator field through a load-torque angle, δ. A Lorentz torque is then produced acting on the rotor, with a consequent energy conversion from electrical to mechanical form. The quantitative details of this process will be considered next in Example 6.3.

6-9 INDUCTION MOTOR SLIP AND LORENTZ TORQUE

Example 6.3

A 60 Hz, 2-pole, 3520 rpm, three-phase, induction motor is running at rated speed. Describe the air-gap stator and rotor fields and the stator and rotor emfs.

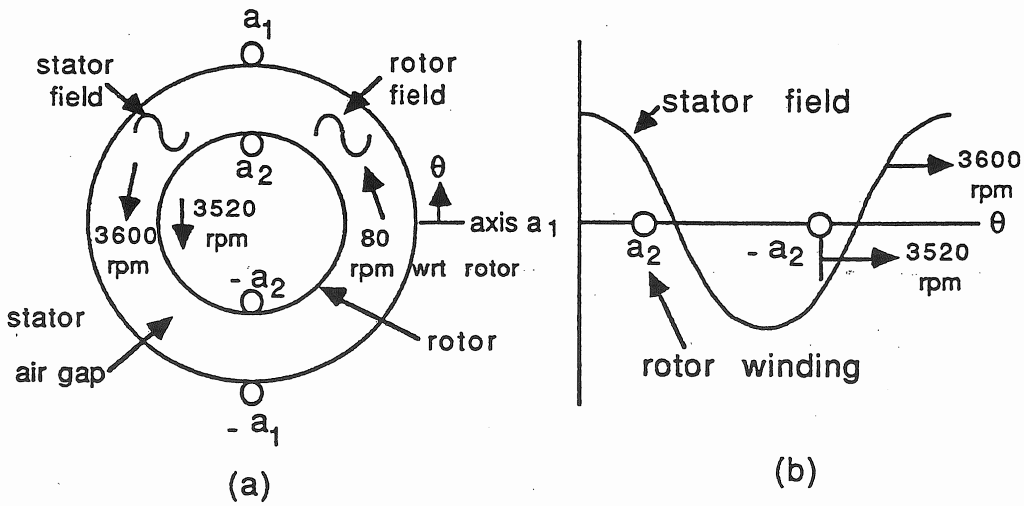


Figure 6.10 Induction Motor Air-Gap Fields

In Fig. 6.10, only the a - phases of the stator and rotor are shown. The b and c phases are omitted for clarity. When the loaded motor is placed on-line, at rated voltage and frequency, the following events take place,

1. Standing in a fixed frame, looking into the airgap, the stator field is seen rotating, with respect to the stator, at 3600 rpm, and the rotor is rotating more slowly at 3520 rpm.

2. In a rotating frame on the rotor, looking into the air-gap, the stator field is seen slowly moving by, counterclockwise, at 80 rpm, with respect to the rotor. The stator field is said to be slipping by the rotor at slip-speed,

$$n_{slip} = n_s - n = 3600 - 3520 = 80 \quad \text{(rpm)}$$

$$\text{where, } n = \text{speed of the rotor}$$

3. A quantity, slip, is defined as the slip speed in per-unit,

$$s = \frac{n_s - n}{n_s} = \frac{3600 - 3520}{3600} = 2.22\ \%$$

$$\text{or,} \qquad n_{slip} = n_s - n = sn_s = 80 \quad \text{(rpm)}$$

The stator field is slipping by the rotor at 2.22% of synchronous speed. (rad/sec)

4. For a machine with P-poles, the angular velocity, rad/sec, corresponding to the above speeds in rpm, is related by a constant,

$$\underset{\frac{rad}{sec}}{\omega_{slip}} = \underset{\frac{rev}{min}}{n_{slip}} \times \underset{\frac{pole\text{-}pair}{rev}}{\frac{P}{2}} \times \underset{\frac{rad}{pole\text{-}pair}}{2\pi} \times \underset{\frac{min}{sec}}{\frac{1}{60}} \; ; \; \underset{\uparrow \text{stator field}}{\omega_s = n_s \times \frac{P}{2} \times 2\pi \times \frac{1}{60}} \; ; \; \underset{\uparrow \text{rotor}}{\omega = n \times \frac{P}{2} \times 2\pi \times \frac{1}{60}}$$

$$\text{therefore,} \qquad s = \frac{n_s - n}{n_s} = \frac{\omega_s - \omega}{\omega_s} = \frac{377 - 368.6}{377} = 2.22\%$$

$$\text{or,} \qquad \omega_{slip} = \omega_s - \omega = s\omega_s = 8.37 \quad \text{rad/sec.}$$

The stator field is slipping by the rotor windings at an angular velocity of 8.37 rad/sec or a speed of 80 rpm.

5. Since the relative motion between the stator field and the rotor windings is 8.37 rad/sec, emfs will be generated in the rotor windings, whose frequency corresponds to this relative angular velocity,

$$\omega_{slip} = s\omega_s$$

$$2\pi f_R \doteq s\, 2\pi f$$

$$f_R = sf = (0.0222)(60) = 1.333 \text{ Hz}$$

Three balanced rotor emfs, 120° apart in time-phase, produce three, balanced, short-circuit rotor currents at slip frequency, 1.333 Hz.

6. The rotor currents flowing through the rotor windings then produce a rotor field, whose velocity depends on the frequency of the currents that produced this rotor field, (refer to p200, but change stator in Fig. 5.14 to this rotor in Fig. 6.9)

$$B_R = \frac{3}{2} B_2 \cos(\theta - s\omega_s t) \qquad (T)$$

$$\uparrow$$

$$2\pi sf = 8.37 \qquad (rad/sec)$$

A sinusoidally-distributed rotor field is created, rotating counterclockwise, at 8.37 rad/sec or 80 rpm, with respect to the rotor.

7. In a fixed frame on the stator, looking into the air gap of Fig. 6.10, the rotor is turning, counterclockwise, at 3250 rpm, and the rotor field is rotating, counterclockwise, at 80 rpm with respect to the rotor, then, the rotor field is rotating at synchronous speed with respect to the stator,

$$_{RF}n_S = n + sn_s = (1-s)\, n_s + sn_s = n_s \qquad (rpm)$$

This equation is true for any value of slip.

In summary, from Fig. 6.10, the stator and rotor fields are both rotating at synchronous speed, regardless of slip, and, therefore, they can be added to produce the resultant field in the airgap as shown in Fig. 6.11.

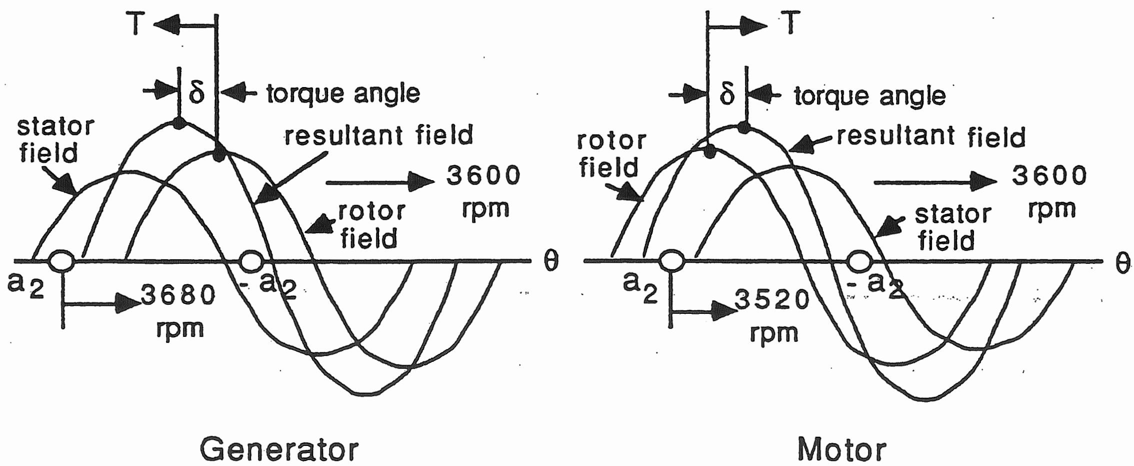


Figure 6.11 Induction Machine Generator and Motor Fields

Whether the machine is a motor or generator, the resultant field always rotates at synchronous speed, from the foregoing discussion.

As a generator, the rotor (represented by its a-phase winding in Fig. 6.11) is driven by a prime mover at greater than synchronous speed, for example, 3680 rpm. The resultant field is, then, slipping negatively with respect to the rotor windings, at a slip,

$$s = \frac{n_s - n}{n_s} = \frac{3600 - 3680}{3600} = -2.22\,\%$$

The rotor field is driven ahead of the resultant field through a torque angle, δ, with a consequent, back-Lorentz torque on the rotor, confirming generator action.

As a motor, the rotor is torqued to a full-load speed of 3520 rpm and the resultant field is slipping positively with respect to the rotor windings at a slip,

$$s = \frac{n_s - n}{n_s} = \frac{3600 - 3520}{3600} = +2.22\,\%$$

The rotor field is torqued behind the resultant field through a torque angle, δ, with a consequent, forward-Lorentz torque on the rotor, confirming motor action.

The operating modes of the induction machine in Example 6.1 can now be summarized.

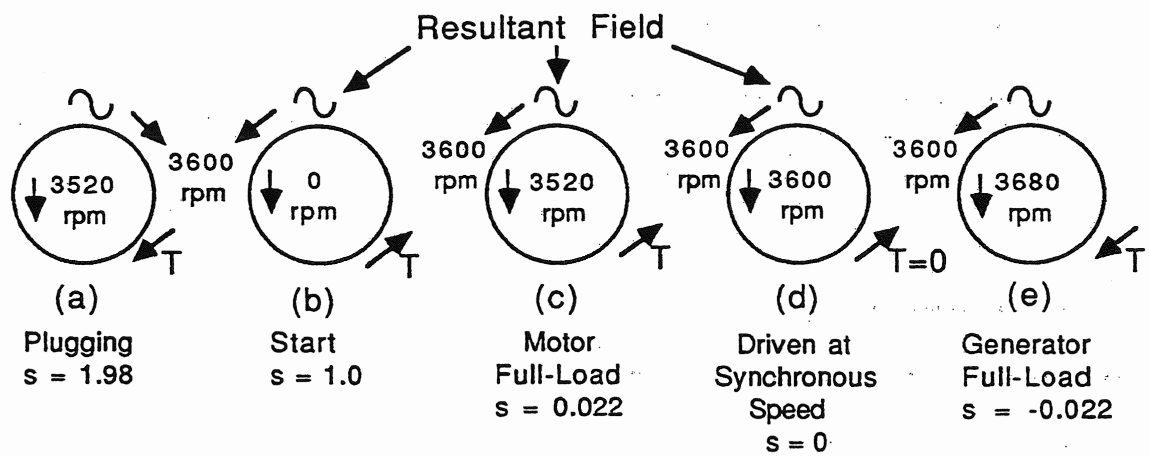


Figure 6.12 Induction Machine Operating Modes

The air-gap resultant fields are shown in Fig. 6.12 for various operating modes.

At <u>start</u>, the rotor is not turning, and the resultant field is slipping 100%,

$$s = \frac{3600 - 0}{3600} = 1.0 \quad ; \quad f_R = (1.0)(60) = 60\ \text{Hz}$$

The interaction between the resultant and rotor fields produces a forward Lorentz torque that accelerates the machine to its steadystate, loaded speed. The induction machine, inherently, has a starting torque.

At <u>full-load</u>, as a <u>motor</u>, the resultant field is slipping 2.22%,

$$s = \frac{3600 - 3520}{3600} = 0.022 \quad ; \quad f_R = (0.022)(60) = 1.33\ \text{Hz}$$

The interaction between the stator and rotor fields, with a consequent resultant field, produces a forward- Lorentz torque equal to the back torques of losses and load in steady state.

When the motor, running at rated speed, is plugged or braked to a halt, two stator leads are suddenly interchanged, thus reversing the resultant field with a consequent slip of 198 %,

$$s = \frac{-3600 - (3520)}{-3600} = 1.98 \quad ; \quad f_R = (1.98)(60) = 118 \text{ Hz}$$

The interaction between the resultant and rotor fields produces a large back-Lorentz torque bringing the machine to a fast halt, whence it is taken off line.

The induction machine cannot rotate at synchronous speed, so it must be driven at synchronous speed, in which case, the resultant field does not slip by the rotor windings, resulting in a consequent zero, Lorentz torque. The back torque of the losses must then be supplied by the prime mover.

At full-load, as a generator, the machine is driven at greater than synchronous speed so that the resultant field slips, –2.22 %,

$$s = \frac{3600 - 3680}{3600} = -0.022 \quad ; \quad f_R = (0.022)(60) = 1.33 \text{ Hz}$$

The interaction between the resultant and rotor fields produces a back-Lorentz torque, which, when added to the back-losses torque, is equal to the forward prime-mover torque, in steady state.

6-10 INDUCTION MACHINE MODEL

The equivalent circuit of the induction machine is greatly facilitated by realizing it is essentially a transformer with a rotating secondary. Transformer theory, then, will be judiciously applied to stator-port a, of this three-port machine. The machine is considered, first, ideal, as in transformer theory, observing that the resultant, air-gap field is rotating at synchronous speed with respect to the stator winding a_1, and, at slip speed with respect to the short-circuited rotor winding, a_2, as in Fig. 6.13,

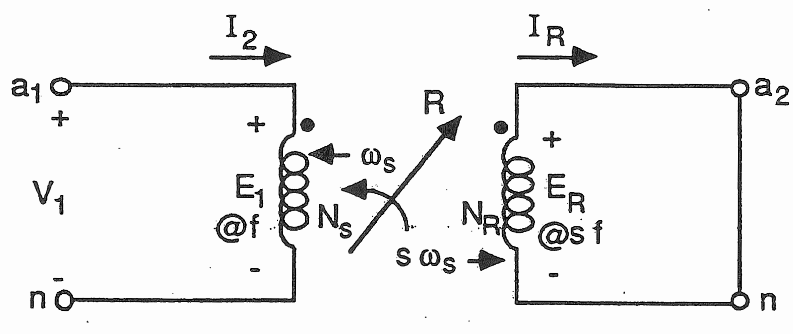


Figure 6.13 Induction-Machine Ideal Model

From Fig. 5.11 and Eqn. (5.13), speed emfs are generated, simultaneously, in the stator and rotor windings at stator and slip-frequencies, respectively , as,

$$\begin{aligned} E_1 &= 4.44\ f N_S \Phi_R \qquad (V) \\ E_R &= 4.44\ sf N_R \Phi_R \qquad (V) \end{aligned} \qquad (6.1)$$

where, N_S = stator turns and N_R = rotor turns

The effective turns-ratio is a function of rotor speed, by dividing Eqns. (6.1),

$$\frac{E_1}{E_R} = \frac{a}{s} \qquad \text{where,} \qquad a \triangleq \frac{N_S}{N_R} \qquad (6.2)$$

From transformer theory, (Fig. 3.9), the stator current, I_2, in Fig. 6.13, consists of an exciting current component which sets up the resultant field in the air-gap and varies with emf, E_1, and a load component, whose mmf exactly opposes the rotor mmf,

$$I_2 = I_E + I_2'$$

The load mmfs must sum to zero so that the air-gap field remains unchanged for a given emf, E_1. The exciting current in an ideal machine is negligibly small, so that,

$$N_S I_2 - N_R I_R = 0$$

or,

$$\frac{I_2}{I_R} = \frac{1}{a} \qquad \text{where,} \qquad a \triangleq \frac{N_S}{N_R} \qquad (6.3)$$

The ideal induction machine is then characterized by Eqns. (6.2), (6.3).

The induction machine is not ideal, however, and the stator and rotor windings, inherently, have resistance and leakage flux as shown in Fig. 6.14.

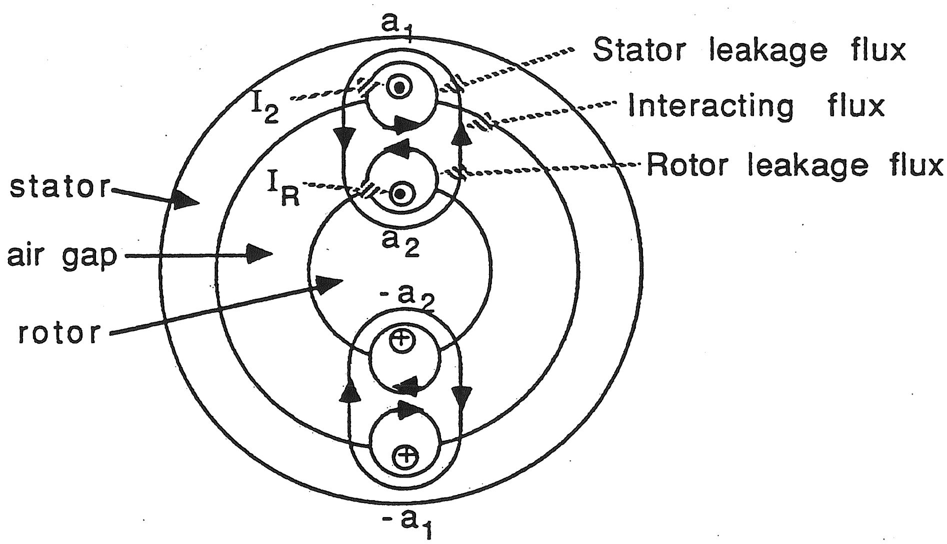


Figure 6.14 Machine Leakage Flux

Furthermore, the exciting current is not negligible, since watts and vars must be supplied to set up the core losses, leakage, and air-gap fields, resulting in Fig. 6.15.

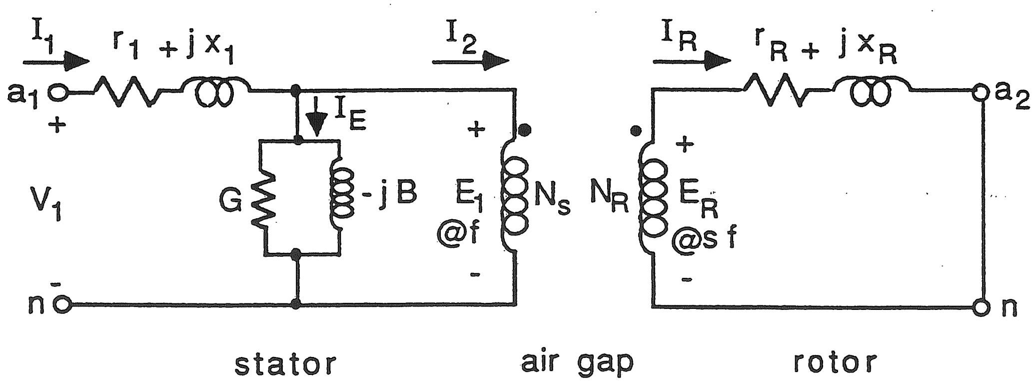


Figure 6.15 Exact Per-Phase Equivalent Circuit

The circuit in Fig. 6.15 now accounts for the core and copper losses, in addition to the vars required to set up the leakage and interacting air-gap fields, where the rotor leakage reactance is at slip frequency,

$$x_R = 2\pi s f L_R \qquad (\Omega)$$

The equivalent circuit is simplified by referring the rotor impedance and terminal short-circuit to the stator.

$$\frac{E_R}{I_R} = r_R + j\,x_R \qquad (\Omega)$$

$$\frac{E_1}{I_2} = \frac{\frac{a}{s}E_R}{\frac{1}{a}I_R} = \frac{a^2}{s}(\,r_R + j\,2\pi sf\,L_R\,) \qquad (\Omega)$$

$$= a^2\frac{r_R}{s} + j\,a^2\,2\pi f\,L_R$$

$$\frac{E_1}{I_2} = \frac{r_2}{s} + j\,x_2 \qquad \text{where,} \quad r_2 \triangleq a^2 R_R \quad (\Omega) \qquad (6.4)$$

$$x_2 \triangleq \frac{a^2}{s}\,x_R$$

The equivalent circuit will be further simplified by omitting the core losses-conductance, G, and by representing the core losses of hysteresis and eddy-currents as a back torque, with little error, later, in the power-flow diagram. The equivalent circuit is redrawn using Eqn. (6.4)

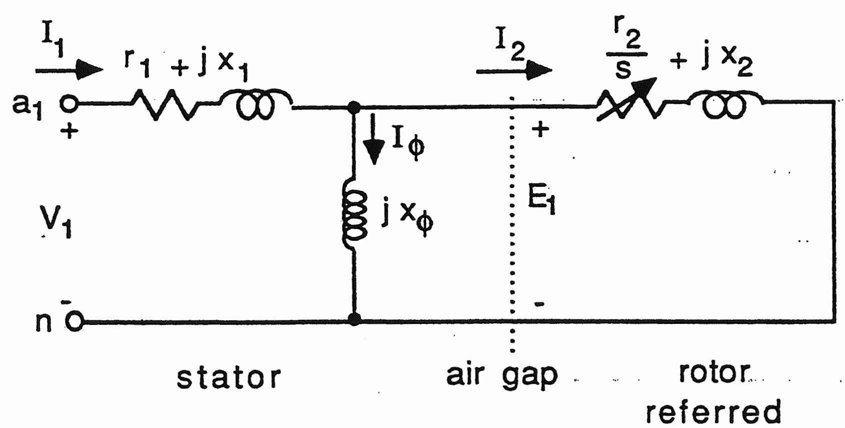


Figure 6.16 Simplified Machine Equivalent Circuit

Observe in Fig. 6.16 and Eqn (6.4), the rotor impedance, referred, is at stator frequency, confirming the fact that the rotor field always rotates at synchronous speed with respect to the stator, regardless of slip. Observe, also, that the rotor resistance, referred, is a function of speed and this cannot be true since the

rotor resistance is constant. This apparent anomaly is resolved by partitioning r_2/s into two parts,

$$\frac{r_2}{s} = r_2 + \left(\frac{1-s}{s}\right) r_2 \qquad (\Omega) \qquad (6.5)$$

This partition is included in Fig. 6.17.

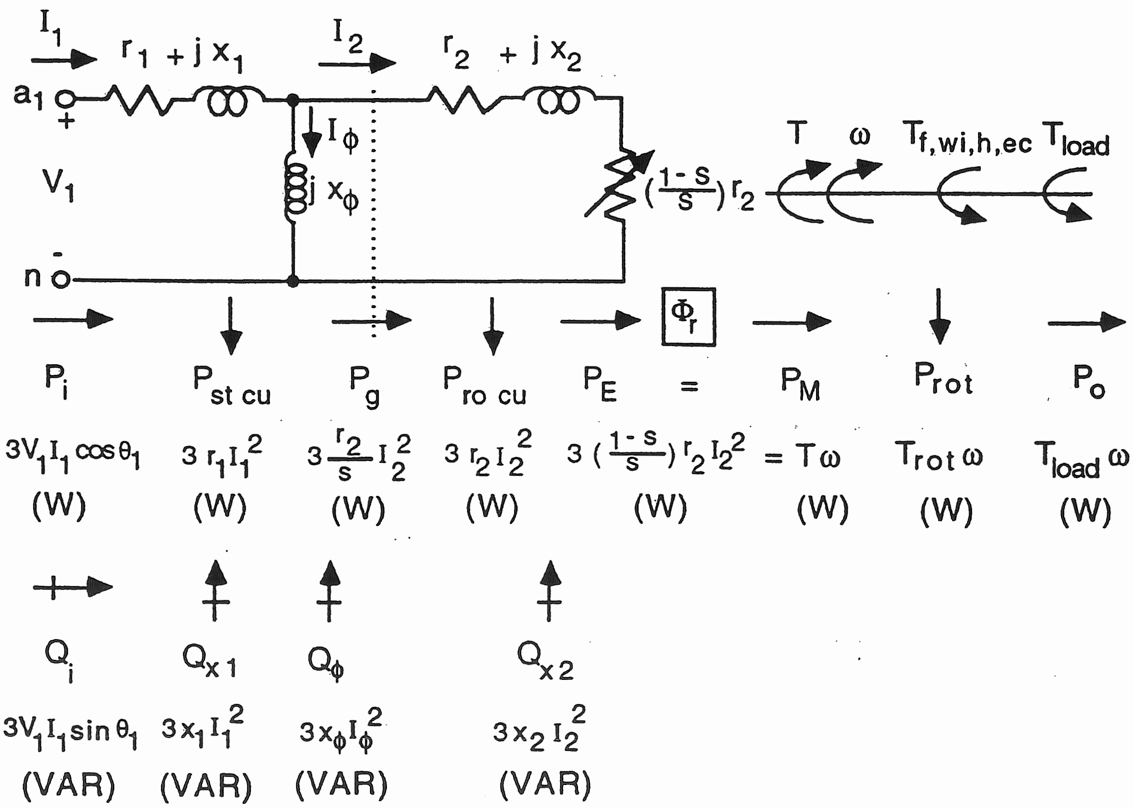


Figure 6.17 Induction Machine Power Flow

The significance of the partitioning in Eqn. (6.5) is now apparent in the power-flow diagram. The real power delivered to the machine is,

$P_i = P_{st\,cu} + P_g$ (w) where, P_g = power delivered across the air-gap

$P_g = P_{ro\,cu} + P_E$ (w) P_E = power delivered to the coupling field

The resistance $(\frac{1-s}{s})\, r_2$ represents a sink or source of energy, P_E, delivered to or taken from the coupling field, depending on whether the slip is positive or negative. Since the energy stored in the coupling field is constant (the magnitude of the resultant field remains constant as it rotates around the air-gap),

$$P_E = P_M \quad \text{(W)} \quad \text{where,} \quad P_M = \text{power taken from the coupling field}$$

$$P_M = P_{rot} + P_o \quad \text{(W)} \qquad P_{rot} = \text{field and mechanical losses}$$

The rotor angular velocity is not synchronous angular velocity, and, since the torques acting on the shaft are independent of the machine pole-pairs, the velocity is calculated as,

$$\omega = \underset{\frac{\text{rev}}{\text{min}}}{n} \times \underset{\frac{\text{rad}}{\text{rev}}}{2\pi} \times \underset{\frac{\text{min}}{\text{sec}}}{\frac{1}{60}} \qquad \text{(rad/sec)}$$

The reactive power delivered to the machine is accounted for in setting up the stator and rotor leakage fields and the interacting field. In contrast to the synchronous generator, the <u>induction machine cannot stand alone as a generator</u> since it requires vars from the line to set up its fields.

In the analysis of this machine, the power delivered across the air-gap, P_g, becomes very important. From Fig. 6.17,

$$P_g = P_i - P_{st\,cu} = 3\, I_2^2 \frac{r_2}{s} \qquad \text{(W)}$$

then, $$P_{ro\,cu} = 3\, I_2^2\, r_2 = s\, P_g \qquad \text{(W)} \qquad (6.6)$$

and, $$P_E = 3\, I_2^2 \left(\frac{1-s}{s}\right) r_2 = (1-s)\, P_g = T\omega \quad \text{(W)}$$

For a given power transferred across the air-gap, the rotor copper losses, in Eqn. (6.6), vary directly as the slip. For this reason, induction machines, at full load, are designed to run at low slip for high efficiency. The power transferred across the air gap, in the third equation of Eqn. (6.6), is rewritten,

$$P_g = \frac{T\omega}{1-s} \quad \text{(W)} \qquad \text{where,} \quad \omega = (1-s)\,\omega_s \quad \text{(rad/sec)}$$

$$P_g = T\,\omega_s \quad \text{(W)} \qquad \text{where,} \quad \omega_s = n_s \times \frac{2\pi}{60} \quad \text{(rad/sec)} \qquad (6.7)$$

The power transferred across the air-gap is directly proportional to the Lorentz torque, since ω_s is a constant for the machine, and P_g is said to be expressed in terms of <u>synchronous watts</u>. The power flow diagram can now be given in terms of Eqns. (6.6), (6.7).

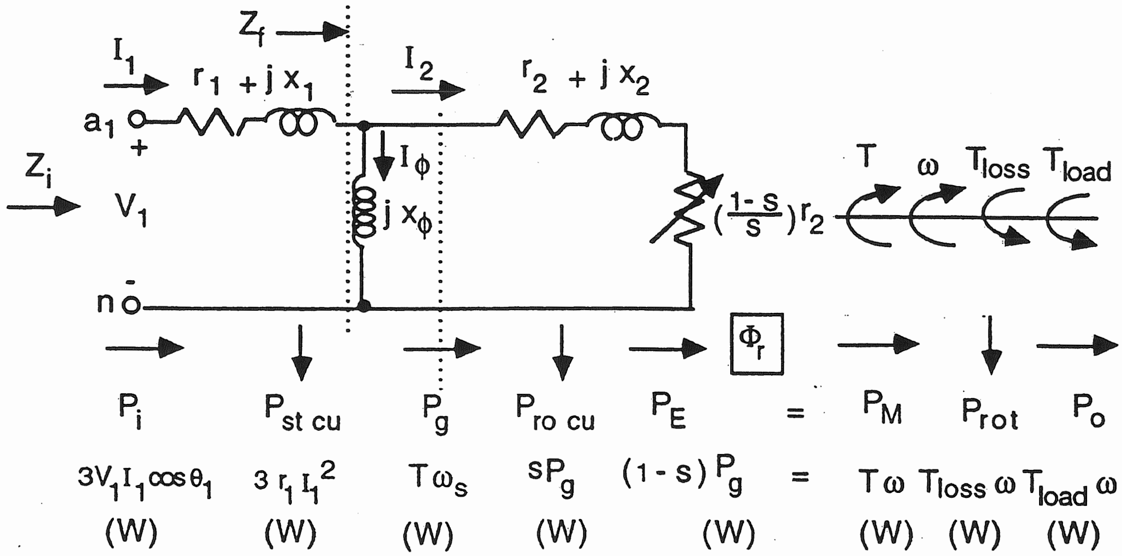


Figure 6.18 Relabled Machine Power Flow

The <u>rating</u> of an induction motor is an important guide to the power flow of this machine since it should not be exceeded for any length of time. For example, a motor rating might be,

3ϕ, 60 Hz, 4-pole, 440 volt, 120 amp, 1746 rpm, 100 hp
(electrical end) (mechanical end)

The electrical and mechanical ratings are always given at the extreme ends of the machine, i.e., at the <u>input</u> line terminals and the <u>load</u> power output.

From this rating, at rated input voltage, when the motor is full load-torqued at 100 hp x 746 W/hp, the input line current will be 120 A, the machine will run at 1746 rpm or will slip 3%, and for a given pf, the full-load efficiency can be determined.

Example 6.4

A 3-phase, Y-connected, 220-volt, 7.5 hp, 19A, 60 Hz, 6-pole, induction motor has the following constants, in ohms per phase, referred to the stator:

$r_1 = 0.294$ $r_2 = 0.144$

$x_1 = 0.504$ $x_2 = 0.209$

$x_\phi = 13.25$

The total friction, windage and core losses is assumed to be constant at 403 watts, independent of load.

For a slip of 2.0 percent, compute the speed, output- torque and power, stator current, power factor and efficiency when the motor is operated at rated voltage and frequency.

Looking towards the left of the magnetizing field in Fig. 6.18,

$$Z_f = \left(\frac{r_2}{s} + j x_2\right) \parallel j x_\phi = R_f + j X_f \qquad (\Omega)$$

For s = 0.02,

$$Z_f = R_f + j X_f = 5.41 + j\,3.11\ \Omega$$

$$Z_i = (r_1 + R_f) + j(x_1 + X_f) = 5.70 + j\,3.61 = 6.75\,\underline{/32.4^\circ} \qquad (\Omega)$$

$$V_1 = \frac{220}{\sqrt{3}}\,\underline{/0^\circ} = 127\,\underline{/0^\circ} \quad (V)$$

$$I_1 = \frac{V_1}{Z_i} = \frac{127\,/0^\circ}{6.75\,\underline{/32.4^\circ}} = 18.8\,\underline{/-32.4^\circ} \quad (A)$$

$$pf = \cos 32.4^\circ = 0.844 \text{ lagging}$$

$$n_s = \frac{120f}{P} = \frac{(120)(60)}{6} = 1{,}200 \quad (rpm)$$

$$\omega_s = (1{,}200)\left(\frac{2\pi}{60}\right) = 125.6 \quad (rad/sec)$$

$$n = (1-s)n_s = (1-0.02)(1{,}200) = 1{,}176 \quad (rpm)$$

$$P_g = 3 I_2^2 \frac{r_2}{s} = 3 I_1^2 R_f = (3)(18.8)^2(5.41) = 5{,}740 \quad (W)$$

$$P_E = (1-s)P_g = P_M = (1-0.02)(5{,}740) = 5{,}630 \quad (W)$$

$$P_{rot} = 403 \quad (W)$$

$$P_o = P_M - P_{rot} = 5{,}630 - 403 = \frac{5{,}230\ (W)}{746\ (W/hp)} = 7.0 \text{ hp}$$

$$T_{load} = \frac{P_o}{\omega} = \frac{5{,}230}{1{,}176 \times \frac{2\pi}{60}} = 42.5 \ (N\text{-}m) \text{ or } 31.4 \ (lb\text{-}ft)$$

$$P_{st\,cu} = 3 I_1^2 r_1 = (3)(18.8)^2(0.294) = 312 \quad (W)$$

$$P_{ro\,cu} = sP_g = (0.02)(5{,}740) = 115 \quad (W)$$

$$P_i = 3 V_1 I_1 \cos\theta_1 = (3)(127)(18.8)\cos 32.4^\circ = 6{,}060 \quad (W)$$

$$\eta = \frac{P_o}{P_o + \text{losses}} = \frac{5{,}230}{5{,}230 + 830} = 86.3 \ (\%)$$

6-11 INDUCTION MACHINE LORENTZ TORQUE

As the induction machine is loaded, the slip increases, and the generated Lorentz torque must vary to accommodate the losses and increased load. We will now consider the T vs slip relationship, in steadystate, as the machine is torqued from start to run. We refer to the equivalent circuit in Fig. 6.19(a) where we concentrate on the fact that P_g is directly proportional to the Lorentz torque, T. The equivalent circuit is simplified in Fig. 619(b) by replacing the circuit to the left of the dashed line by its Thevenin equivalent,

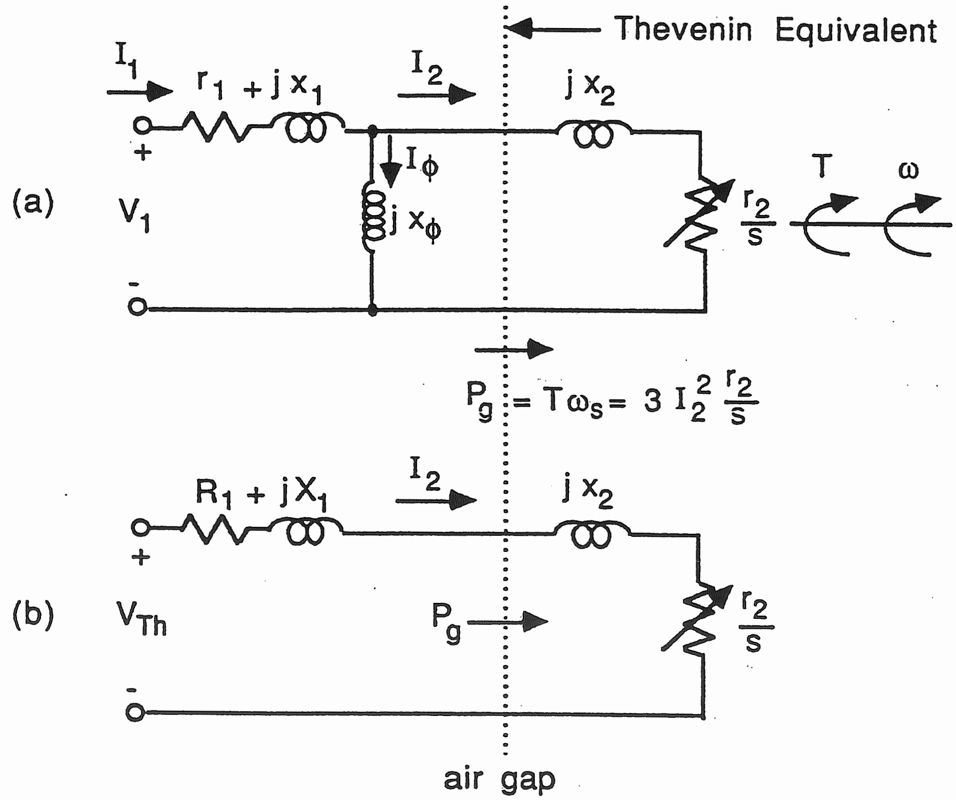


Figure 6.19 Machine Thevenin Equivalent

where,
$$|V_{Th}| = V_{oc} = \left| \frac{j\, x_\phi}{r_1 + j\,(x_1 + x_\phi)} \times V_1 \right| \quad (V)$$

$$Z_{Th} = R_1 + j X_1 = (r_1 + j x_1) \;||\; j x_\phi \quad (\Omega)$$

when V_1 is replaced by a short-circuit

From Fig. 6.19 (b),

$$|I_2| = \frac{V_{Th}}{\sqrt{(R_1 + \frac{r_2}{s})^2 + (X_1 + x_2)^2}} \qquad \text{(A)}$$

The Lorentz torque is then,

$$T = \frac{P_g}{\omega_s} = \frac{3\, I_2^2 \frac{r_2}{s}}{\omega_s} = \frac{1}{\omega_s} \times \frac{3\, V_{Th}^2 \frac{r_2}{s}}{(R_1 + \frac{r_2}{s})^2 + (X_1 + x_2)^2} \quad \text{(N-m)} \qquad (6.8)$$

If Eqn. (6.8) is plotted for $0 \le s \le 1.0$, Fig. 6.20 results,

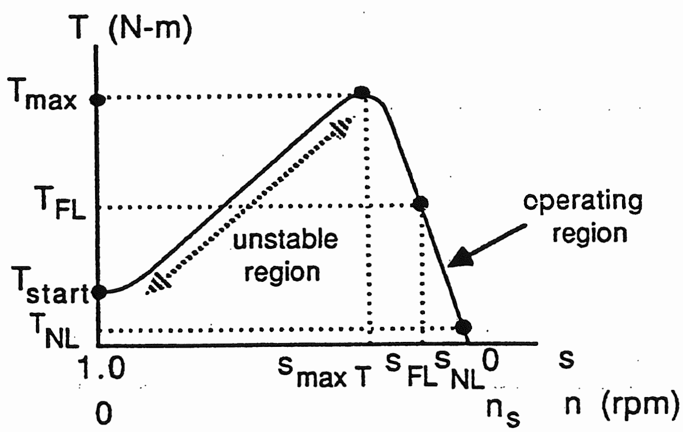


Figure 6.20 Induction Machine Torque vs Slip Curves

The induction motor has a starting torque which accelerates the rotor to its no-load speed, which is almost synchronous, since its no-load slip is quite small.

The motor can then be loaded to full-load with a consequent small increase in slip and this is called its operating region.

If the motor is loaded beyond full-load, maximum generated torque is reached, beyond which, the motor stalls.

The region between T_{max} and T_{start} is, therefore, an unstable region where the motor cannot operate in steadystate.

A well-designed induction motor operates at almost constant speed and relatively high efficiency, since its full-load slip is usually small.

6-12 INDUCTION MACHINE MAXIMUM GENERATED TORQUE

Further insight into the motor characteristics of Fig. 6.20 is obtained by deriving the conditions for maximum, generated torque. From Fig. 6.19,

$$P_g = T\,\omega_s \qquad \text{(W)}$$

Since ω_s is constant, T is maximum when P_g is maximum. From the maximum power transfer theorem, P_g is maximum when,

$$\frac{r_2}{s} = \sqrt{R_1^2 + (X_1 + x_2)^2} \qquad (\Omega)$$

Therefore, the slip at maximum torque is,

$$s_{max\,T} = \frac{r_2}{\sqrt{R_1^2 + (X_1 + x_2)^2}} \tag{6.9}$$

For a given machine, the slip at maximum torque varies with rotor resistance, r_2, only, i.e., if r_2 doubles, $s_{max\,T}$ doubles.

If Eqn. (6.9) is substituted in Eqn. (6.8),

$$T_{max} = \frac{1}{\omega_s} \times \frac{\frac{3}{2} V_{Th}^2}{R_1 + \sqrt{R_1^2 + (X_1 + x_2)^2}} \quad \text{(N-m)} \tag{6.10}$$

For a given machine, the maximum torque is independent of rotor resistance, r_2, and varies only with the input voltage squared. Thus, if, the machine is undervoltaged by one-half, the maximum torque decreases by one quarter. This is devastating for undervoltaged air conditoners, motors, etc. since the line currents must increase, with corresponding increased I^2R losses, to compensate for decreased voltage, thus shortening the working lives of these devices.

Example 6.5

The machine of Example 6.4 has the following parameters,

$$r_1 = 0.294\ \Omega \qquad r_2 = 0.144\ \Omega$$
$$x_1 = 0.504\ \Omega \qquad x_2 = 0.209\ \Omega$$
$$x_\phi = 13.25\ \Omega$$

Calculate the following points on its torque-slip curve,

a) Starting torque.
b) Maximum torque.
c) Running torque at a slip of 3%.

The Thevenin equivalent circuit is,

$$V_{Th} = \left| \frac{jx_\phi}{r_1 + j(x_1 + x_\phi)} \times V_1 \right| = 122.3 \quad V$$

$$R_1 + j X_1 = \frac{(r_1 + jx_1)(jx_\phi)}{r_1 + j(x_1 + x_\phi)} = 0.273 + j0.491 \quad \Omega$$

The slip at maximum torque is,

$$s_{maxT} = \frac{r_2}{\sqrt{R_1^2 + (X_1 + x_2)^2}} = 0.192$$

The rotor currents are,

$$I_2^2 \text{ (start T)} \Big|_{s=1.0} = \frac{V_{Th}^2}{(R_1 + \frac{r_2}{s})^2 + (X_1 + x_2)^2} = 22{,}526 \quad (A^2)$$

$$I_2^2 \text{ (max. T)} \Big|_{s=0.192} = \quad " \quad = 9{,}732 \quad (A^2)$$

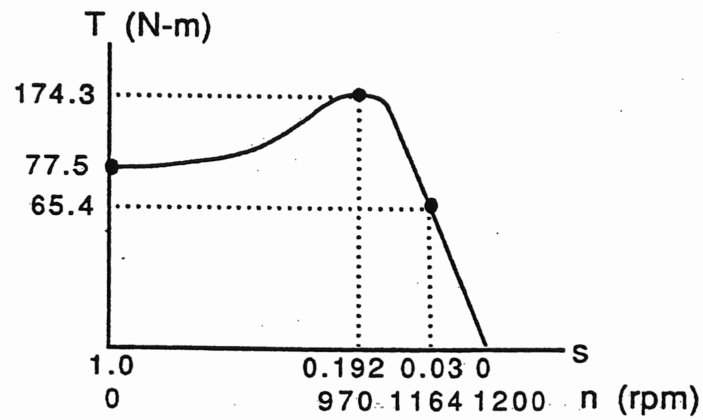
$$I_2^2 \text{ (run. T)} \Big|_{s=0.03} = \quad " \quad = 570 \quad (A^2)$$

$$n_s = \frac{120f}{P} = 1200 \text{ rpm} \quad ; \quad \omega_s = n_s \times \frac{2\pi}{60} = 125.6 \quad \text{(rad/sec)}$$

$$T_{start} = \frac{P_g}{\omega_s} = \frac{3\, I_2^2}{\omega_s} \times \frac{r_2}{s} = 77.5 \quad \text{(N-m)}$$

$$T_{max.} = \;" \; = \; " \; = 174.5 \quad \text{(N-m)}$$

$$T_{run.} = \;" \; = \; " \; = 65.4 \quad \text{(N-m)}$$



$$n = (1 - s) n_s$$

$$n \text{ (max. T)} = 970 \quad \text{(rpm)}$$

$$n \text{ (run. T)} = 1164 \quad \text{(rpm)}$$

6-13 ROTOR LEAKAGE FLUX AND DOUBLE-CAGE ROTORS

Equations (6.9) and (6.10) determine the shape of the T-slip curves as the rotor resistance is varied, as is shown in Fig. 6.21.

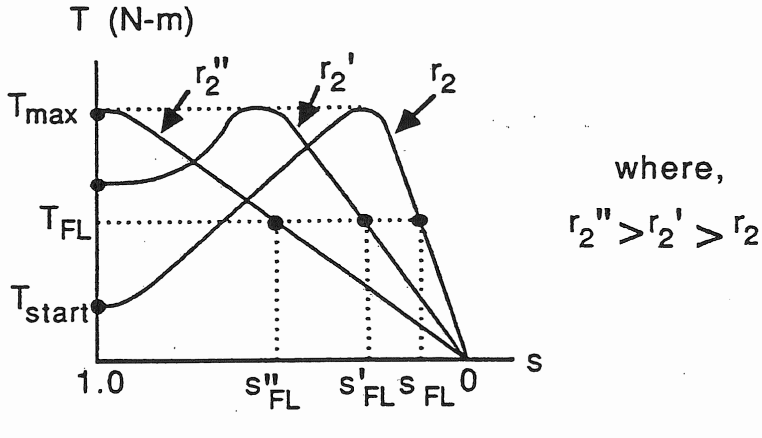


Figure 6.21 Torque Curves for Varying Rotor Resistances

Observe, in Fig. 6.21, as the rotor resistance is increased, the maximum torque remains constant, but the starting torque increases. Also observe, that if r_2 is the inherent resistance of the rotor windings, the starting torque is less than the generated full-load torque, so the machine cannot be started under full-load. It would then be desirable to increase the starting torque by increasing the rotor resistance, which can be done with a wound-rotor machine (Fig. 6.8), by increasing the resistance bank at start. However, as Fig. 6.21 indicates, at large rotor resistances, the full-load slip is large, resulting in lower running speeds and decreased efficiency. This can be corrected, with a wound rotor machine, by bringing the machine up to speed with a large rotor resistance and then reducing the resistance bank to zero, resulting in minimum r_2 and low running slip at rated load.The rotor resistance, however, of a squirrel-cage induction motor, cannot be externally varied, so machine designers take advantage of the rotor leakage flux to obtain large rotor resistance at start and low rotor resistance at run. This is shown in Fig. 6.22 for a rotor with deep bars.

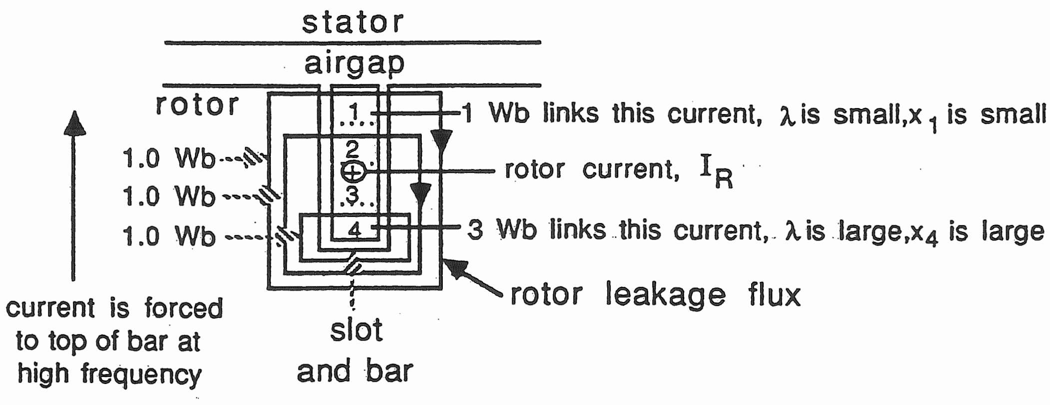


Figure 6.22 Deep Bar Rotor Leakage Flux

As seen in Fig. 6.22, the flux linkage and corresponding inductive reactance to rotor current flow is small at the top of the bar and greater at the bottom of the bar. As the frequency of the rotor current increases, more of the current is forced to flow through the top of the bar and this phenomenon is called skin-effect. Advantage is taken of this phenomenon by constructing double-cage rotors as shown in Fig. 6.23.

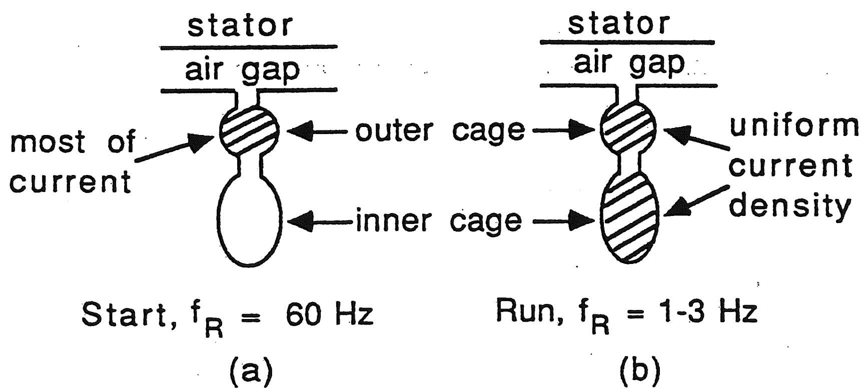


Figure 6.23 Double-Cage Rotor

At start in Fig. 6.23(a), the slip is 100% and the rotor frequency is high, forcing the current through the outer cage, of small cross-section, with consequent large r_2, and large starting torque. In Fig. 6.23(b), when the machine is up to loaded speed, the slip and rotor frequency are small with the rotor current now flowing through both cages with large cross-section, and consequent small r_2, resulting in a low running slip.

6-14 SINGLE-PHASE INDUCTION MOTORS

With auxiliary starting windings, single-phase induction motors are used in enormous quantities, primarily in residences, to power small fans, air conditioners, blowers, etc. It is a cylindrical device as shown in Fig. 6.24.

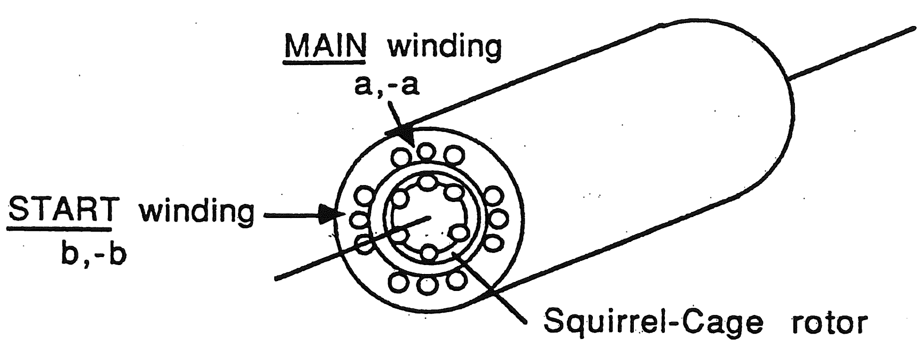


Figure 6.24 Single-Phase Induction Motor Construction

A distributed main winding, and in space quadrature, a start winding, are placed on the inner surface of the stator. The rotor is squirrel-caged, injection molded, with shorting end-rings. The analysis begins by considering the machine with its main winding only as in Fig. 6.25.

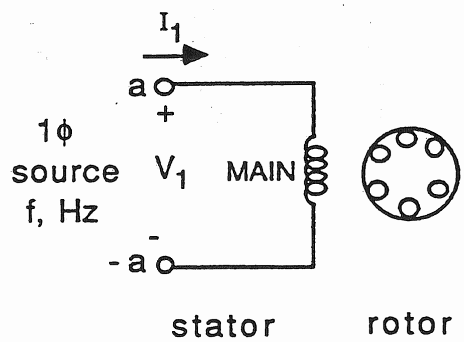


Figure 6.25 Single-Phase Motor Symbolic Diagram

When the machine is connected to a single-phase source of frequency, f Hz, a main-winding current will flow,

$$i_1 = I_m \cos \omega t \qquad \text{(A)}$$

A stationary, pulsating stator field is then created, (Fig. 5.14),

$$B_s = (k\, i_1) \cos \theta$$

$$= (k\, I_m \cos \omega t) \cos \theta$$

$$B_s = B \cos \omega t \cos \theta \qquad \text{(T)} \qquad (6.11)$$

which, using trignometric identity,

$$\cos \alpha \cos \beta = \frac{1}{2} \cos (\alpha + \beta) + \frac{1}{2} \cos (\alpha - \beta)$$

can be resolved into two components,

$$B_s = \underbrace{\frac{B}{2} \cos(\theta - \omega_s t)}_{\text{forward rotating field}} + \underbrace{\frac{B}{2} \cos(\theta + \omega_s t)}_{\text{backward rotating field}} \qquad \text{(T)} \qquad (6.12)$$

The forward and backward rotating fields sum, at any instant of time, to the stationary, pulsating field, in Eq. (6.11), as shown in Fig. 6.26.

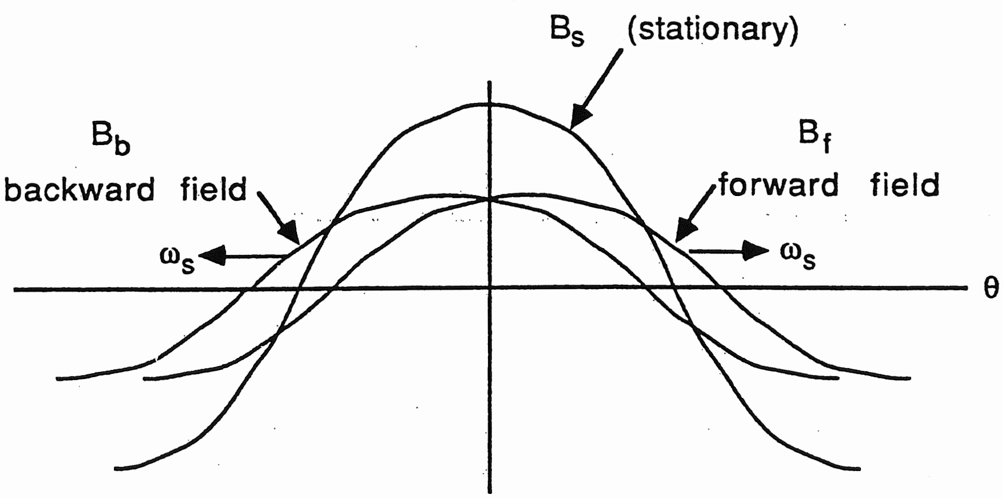
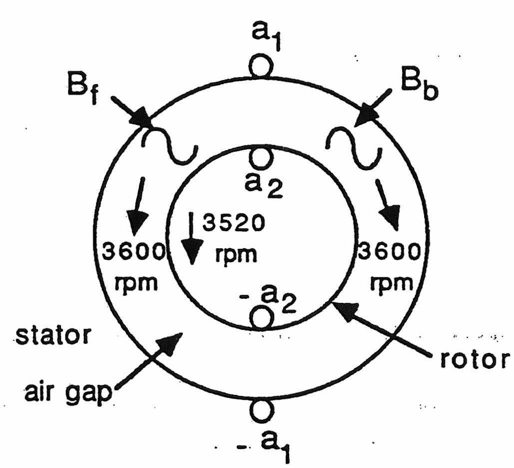


Figure 6.26 Double Revolving Fields

The forward and backward fields rotate at synchronous speed, in opposite directions with respect to the stator, and at slip speed with respect to the short-circuited rotor winding as shown in Example 6.6.

Example 6.6

A 60 Hz, 2-pole, 3520 rpm, single-phase induction motor is running at rated speed. Looking into the air-gap, the stator forward and backward fields rotate past the moving rotor winding,



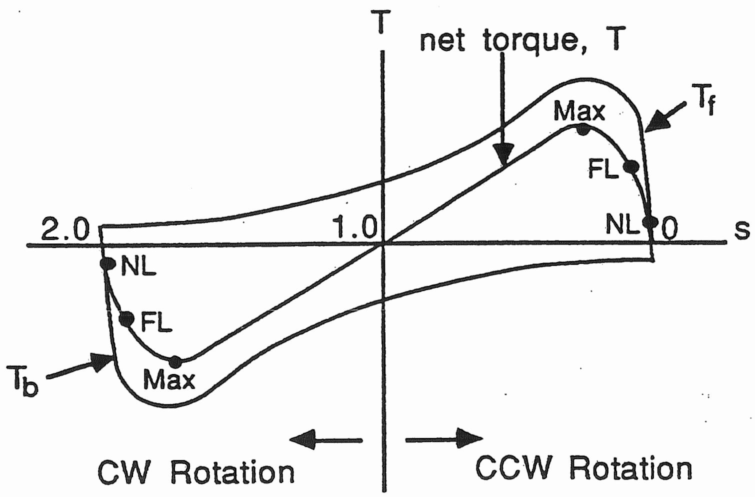
with a forward and backward slip,

$$s_f = \frac{3600 - 3520}{3600} = 0.0222 \quad ; \quad s_b = \frac{-3600 - 3520}{-3600} = 1.98$$

Emfs will simultaneously be induced in the rotor winding at slip frequencies,

$$f_f = (0.0222)(60) = 1.33 \text{ Hz} \quad ; \quad f_b = (1.98)(60) = 119 \text{ Hz}$$

The consequent rotor, short-circuit currents at frequencies 1.33 Hz and 119 Hz produce forward and backward rotor fields that track the stator forward and backward fields, generating forward and backward torques that vary with slip as,



The net torque that acts on the rotor is the algebraic sum of the forward and backward torques at any value of slip. Observe, at start, the slip is 100%, and the forward and backward torques sum to zero, therefore, this machine, inherently, has no starting torque, using its main winding only. If, however, the machine, with its main winding only, is externally brought up to speed, in either direction, the motor can be loaded from no-load to full-load to maximum, beyond which it stalls. This motor, with its main winding only, is not very practical since it has no starting torque, and starting methods for this motor and other types of single-phase motors will be discussed in the next section.

6-15 PRACTICAL SINGLE-PHASE MOTORS

The methods used to start single-phase induction motors use the principles of the rotating field established in Problem 5.6, ie., if two currents, 90° apart in time phase, flow through two stator windings in space quadrature, a rotating field will be created, moving at synchronous speed around the air gap. Therefore, <u>two</u> windings are placed on the stator, as shown in Fig. 6.24, – a MAIN or RUN winding, in space quadrature with a START winding. The symbolic diagram for this machine is shown in Fig. 6.27.

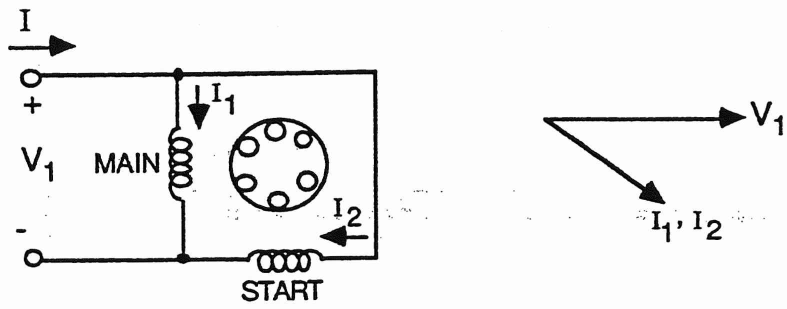


Figure 6.27 Two-Winding Induction Motor

If the two windings, in Fig. 6.27, are identical, no rotating field is established since the currents are not split in time phase as indicated in the phasor diagram.

SPLIT-PHASE INDUCTION MOTOR

If, however, the resistance of the START winding is increased (by using smaller cross-section wire), I_2 will be smaller in magnitude and more in-phase with V_1, as indicated in the phasor diagram of Fig. 6.28.

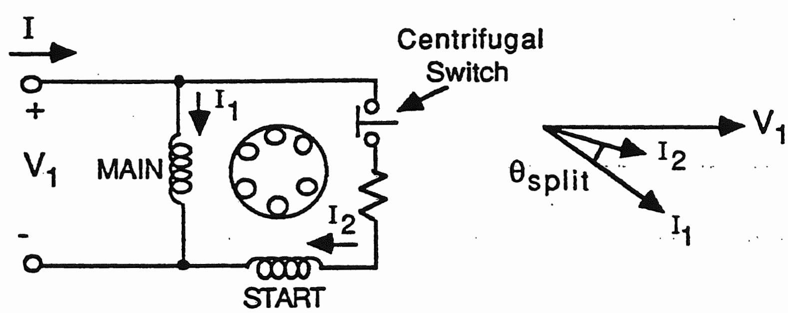


Figure 6.28 Split-Phase Induction Motor

The stator currents are now split in time-phase and, at start, a rotating field will be created, of varying magnitude, because the currents are not balanced nor 90° apart. The starting torque is low because of small θ_{split}, but the machine is relatively inexpensive and is used to drive low starting-torque devices such as fans, blowers, etc., where cost is a factor. Because of the increased I^2R loss, the START winding is removed with a centrifugal switch when approximately 75% synchronous speed is reached. The motor then operates with the MAIN winding, only, according to the principles in the preceding section.

CAPACITOR-START INDUCTION MOTOR

The stator currents can also be split in time-phase by placing a capacitor in series with the START winding, as shown in Fig. 6.29.

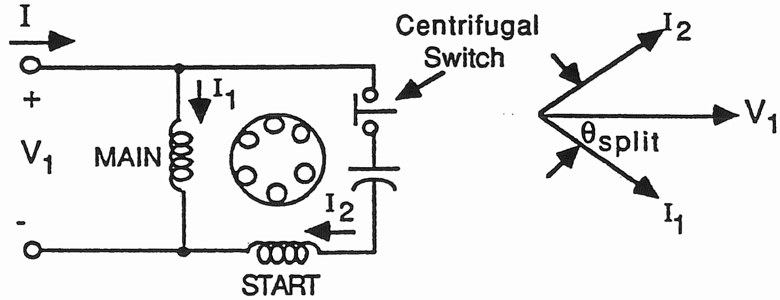


Figure 6.29 Capacitor-Start Induction Motor

By placing a large electrolytic capacitor in series with the start winding, I_2, can be made to lead V_1 with a resulting large θ_{split}. The starting torque is, therefore, much higher than the split-phase machine but it is, relatively, much more expensive and it is used to power high starting-torque devices such as bench grinders, air-conditoners, etc. A capacitor-start machine can always be distinguished from a split-phase machine because there is no place to hide the capacitor except in a large tube mounted on top of the motor. Either type motor can be reversed in direction by reversing the run winding with respect to the start winding, which requires a small box and extra expense, mounted on the outside of the motor to have access to the winding leads.

Other types of single-phase motors are also often used to power household and industrial devices such as the shaded-pole motor shown in Fig. 6.30.

SHADED POLE SINGLE-PHASE MOTOR

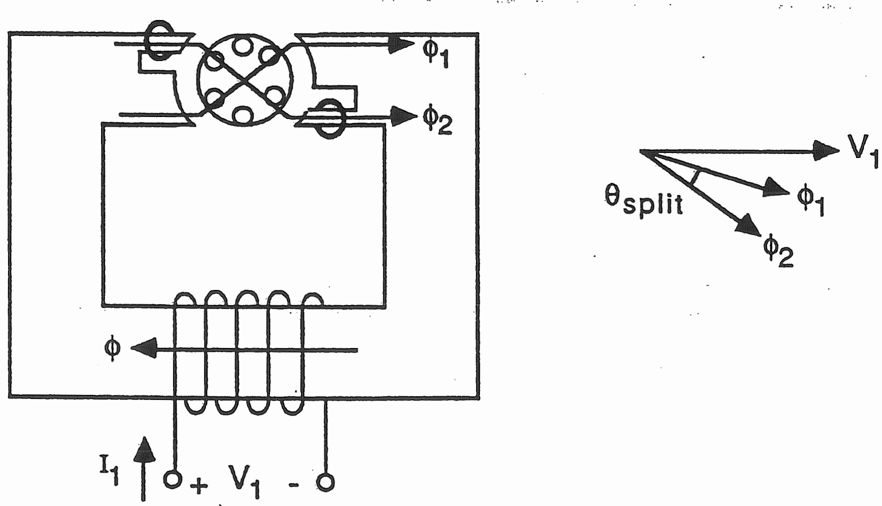


Figure 6.30 Shaded-Pole Single-Phase Motor

The shaded-pole induction motor has two slots cut in opposite pole tips. Placed in each slot is a heavy, shorted turn of copper wire. The portion of the total flux that passes through the shorted turns is delayed in time, (because of Faraday induction) with respect to the portion of total flux that does not pass through the shorted turns. Since θ_{split} is small the starting torque is small and this motor is used extensively to power small toys, electric clocks and electric timers. The motor always turns from the unshaded pole tip to the shaded pole tip.

UNIVERSAL SINGLE-PHASE MOTOR

Also extensively used, is the universal motor. It looks like a dc motor, to be discussed in the next section, except that both the stator and rotor are laminated to reduce eddy-current loss. This machine is shown in Fig. 6.31.

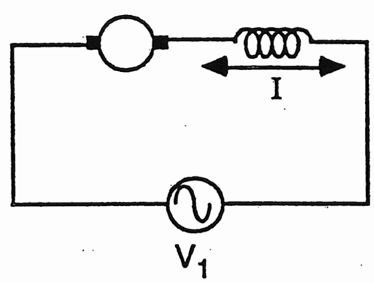


Figure 6.31 Universal Single-Phase Motor

The universal motor can be connected to either an ac or dc source, hence its name. The armature and field winding are always connected in series so that when the current flows in either direction, both the armature and field reverse together with a resultant unidirectional torque. The motor has all of the characteristics of the series, dc motor– high starting torque, high running speed and it must be geared to its load because if unloaded it will run away. The motor is used extensively in hand-electric drills, vacuum cleaners, electric razors, etc.

There are many other single-phase motors also used, such as, the reluctance motor, the hysteresis motor, the stepper motor, the repulsion motor, etc., but the single-phase motors described above are the motors most often encountered in practice.

STARTING THE THREE-PHASE SYNCHRONOUS MOTOR

As described in Section 6-6, the three-phase synchronous motor, inherently, does not have a starting torque. For this reason an Amortisseur winding, which is simply a squirrel-cage winding, is added to the rotor as shown

in Fig. 6.7. At start, the rotating stator field slips 100% with respect to the Amortisseur winding, whose consequent short-circuit currents create an accelerating starting torque, bringing the machine up to near-synchronous speed as shown in Fig. 6.20. The rotor field, in Fig. 6.7, is then energized with a direct current, I_f, and the rotor field locks in with the stator field at synchronous speed. At run, then, the Amortisseur winding is turning, with the rotor, at synchronous speed, with zero slip, and it remains inoperative in running steadystate.

6-16 DIRECT CURRENT MOTORS

The dc machine is presently used, extensively, in the electric-power industry, in the fields of locomotion, hoisting and precision-speed and position control. Because of its importance in this widespread industry, it will be described, modeled and analyzed in this section. The machine construction is relatively complicated, but its analytic model is quite simple. The elementary dc machine will be considered first, together with a more-practical machine, from which its model will be inferred.

6-17 DC MOTOR STATOR FIELD

The machine is cylindrical, and its cross-section is shown in Fig. 6.32.

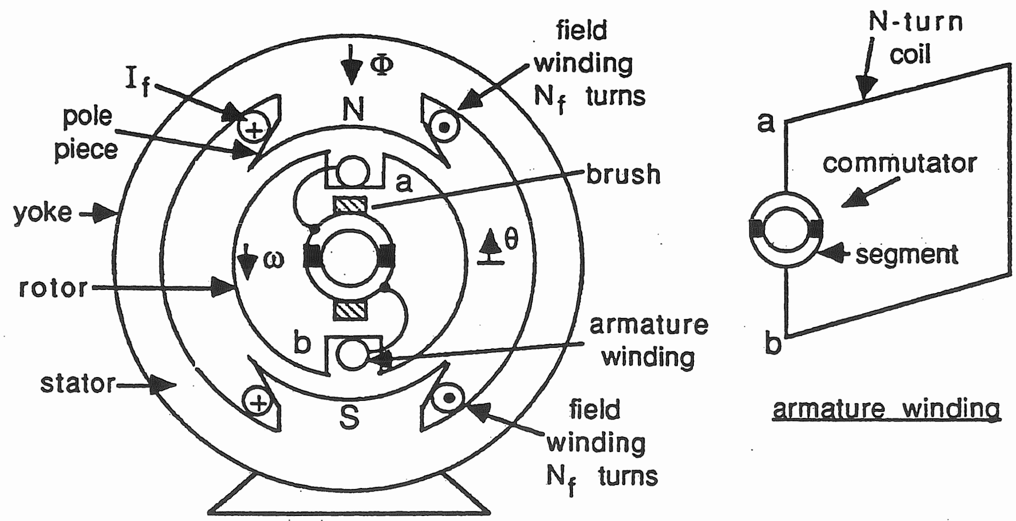


Figure 6.32 Elementary Two-Pole DC Machine

The stator consists of a yoke and <u>salient</u> pole-pieces, (they project into the air-gap), where on <u>each</u> pole piece, an N_f-turn, winding is placed. The field windings are connected in series and a direct current, I_f, is allowed to flow

through the windings in the direction indicated for each pole-pair. For the pole-pair indicated in the Figure, using the right-hand mmf rule, the top and bottom poles are North and South poles, respectively. For a well-designed machine, negligible mmf is dropped along the iron portions of the flux path, so the mmf of each pole-piece is dropped uniformly across its gap. The flux will take the path of least reluctance across each gap, so the corresponding flux density distribution is radial and uniform under each pole piece as shown in Fig. 6.33.

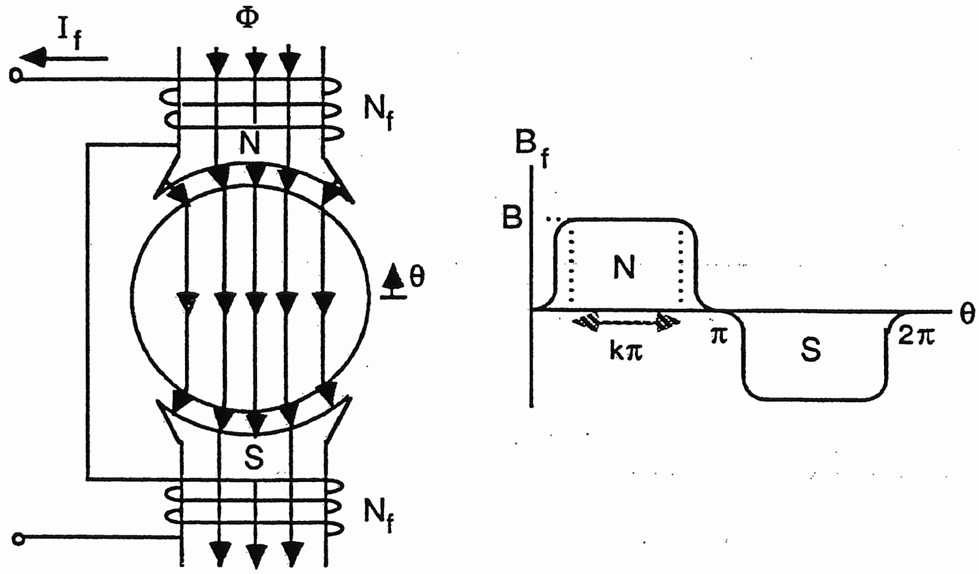


Figure 6.33 Two-Pole DC Machine Stator Field

The total flux passing through each pole is found by integrating the flux density, which is constant axially, as well as with θ, under the cylindrical area of each pole piece with pole span, $k\pi$ rad., where k is usually 0.7-0.8. The leakage flux density beyond the pole tips is considered negligible.

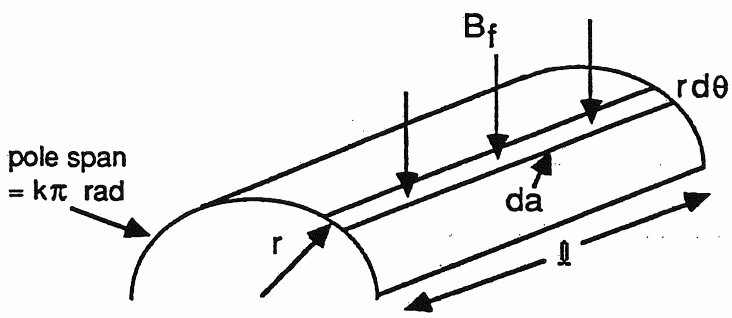


Figure 6.34 Air-Gap Surface of Integration

$$\Phi = \int B_f \, da = \int_0^{\pi} B \, \ell \, r \, d\theta = B \, \ell \, r \, k\pi \quad \text{(Wb/pole)} \qquad (6.13)$$

where, B is the magnitude of the flux density distribution in Fig. 6.33 and varies with the field current that produced it, as shown in Fig. 6.35.

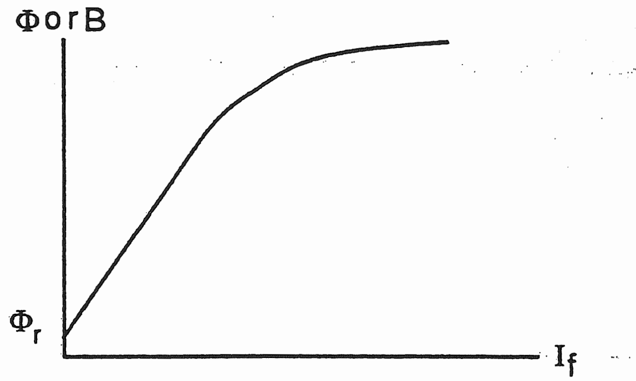


Figure 6.35 Machine Magnetization Curve

The flux can be varied, with the field current, from residual to saturation where small increase takes place. This machine magnetization curve will become important in later analysis.

6-18 DC ROTOR CURRENT-CARRYING CONDUCTORS

The rotor of the elementary dc motor is shown in Fig. 6.36, with its N-turn coil placed in two opposite slots cut axially on the rotor surface.

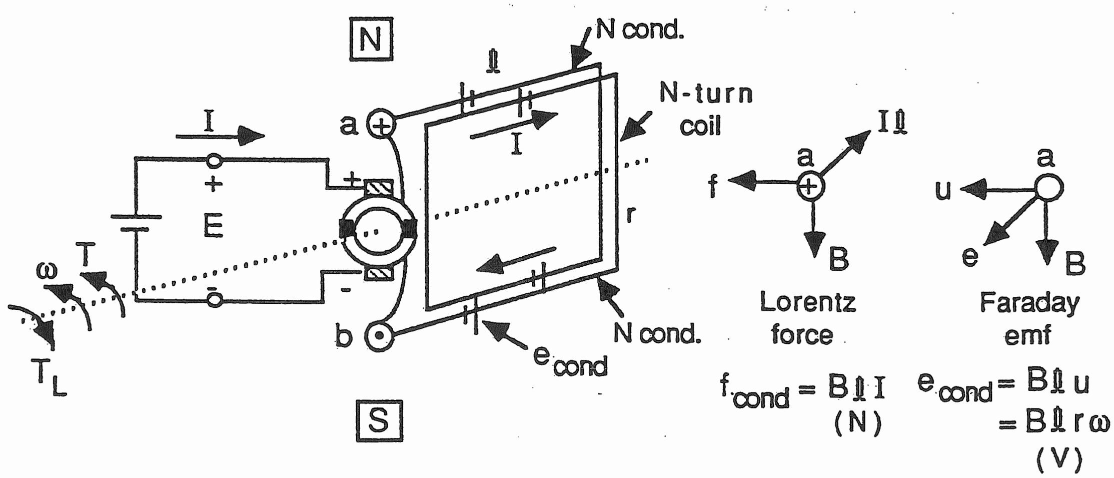


Figure 6.36 Rotor of the Elementary DC Machine

The ends of the rotor coil are brought out and connected to two annular, copper, cylindrical segments, insulated from each other and the shaft. This assembly is called the commutator, which rotates with the rotor and its winding, under stationary, spring-loaded, carbon brushes, thus allowing continuous current flow with rotor rotation.

A dc source is connected to the brushes so that current flows in the direction indicated. A counterclockwise, (forward), Lorentz force is then created, on each of the 2N-conductors in both slots, acting through a radius, r meters, on the rotor. The consequent forward torque on each conductor, while under a pole piece, is,

$$T_{cond} = f_{cond}\ r = B\ \ell\ I\ r \qquad \text{(N-m)} \qquad (6.14)$$

A Faraday back-emf is generated across each of the conductors, in the directions indicated, and these conductor emfs are additive, in series, for the 2N-conductors in both slots. The emf across each conductor, while under a pole piece, is,

$$e_{cond} = B\ell u = B\ell r\omega \qquad \text{(V)} \qquad (6.15)$$

If θ is measured around the air-gap, from interpolar space, (Fig. 6.32), to a conductor in slot a, then θ will increase as,

$\theta = \omega t$ (rad) where, ω = angular velocity of the rotor (rad/sec)

and the conductor in slot a can then be tracked, in Fig. 6.37, as the rotor rotates.

Since the conductor moves through the rotor field in Fig. 6.33, the conductor, emf and current, in slot a, reverse as the conductor moves past the North pole ($0 \le \theta \le \pi$), and then the South pole ($\pi \le \theta \le 2\pi$), as indicated in Fig. 6.36. This conductor emf and torque are plotted, in Fig. 6.37, for one revolution of the rotor.

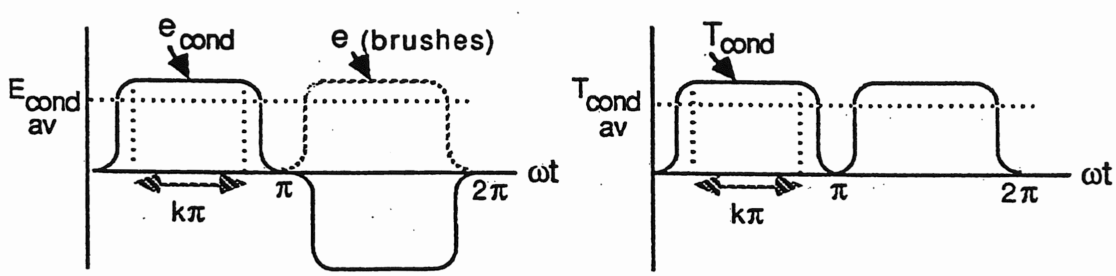


Figure 6.37 Generated Conductor Emf and Torque

Observe in Fig. 6.37, the conductor emf and current are alternating, however, as the rotor turns, the conductor in slot a with its commutator segment will leave the field of the North pole and enter the South pole where its emf and current reverses.

Since the commutator segment of the conductor is now connected to the bottom brush, the brush- emf polarity remains unchanged and the conductor torque remains unidirectional.

The commutator, then, acts as a switch or mechanical rectifier, resulting in a unidirectional, generated, conductor- torque and brush conductor- emf.

If the emf and torque, in Fig. 6.37, are assumed negligible while the conductor is not under a pole-piece, the average generated-conductor emf and torque is, from Eqns. (6.14),(6.15) -

$$E_{cond}\,(av) = \frac{1}{\pi}\int_0^{\pi} B\,\ell\,r\,\omega\,d(\omega t) \quad ; \quad T_{cond}\,(av) = \frac{1}{\pi}\int_0^{\pi} B\,\ell\,I\,r\,d(\omega t)$$

$$= \frac{1}{\pi} B\,\ell\,r\,\omega\,(k\pi) \quad (V) \qquad\qquad = \frac{1}{\pi} B\,\ell\,I\,r\,(k\pi) \quad (N\text{-}m) \quad (6.16)$$

From Eqn. (6.13),

$$B = \frac{\Phi}{\ell\,r\,k\pi} \qquad (T)$$

which, when substituted in Eqn. (6.16),

$$E_{cond}\,(av) = \frac{1}{\pi}\Phi\,\omega \quad (V) \quad ; \quad T_{cond}\,(av) = \frac{1}{\pi}\Phi\,I_{cond} \quad (N\text{-}m) \qquad (6.17)$$

Since the total flux varies with I_f, (Fig. 6.35), the generated emf and torque can be varied from residual to saturation, externally, with a field rheostat.

Example 6.7

The motor in Fig. 6.36 is running at 1800 rpm, since the field current is adjusted so that a flux density of 1.0 T exists in the air-gap under each pole. The rotor has diameter, 0.152 m, axial-length, 0.3 m, and a 16–turn coil is wound in its slots. The stator has two poles, each with a pole span of 0.8π rad. The motor is torqued until 12 A is drawn from the source.

(a) What is the instantaneous back-emf and forward-torque generated on each conductor while the coil is under a pole piece?

$$e_{cond} = B\,\ell\,r\,\omega = (1.0)(0.3)\left(\frac{0.152}{2}\right)\left(\underset{\frac{rev}{min}}{1800} \times \underset{\frac{rad}{rev}}{2\pi} \times \underset{\frac{min}{sec}}{\frac{1}{60}}\right) = \underline{4.3}\ \text{(V/cond)}$$

$$T_{cond} = B\,\ell\,I\,r = (1.0)(0.3)(12)\left(\frac{0.152}{2}\right) = \underline{0.274}\ \text{(N-m/cond)}$$

(b) What is the flux per pole?

$$\Phi = B\,\ell\,r\,k\pi = (1.0)(0.3)\left(\frac{0.152}{2}\right)(0.8\pi) = \underline{0.0573}\ \text{(Wb/pole)}$$

(c) What is the average, generated, back-voltage across the brushes and the total average forward- torque on the rotor?

$$E = 2N\left(\frac{1}{\pi}\Phi\,\omega\right) = (2)(16)\left(\frac{1}{\pi}\right)(0.0573)\left(1800 \times \frac{2\pi}{60}\right) = \underline{110}\ \text{V}$$

$$T = 2N\left(\frac{1}{\pi}\Phi\,I\right) = (2)(16)\left(\frac{1}{\pi}\right)(0.0573)(12) = \underline{7.0}\ \text{N-m}$$

Practical DC Machine

The elementary dc machine described thus far does not make full use of the rotor for generating emf and torque.

In a practical machine, many slots are cut axially around the surface of the rotor and coils placed in these slots are connected in a variety of ways to obtain high voltage-low current or low voltage-high current machines or machines with any combination in between these extremes.

However, regardless of the winding scheme used, all dc machines have certain characteristics in common, as illustrated in Fig. 6.38, where eight coils are placed in eight slots on the rotor.

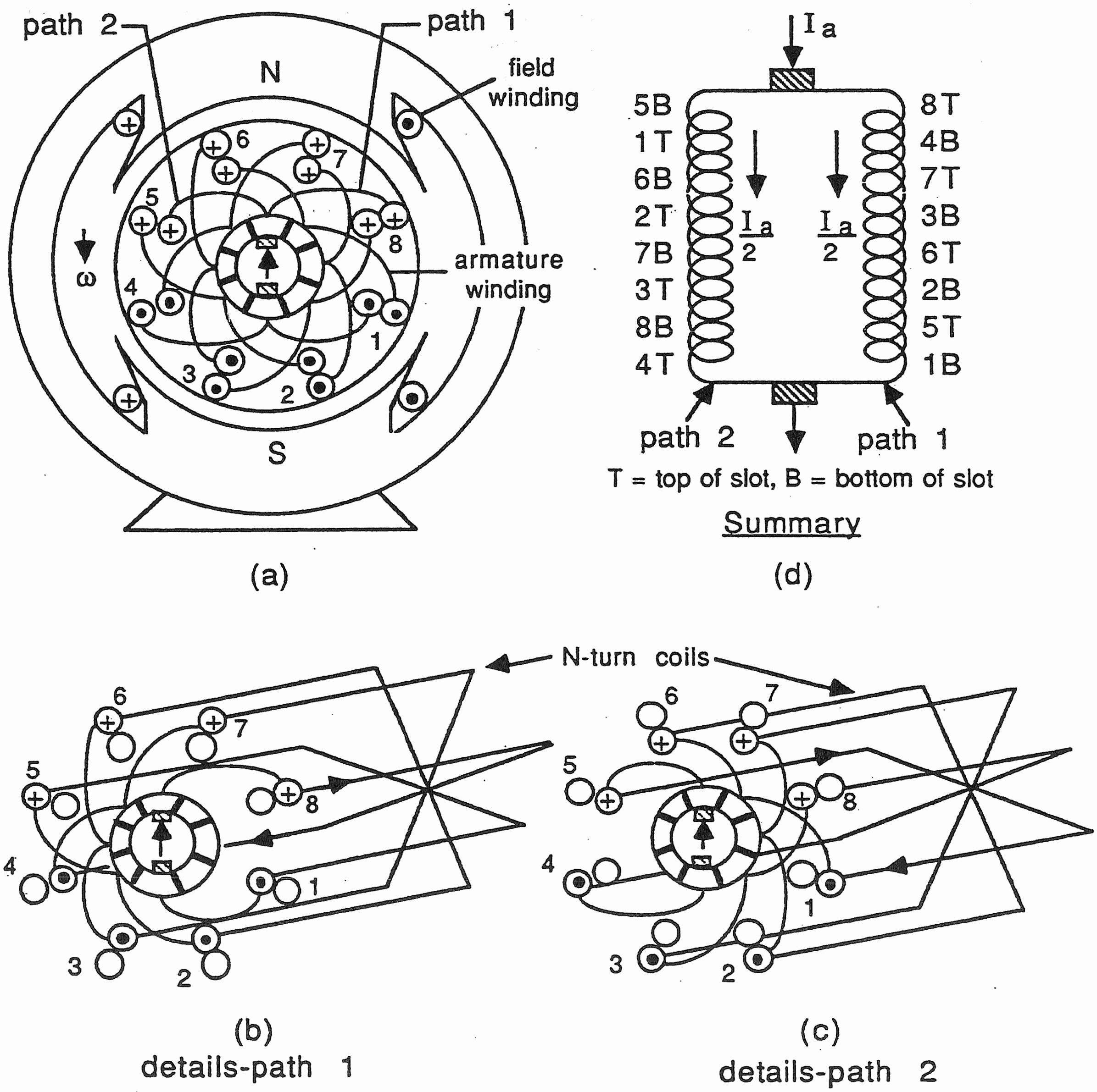


Figure 6.38 Practical DC Machine, Rotor (Armature) Winding

Each N-turn coil has one coil-side placed in the bottom of a slot and the opposite coil-side is placed in the top of the opposite slot, as illustrated in Fig. 6.38 (b),(c). The coils are brought out and connected to commutator segments, as indicated, resulting in two paths for current flow for the total armature current leaving the top brush. In Fig. 6.38(d) all of the conductors are connected equally, in series, via the commutator segments, between two paths as indicated.

All dc machines have the following characteristics in common, i.e., if,

Z = total conductors in all of the rotor slots

a = number of paths for current flow

there will be Z/a conductors in each path and the total armature current, I_a, will divide equally, I_a/a, through each path. Furthermore, the current in these conductors will flow in one direction under one pole and in the opposite direction under the other pole regardless of the rotor position, i.e., conductors passing through interpolar space are commutated - their emfs and currents reverse, assuring unidirectional torque on the rotor by all of the conductors, and unidirectional emf across the brushes by the conductors in any one path, as shown in Fig. 6.39.

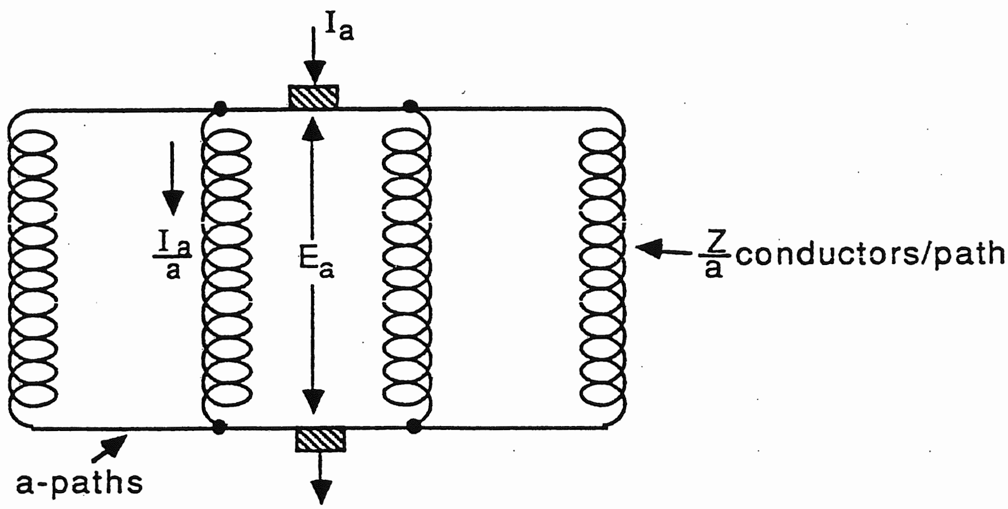


Figure 6.39 Generalized DC Machine Armature Winding

6-19 DC MOTOR GENERATED EMF AND TORQUE

From Eqn (6.17) and Fig. 6.39, the emf generated across the brushes is the sum of the emfs generated in the conductors in any one path,

$$E_a = \left(\frac{Z}{a}\right)\left(\frac{1}{\pi}\Phi\omega\right) = K\Phi\omega \quad \text{(V)} \tag{6.18}$$

and is shown in Fig. 6.40,

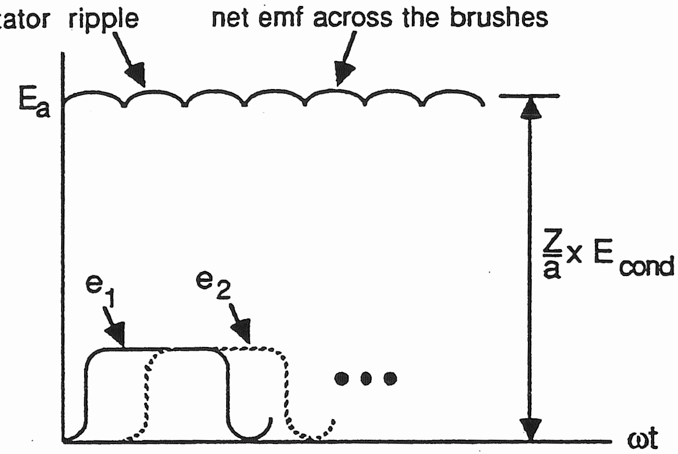


Figure 6.40 Generated Voltage Across the Brushes

As each commutator segment leaves the brush, for the moving rotor in Fig. 6.38, a slot conductor will move through interpolar space and the emf of the conductor will go to zero, as indicated in Fig. 6.40, resulting in a slight dip in the total generated emf. This variation is called commutator ripple, and the periodic frequency of this ripple is usually high, being given by the number of rotor segments and the rotor speed. It is easily filtered, if necessary, but its amplitude is usually quite small.

The total rotor torque is generated by the current flowing through all the slot conductors and from Eqn. (6.17) and Fig. 6.39,

$$T = (Z)\left(\frac{1}{\pi}\Phi\frac{I_a}{a}\right) = K\Phi I_a \qquad \text{(N-m)} \qquad (6.19)$$

where, $K = \dfrac{Z}{\pi a}$, is the machine constant, and has the same magnitude for both emf and torque.

Example 6.8

The dc machine in Fig. 6.38 has its field current adjusted so that the flux per pole is 0.1 Wb. If there are 2 turns/coil and 2 coil sides per slot, what is the generated voltage across the brushes, E_a, and the generated shaft torque, T? The machine is rotating at 1800 rpm with an armature current, I_a, of 10 A.

$$Z = \underset{\frac{\text{cond}}{\text{slot}}}{4} \times \underset{\text{slots}}{8} = 32 \text{ cond.}$$

$$K = \frac{Z}{\pi a} = \frac{(32)}{(\pi)(2)} = 5.09$$

$$E_a = K\Phi\omega = (5.09)(0.1)\left(1800 \times \frac{2\pi}{60}\right) = \underline{96} \text{ V}$$

$$T = K\Phi I_a = (5.09)(0.1)(10) = \underline{5.09} \text{ N-m}$$

6-20 DC MACHINE MODEL

The armature of the dc machine as shown in Fig. 6.38 is modeled by its Thevenin equivalent circuit between the brushes. The Thevenin or open-circuit voltage, E_a, across the brushes, is the average of the sum of the emfs generated in any <u>one</u> path. The Thevenin or armature resistance, R_a, is the <u>parallel</u> combination of the series-conductor resistance in each path. The mechanical system is represented by the Lorentz torque, T, which is the sum of the generated torques acting on <u>all</u> of the rotor conductors, together with the losses- torque of friction, windage, hysteresis and eddy-currents and the drive (gen) or load torque (motor). The stator field coil is also included since the field current establishes the coupling field in the gap, a measure of which, is the total flux per pole, Φ. The resistance of the field circuit, <u>including</u> the rheostat, is R_f ohms. The machine model is given in Fig. 6.41.

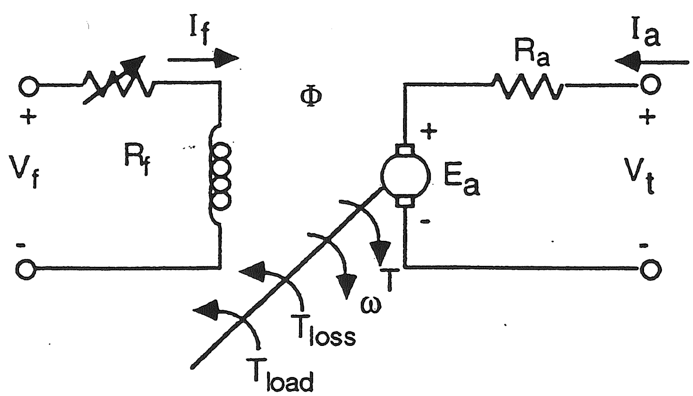


Figure 6.41 DC Machine Model (Motor)

The motor power flow is given in Fig. 6.42,

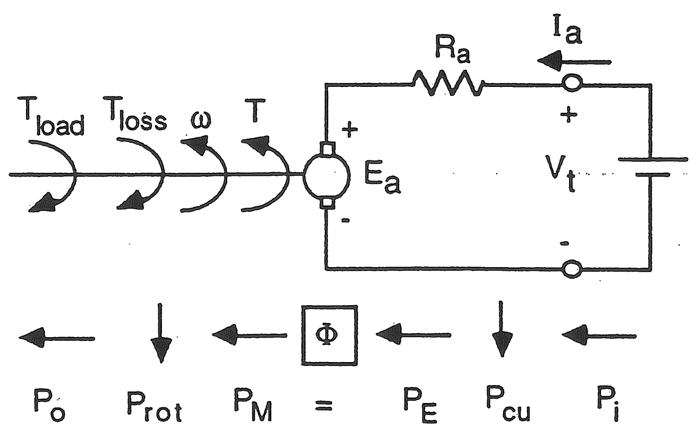


Figure 6.42 DC Motor Power Flow

where, $P_i = V_t I_a = I_a^2 R_a + E_a I_a \quad (W)$

$$E_a I_a = T\omega \quad (W) \quad (6.20)$$

$$T\omega = T_{loss}\,\omega + T_{load}\,\omega \quad (W)$$

The dc machine magnetization curve is found by driving the machine as an unloaded generator at rated speed, ω_o, and realizing, that

$$V_t = E_a = K\Phi\omega_o \qquad \text{when, } I_a = 0 \text{ (A)} \quad (6.22)$$

The generated emf, E_a, then, varies as Φ, since the speed, ω_o, is constant. The flux per pole varies nonlinearly with the field current as in Fig. 6.35, therefore, the generated emf is nonlinearly related to I_f, as given in Fig. 6.43.

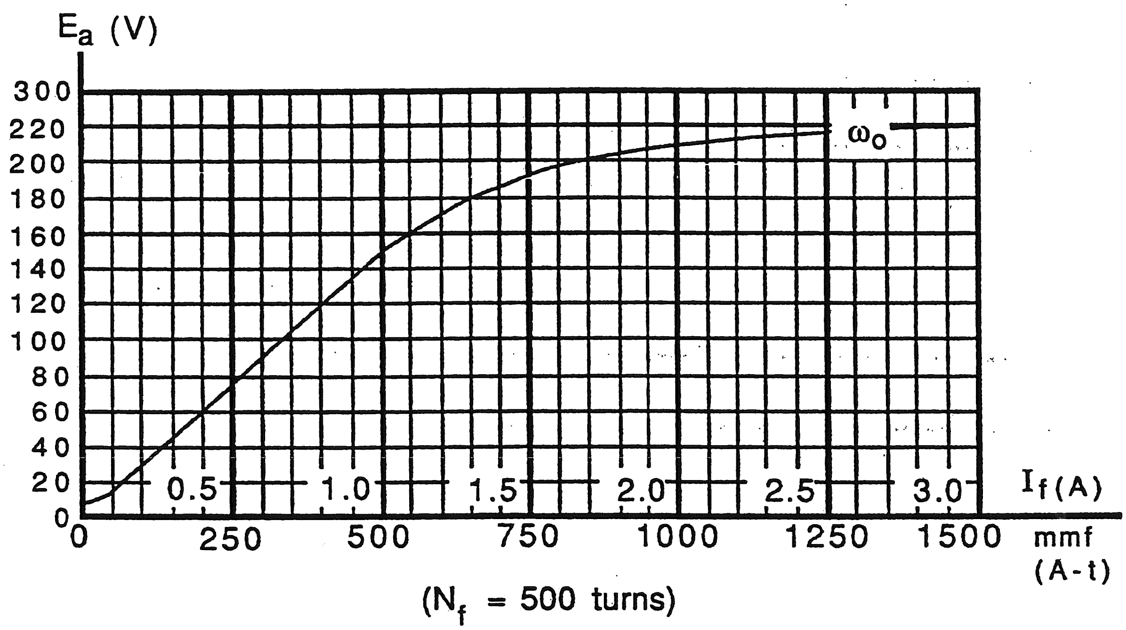


Figure 6.43 DC Machine Magnetization Curve

Recall, that the driving mmf of each field winding is $N_f I_f$ (A-t), which determines the flux per pole, and therefore, the generated emf in Fig. 6.43. If the field current flows through 500 turns then the emf is known not only for each field current but for each corresponding driving mmf of the field winding. This observation will become important when pole windings are discussed.

Once the magnetization curve for a dc machine is obtained at ω_o, then magnetization curves at other speeds can be obtained,

$$\begin{aligned} E_0 &= K\,\Phi_o\,\omega_o \\ E_1 &= K\,\Phi_1\,\omega_1 \end{aligned} \qquad (6.21)$$

If Eqns. (6.21) are divided, and the emfs are determined at the same field current, the constants and the fluxes cancel, yielding,

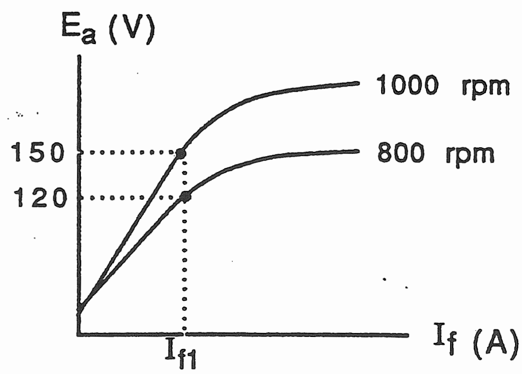
$$E_1 = \frac{\omega_1}{\omega_o} E_0 \qquad (V) \qquad (6.22)$$

A point, E_1, is found on the new magnetization curve at speed, ω_1.

Example 6.9

Assume that the magnetization curve in Fig. 6.43 is given at 1000 rpm. At a field current of 1.0 A, find the corresponding emf at 800 rpm.

$$E_1 = \frac{\omega_1}{\omega_o} E_o = \frac{800}{1000} X\, 150 = 120\ V$$



6-21 FIELD WINDINGS AND CONNECTIONS OF A DC MOTOR

The versatility of dc machine performance is obtained by placing two windings on each pole-piece as shown in Fig. 6.44.

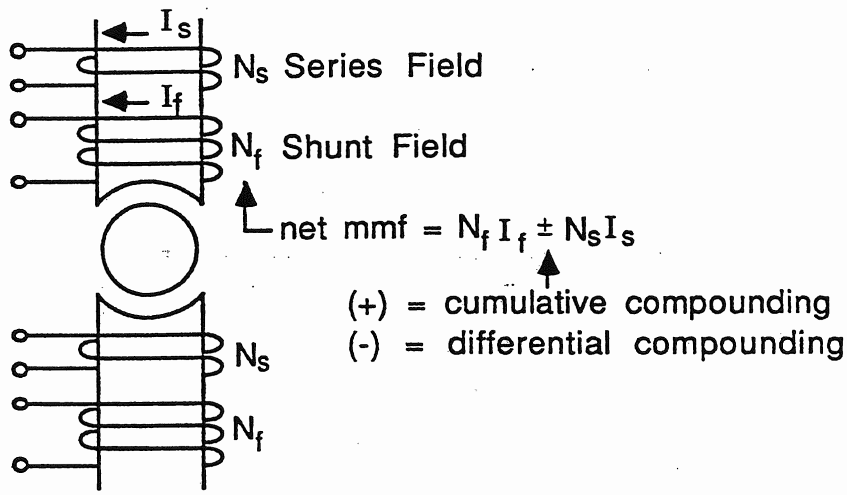


Figure 6.44 Shunt and Series Field Windings

The field winding of N_f-turns, discussed to this point, is called the shunt-field winding, since it is always connected in parallel with the armature. A series-field winding is added to each pole-piece, of N_s turns, and is always connected in series with the armature. The net driving-mmf, creating the flux per pole or generated emf, is then,

$$\text{net mmf} = N_f I_f \pm N_s I_s \qquad \text{(A-t)} \qquad (6.23)$$

and because this is true, the mmf in the magnetization curve of Fig. 6.43 can be interpreted as the net mmf of Eqn. (6.23). When both windings are used, with the mmfs aiding in Fig. 6.44, the machine is said to be <u>cumulatively</u> compounded. The negative sign in Eqn. (6.23) indicates <u>differential</u> compounding, when the mmfs are connected subtractive.

The two most common connections of the dc motor are the shunt and series motors shown in Fig. 6.45,

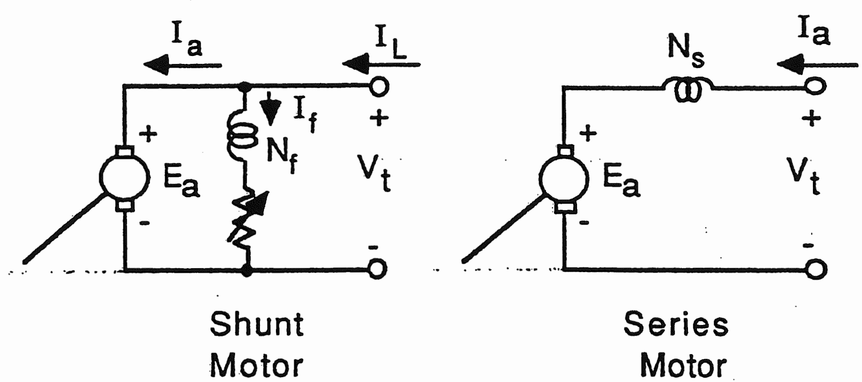


Figure 6.45 DC Motor Connections

From Eqn. (6.23), the net mmf creating E_a, is $N_f I_f$ (A-t) for the shunt motor and $N_s I_a$ (A-t) for the series motor, and the mmf, in Fig. 6.43, is interpreted accordingly when evaluating E_a.

Since the terminal voltage of either motor is considered constant, the variables of interest are motor speed and motor load torque. These external characteristics are shown in Fig. 6.46.

The motor rating, which determines these characteristics, is very important, since in practice, these ratings should not be exceeded for any length of time. A typical motor rating might be given, as,

1/4 hp, 120 V, 2.8 A, 1800 rpm

The horsepower rating, when multiplied by, $746 \frac{W}{hp}$,

is the rated or full-load mechanical power, $T_L\omega$, (W) at the shaft output. When torqued to rated power output, above, rated current at rated voltage will exist at the input terminals of the machine (V_t, I_L). This, then, gives a measure of the machine efficiency at rated or full-load. When loaded and operated under rated conditions, the machine will efficiently run, continuously, for many years provided the brushes, commutator, and bearings are maintained periodically.

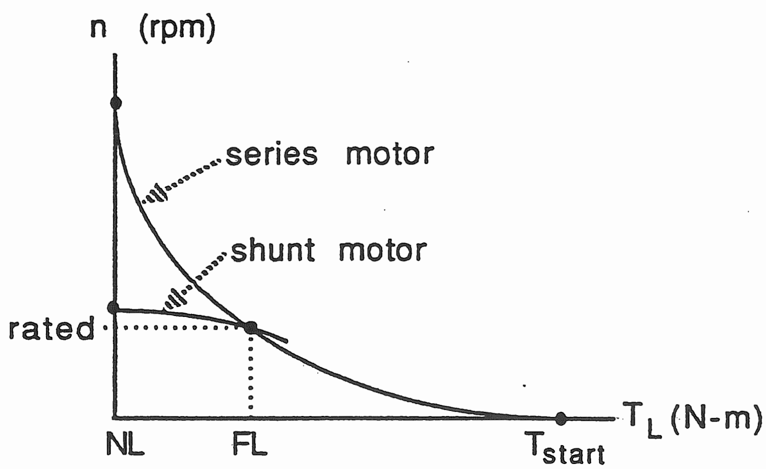


Figure 6.46 DC Motor External Characteristics

The shunt motor has a drooping speed characteristic, with loading, which is usually desirable for safety and other reasons. The series motor is used for its very large starting torque but it cannot be unloaded since it will run away as indicated. For this reason a series dc motor is always directly connected or geared to its load.

6-22 DC MOTOR ANALYSIS

DC SHUNT MOTOR

A shunt motor will be considered first and its diagram and power-flow are given in Fig. 6.47,

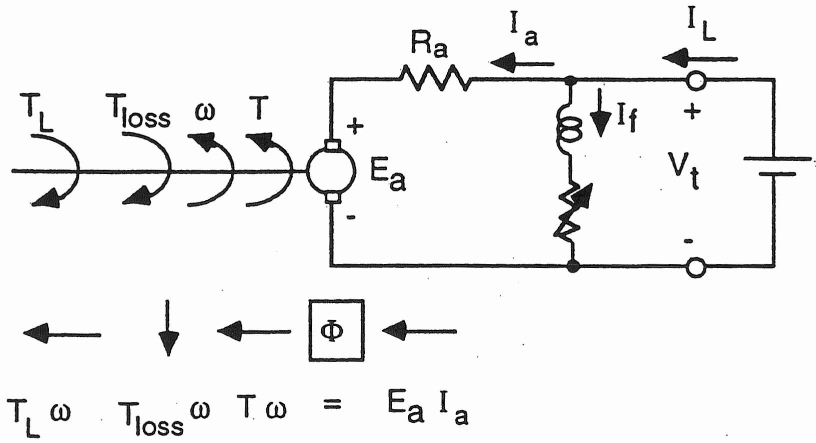


Figure 6.47 DC Shunt Motor

The equations that define the performance of this motor are given from Fig. 6.47,

$$E_a = V_t - R_a I_a = K\Phi\omega \qquad \text{(V)}$$

$$I_L = I_f + I_a \qquad \text{(A)} \qquad (6.24)$$

$$\text{net mmf} = N_f I_f \qquad \text{(A-t)}$$

From Eqn. (6.24), the motor speed is,

$$\omega = \frac{V_t - R_a I_a}{K\Phi} \qquad \text{(rad/sec)} \qquad (6.25)$$

where, $$T = K\Phi I_a \qquad \text{(N-m)}$$

Several important observations can be made concerning Eqn. (6.25),

1. V_t is constant, since the motor is considered connected to an infinite bus, therefore, I_f is constant, for a given rheostat setting, when the motor is torqued from no-load to full-load.

2. The flux per pole, Φ, is directly variable with the field current,only, and is independent of motor loading.

3. From Fig. 6.47, as the load torque increases, the Lorentz torque, T, must increase with a consequent increase of armature current, I_a. For a given I_f, therefore, the load torque determines the armature current.

4. The motor speed varies inversely with the flux or field current, for a given terminal voltage and load torque.

Shunt motor analysis now continues with Example 6.10.

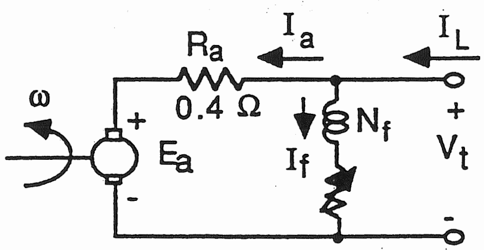
Example 6.10

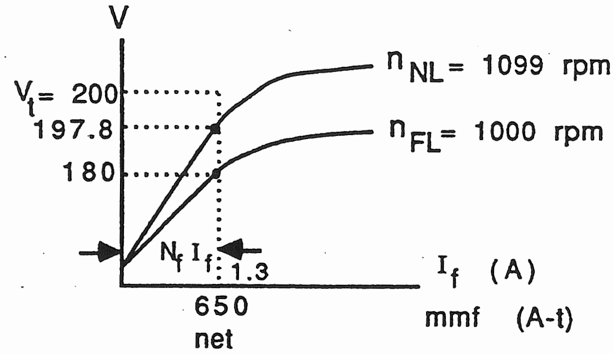
A two-pole dc machine is rated 200 V, I_a = 50 A, 1000 rpm, $R_a = 0.4\ \Omega$ and its magnetization curve is given at 1000 rpm in Fig. 6.43. The rotational losses are measured at no-load as 1100 W and are assumed to remain constant with speed.

If the machine is operated as a shunt motor, at rated terminal voltage,

(a) What is the field current required to obtain rated speed, when the motor is torqued to I_a = 50 A?

(b) Find the no-load speed and speed regulation.





The no-load and full-load magnetization curves are shown in the volts-mmf plane, where the net mmf (horizontally) determines the emf, E_a, which must lie on a magnetization curve, and is is calculated from KVL, around the outside circuit-loop, ($E_a = V_t - R_a I_a$), and is displayed vertically.

a) At full-load, $I_a = 50$ A,

$$E_a = V_t - R_a I_a = 200 - (0.4)(50) = 180 \text{ V @ } 1000 \text{ rpm}$$

$$\text{net mmf} = N_f I_f = 650 \text{ A-t} \quad ; \quad I_f = \frac{650}{500} = \underline{1.3 \text{ A}} \quad \text{(Fig. 6.43)}$$

b) At no-load, $T_L = 0$, and from Fig. 6.47,

$$T_{loss}\,\omega = E_a I_a = 1100 \text{ W}$$

Initially assume $E_a \cong V_t = 200$ V,

$$I_a\,NL = \frac{1100}{200} = 5.5 \text{ A}$$

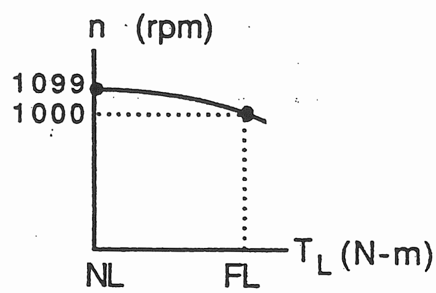
First correction for E_a,

$$E_a = 200 - (0.4)(5.5) = 197.8 \text{ V @ } n_{NL} \text{ rpm}$$

Since the field current remains constant,

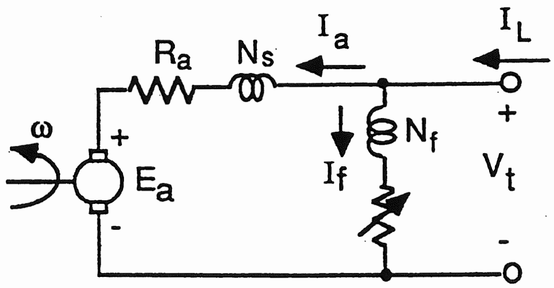
$$\frac{E_a\,NL}{E_a\,FL} = \frac{n_{NL}}{n_{FL}} = \frac{197.8}{180}$$

$$n_{NL} = \frac{197.8}{180} \times 1000 = \underline{1099 \text{ rpm}}$$

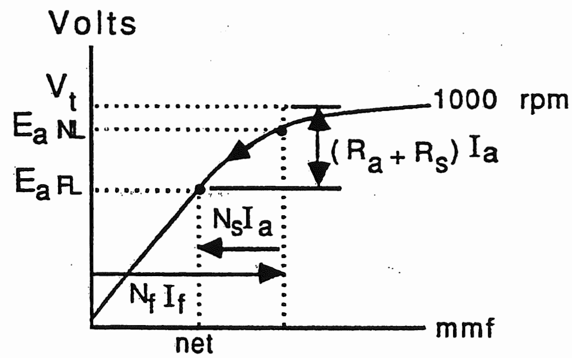


c) $\quad S\,R = \dfrac{n_{NL} - n_{FL}}{n_{FL}} = \dfrac{1099 - 1000}{1000} = 9.9\%$

To obtain zero speed regulation, ($n_{FL} = n_{NL}$), a series field must be connected, differentially, in series with the armature, and the field current must be readjusted to obtain rated speed at no-load, as indicated in the volts-mmf plane,

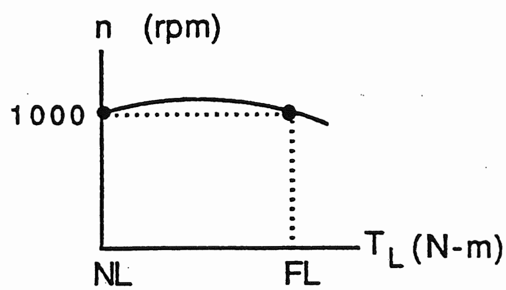


compound motor



net mmf = $N_f I_f - N_s I_a$

At no-load, $N_s I_a$ is negligibly small and the emf is determined by $N_f I_f$, where I_f remains constant. As the motor is torqued to full-load, the armature current increases, and the series mmf subtracts from the shunt mmf so that the emf, determined by the consequent net mmf, falls with the full-load, armature current, resulting in rated speed at full-load. The speed characteristic is then flat,



DC SERIES MOTOR

The series motor will be considered next, and the equations that define its performance are given from Fig. 6.45,

$$E_a = V_t - (R_a + R_s)\, I_a = K\,\Phi\,\omega \quad (V) \tag{6.26}$$

$$\text{net mmf} = N_s I_a \quad (A\text{-}t)$$

From Eqn (6.26) the motor speed is,

$$\omega = \frac{V_t - (R_a + R_s)\, I_a}{K\,\Phi} \quad \text{(rad/sec)}$$

(6.27)

where, $$T = K\,\Phi\, I_a \quad \text{(N-m)}$$

Several important observations can be made concerning Eqn. (6.27),

1. The driving mmf that determines the flux, Φ, and the consequent generated emf, is the armature current, I_a, flowing through N_s-turns on each pole-piece, therefore,

$$\Phi = K'\, I_a$$

2. Equations (6.27), for a series motor, can then be rewritten,

$$\omega = \frac{V_t - (R_a + R_s)\, I_a}{K'' I_a} \quad \text{(rad/sec)} \qquad (6.28)$$

where, $$T = K''\, I_a^2 \quad \text{(N-m)}$$

3. At no-load, the armature current is quite small, so the flux becomes essentially residual, and the speed becomes enormously large or the motor runs away with disastrous mechanical consequences.

4. At start, from Fig. 6.45, the generated emf is zero, and the armature current is limited only by the small armature and series – field resistance. The generated starting torque, therefore, from Eqn.(6.28), is very large.

Series motor analysis now continues with Example 6.11.

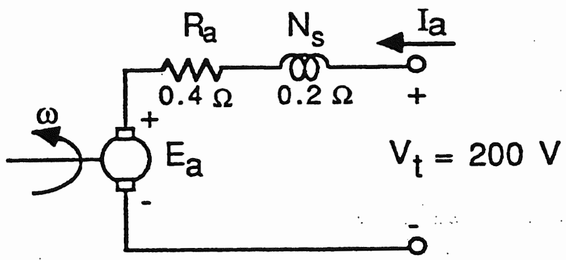
Example 6.11

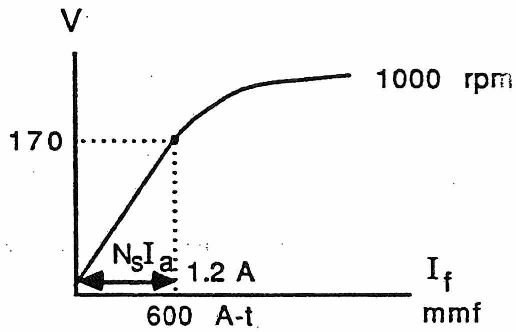
The machine of Example 6.10 is operated as a series motor. The resistance of the series winding is estimated to be 0.2 ohm.

(a) Design the series winding for rated speed at rated load and rated terminal voltage.

(b) What is the torque generated at rated speed?

(c) What is the speed at no-load?





a) At full-load, $I_a = 50$ A

$$E_a = V_t - (R_a + R_s)\, I_a = 200 - (0.4 + 0.2)(50)$$

$$= 170 \text{ V} \;\; @\; 1{,}000 \text{ rpm}$$

$$\text{net mmf} = 600 \text{ A-t} \;\; (\text{Fig. } 6.43) = N_s I_a$$

$$N_s = \frac{600}{50} = 12 \text{ turns}$$

b) From (a), $E_a = 170 = K\Phi\omega \;@\; \text{mmf} = 600$ A-t

$$K\Phi = \frac{170}{(1{,}000)\left(\frac{2\pi}{60}\right)} = 1.62 \;@\; 600 \text{ A-t mmf}$$

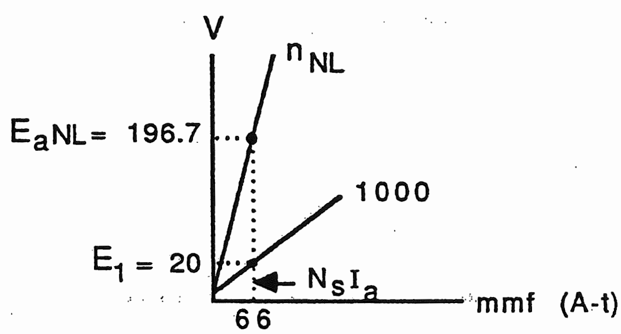
$$T = K\Phi I_a = (1.62)(50) = 81 \text{ N-m}$$

c) At no-load, $T_L = 0$,

$$T_{loss}\,\omega = E_a I_a = 1100 \text{ W}$$

Initially assume $E_a \cong V_t = 200$ V,

$$I_a NL = \frac{1100}{200} = 5.5 \text{ A}$$



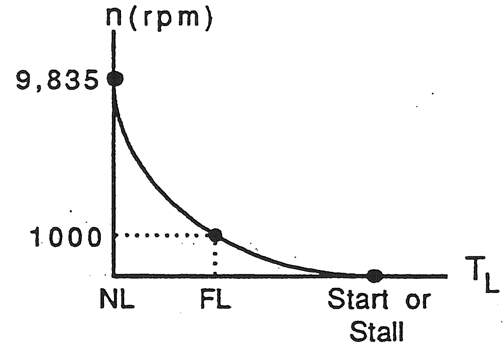
First correction for E_a,

$$E_a = 200 - (0.4 + 0.2)(5.5) = 196.7 \text{ V} \;@\; n_{NL} \text{ rpm}$$

$$N_s I_a = (12)(5.5) = 66 \text{ A-t} \;\; ; \;\; E_1 = 20 \text{ V} \;\; (\text{Fig. } 6.43)$$

$$\frac{E_aNL}{E_1} = \frac{n_{NL}}{1{,}000}$$

$$n_{NL} = \frac{196.7}{20} \times 1{,}000 = 9{,}835 \text{ rpm}$$



6-23 DC MOTOR STARTING AND SPEED CONTROL

In general, dc motors are started and speed controlled as in Fig. 6.48.

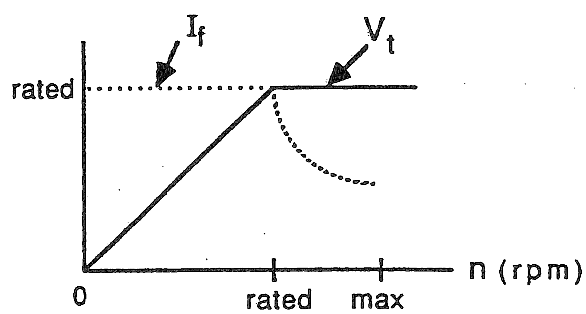


Figure 6.48 DC Motor Speed Control

From start to rated speed, the field current is set at maximum value and the dc terminal voltage is gradually brought from minimum to rated, without the armature current ever exceeding rated. For speeds greater than rated, the terminal voltage is kept constant at rated and the field is weakened to obtain maximum speeds as high as ten times rated. The problem, here, is two-fold–

1. In an ac power system, how is a dc armature voltage obtained?

2. If a dc voltage is available, how can it be varied from minimum to rated value?

Both of these questions are answered with a phase-controlled SCR, connected in series with the armature across a single-phase, ac, source as in Fig. 6.49.

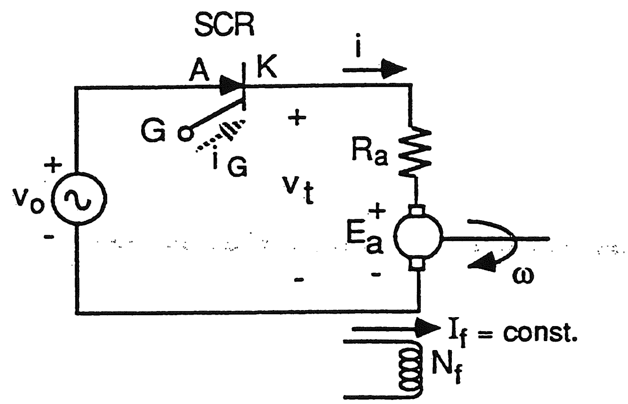


Figure 6.49 Phase-Controlled SCR Motor Drive

The silicon-controlled rectifier (SCR) is simply a diode, which, when forward-biased, will not conduct, (fire) until a positive current, i_G, is injected into its gate terminal. Assuming an ideal SCR with no forward voltage drop, the waveforms, at start, are given in Fig. 6.50,

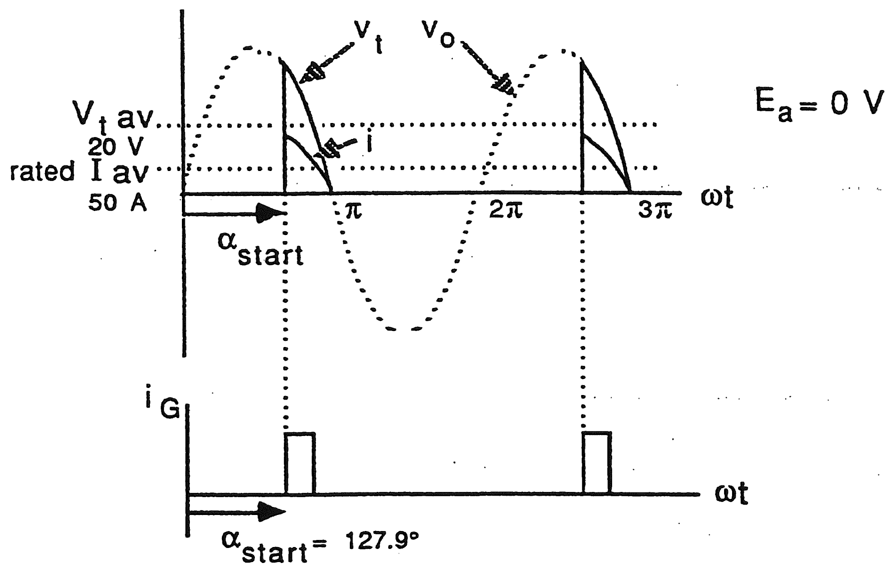


Figure 6.50 SCR Drive Waveforms at Start

The gate-current pulses are synchronized to the source and the firing angle, α, is set so that the average, (dc), value of the terminal voltage, V_t av, is exactly the value required to limit the average, (dc), value of the armature current, I av, to rated. This, then, creates an accelerating torque which starts the motor rotating, resulting in a back emf that helps limit the current.

The firing angle is then gradually decreased until the average, (dc), terminal voltage is rated and the machine is running at rated speed and rated load. The waveforms, at run, are shown in Fig. 6.51.

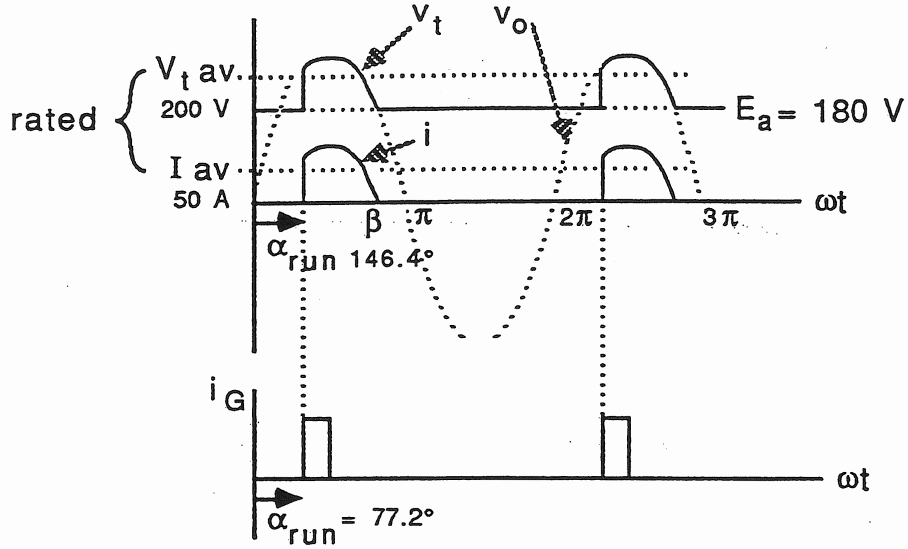


Figure 6.51 SCR Drive Waveforms at Run

Example 6.12

The machine of Example 6.10 is connected, as in Fig. 6.49, to a 230V, 60 Hz source.

(a) What must the terminal voltage be, at start, to limit the armature current to rated? (Fig. 6.50)

Since $E_a = 0$, at start,

$$V_t = R_a I_a = (0.4)(50) = 20 \text{ V}$$

The average value of the terminal voltage, then, must be,

$$V_{t\,av} = \frac{1}{2\pi} \int_{\alpha_{start}}^{\pi} V_m \sin \omega t \, d(\omega t) = 20 \qquad \text{(V)}$$

then, $\underline{\underline{\alpha_{start} = 127.9°}}$

and, $i = \frac{V_t}{R_a}$; $I_{av} = \frac{1}{2\pi} \int_{\alpha_{start}}^{\pi} \frac{V_m}{R_a} \sin \omega t \, d(\omega t) = \underline{\underline{50}} \qquad \text{(A)}$

(b) What must the firing angle, α, be, at run, so that the terminal voltage, armature current, and speed are rated? (Fig. 6.51)

now, $$E_a = 200 - (0.4)(50) = 180\text{ V} \quad @\ 1000\text{ rpm}$$

and, $$\beta = 180° - \sin^{-1}\frac{180}{\sqrt{2}\,(230)} = 146.4° \text{ or } 2.555 \text{ rad}$$

and, $$V_{t\,av} = \frac{1}{2\pi}\left[\int_0^{\alpha_{run}} E_a\, d(\omega t) + \int_{\alpha_{run}}^{\beta} V_m \sin\omega t\, d(\omega t) + \int_{\beta}^{2\pi} E_a\, d(\omega t)\right] = 200 \quad (V)$$

then, $\underline{\underline{\alpha_{run} = 77.2°}}$ (solution is a transcendental equation in α)

and, $$i = \frac{v_t - E_a}{R_a} \quad ; \quad I_{av} = \frac{1}{2\pi}\int_{\alpha_{run}}^{\beta}\left(\frac{V_m}{R_a}\sin\omega t - \frac{E_a}{R_a}\right) d(\omega t) = \underline{\underline{50}} \quad (A)$$

<u>Starting</u> and <u>speed-control</u> of a dc motor can be <u>summarized</u> in the speed equation,

$$\omega = \frac{\overset{\overset{\alpha}{\downarrow}}{V_t} - R\,\overset{\overset{T_L}{\downarrow}}{I_a}}{\underset{\underset{I_f \text{ or } I_a}{\uparrow}}{K\Phi}} \quad \text{(rad/sec)} \qquad (6.29)$$

Starting current and speed can be varied by varying any one of the variables on the right side of Eqn. (6.29),

1. The field current (shunt motor) or armature current (series motor) determines Φ.

2. Since, $T_L \approx T = K\,\Phi\, I_a$, the armature current is variable with the load torque for a given flux.

3. The armature resistance can be varied by inserting a variable external resistance in series with the armature.

4. V_t can be varied by changing the firing angle of Fig. 6.49.

6-24 SUMMARY

By far, the major load on existing power systems are motors, consisting primarily of ac-synchronous, induction motors and dc motors.

The synchronous motor is used in constant-speed load applications such as fans, air compressors, etc. It requires real power, continuously, which is converted to useful mechanical form, but is capable of delivering reactive power to the line (over excited), where it is quite useful in power-factor correction.

It is limited by the maximum amount of real power it can draw from the line where the torque-angle, δ, opens to 90°, and if further torqued it will snap out of synchronism and stall. Its performance is best summarized by its V-curves, in Fig. 6.5, for constant loads ranging from no-load to full-load.

The induction machine, in contrast to the dc-excited synchronous machine, is excited by the rotor windings slipping by the synchronously, rotating resultant field. As a consequence, the induction machine is asynchronous, and cannot, as a motor, rotate at synchronous speed.

With the perception that an induction motor looks like a transformer with a rotating secondary, its per-phase equivalent circuit resembles that of a transformer with an effective turns-ratio that varies with speed,

$$\frac{E_1}{E_R} = \frac{a}{s}$$

It is essentially a constant speed machine, from no-load to full-load, where it slips only a few per-cent, and its performance is best summarized in Fig. 6.21, where high starting torque and low running slip can be achieved by varying its rotor resistance.

As a single-phase machine, it requires a start-winding in series with a resistance (split-phase) or a capacitor (capacitor-start), and is used extensively in low-power, load applications. Other single-phase motors such as the shaded-pole, universal, reluctance, hysteresis, stepper motors, etc. supply a myriad of remaining load applications.

The dc motor, because of its various field connections, is used extensively in traction, hoisting, and automatic velocity and position-control, load applications.

It is used as a series-motor primarily in hoisting and traction, because of its large starting torque but it must be directly-connected or geared to its load, since it will run away at no-load.

The shunt motor is used primarily in automatic-control, steel rolling mills, etc. because its speed or torque can be continuously controlled by suitably varying its field current.

With the advent of the SCR, dc motors can be powered directly from ac lines with the consequence that dc motor loads are an appreciable percentage of the motor load on modern electric power systems.

Perhaps this chapter can be best concluded by asking a question – What basically makes these machines run as motors?

For the synchronous motor, this question is best answered by referring to Fig. 5.15. The resultant field, rotating at synchronous speed, hauls the dc-excited rotor field behind it through a torque angle, thus exerting a forward Lorentz torque on the rotor.

For the three-phase induction motor, this question is best answered by referring to Fig. 6.11. The rotor windings slip by the synchronously rotating resultant field, creating a rotor field that is hauled behind the resultant field through a torque angle, thus exerting a forward Lorentz torque on the rotor.

For the dc machine, this question is best answered by referring to Figs. (6.33) and (6.38). The field of a dc machine is stationary as in Fig. (6.33), and because of its commutator, the rotor-current distribution in Fig. 6.38 is also stationary, resulting in a unidirectional, forward, Lorentz torque acting on the rotor, making it a motor.

These three machines, then, create most of the demand on our nation-wide power-grid, and this demand must be supplied by synchronous generators, on a continuing basis, as the demand varies throughout each day. This problem will be described and analyzed in the next chapters.

PROBLEMS

6.1 The synchronous machine of Problem 5.7 is operated as a motor drawing rated kVA, 0.8 pf lagging, at rated voltage from the line.

a) Draw and label the per-phase, equivalent circuit.

b) What approximate I_f (A) is required to operate this motor with the above load? Sketch the phasor diagram and label completely with numerical values, in per-unit, using the machine rating as a base.

c) What is the maximum power, for I_f in (b), that can be drawn by this motor? Sketch and label the phasor diagram for this condition, with numerical values, in per-unit, using the machine rating as a base.

6.2 A 3-phase, 60 Hz, 6- pole, Y-connected synchronous machine, when driven as an open-circuited generator, has a terminal voltage of 2,340 V(line to line) with a field current, $I_f = 20$ A. When the machine is operated as a motor from a 2,200 V, 60 Hz line with $I_f = 20$ A, unity power factor, it delivers 890 N-m to a mechanical load. If the rotational and electrical losses are negligible, find,

a) Motor speed, rpm

b) Armature current, I_a (A)

c) Torque angle, δ, degrees

d) Synchronous reactance, Ω/ϕ.

6.3 A 3-phase, 60 Hz, 4-pole, 10 MVA, 13.8 kV, Y-connected, $X_s = 16.2\ \Omega/\phi$, synchronous motor is drawing rated MVA, unity power factor, at rated voltage from the line. If all losses are negligible,

a) At what speed, rpm, does the motor run?

b) Sketch the phasor diagram and label completely with numerical values, in per-unit, using the machine rating as a base.

c) With constant field current, the motor is unloaded. Sketch the phasor diagram and label completely with numerical values in per-unit, using the machine rating as a base.

6.4 A 3-phase, 60 Hz, 6-pole, 10 kVA, 240 V, $X_s = 4.0\ \Omega/\phi$, synchronous motor, with negligible losses, is drawing rated kVA, 0.85 pf lagging from a 240-volt infinite bus.

a) Draw and label completely, with numerical values, the phasor diagram for this motor.

b) If the mechanical loading in (a) is kept constant, and the rotor field excitation is changed, what is the minimum line current, I_a, (polar form) that can be drawn ? Draw and label the phasor diagram, with numerical values, for this condition.

6.5 A three-phase, 60 Hz, 1,140 rpm induction motor is connected to a three-phase bus at rated voltage.

a) How many poles does the motor have?
b) What is the percent slip at full-load?
c) What is the corresponding frequency of the rotor voltages?
d) What is the corresponding speed (rpm),
(1) of the rotor field with respect to the rotor?
(2) of the rotor field with respect to the stator?
(3) of the rotor field with respect to the stator field?
(4) of the resultant field (stator field plus the rotor field) with respect to the stator?
(5) of the resultant field with respect to the rotor?

6.6 Do Problem 6.5 for a slip of –5%.

6.7 Do Problem 6.5 for a slip of 195%.

6.8 A 100 hp, 3-phase, Y-connected, 440 volt, 60 Hz, 8-pole, squirrel-cage induction motor has the following parameters (ohms/phase) referred to the stator,

$$r_1 = 0.085 \qquad r_2 = 0.067$$

$$x_1 = 0.196 \qquad x_2 = 0.161 \qquad x_\phi = 6.65$$

No-load rotational loss = 3.2 kW, and remains constant with speed.

a) Compute I_1, pf, hp_o, efficiency at rated voltage and frequency for s = 3%. (This slip is <u>not</u> at rated hp)
b) Compute input starting current and generated starting torque.

6.9 Do Problem 6.8 for s = 3.5%.

6.10 A 10 hp, 3-phase, 60 Hz, 6-pole, induction motor runs at a slip of 3% at <u>full-load</u> horse power. Rotational losses at full-load are 4% of output power. Compute:

a) The power delivered across the air-gap at full-load.
b) The rotor copper loss at full-load.
c) The generated torque at full-load.

6.11 Do Problem 6.10 if the motor runs at a slip of 3.5% at <u>full-load</u> horsepower.

6.12 A 10 hp, 230 volt, 3-phase, Y-connected, 60 Hz, 4-pole, squirrel cage induction motor develops full-load generated torque at a slip of approximately 4%, at rated voltage and frequency. The rotational losses are 4% of rated horsepower and remain constant with speed. The machine parameters in ohms per phase, are,

$r_1 = 0.36$ $\quad x_1 = x_2 = 0.47$

$r_2 = 0.223$ $\quad x_\phi = 15.5$

a) Determine slip at maximum torque
b) Determine the maximum, and full-load, and starting torques.
c) Sketch the torque-slip curve and label all important torques and corresponding slips with numerical values.

6.13 Do Problem 6.12 for the machine in Problem 6.8 ($s_{FL} \approx 3\%$)

6.14 Sketch and completely label all torques, velocity and power flow, with numerical values, on a power- flow diagram, for the machine in Problem 6.12, when the machine is,

a) a generator at full-load slip. (Power input is not rated!)
b) a motor at full-load slip.
c) plugged as a motor in (b).

6.15 Do Problem 6.14 for the machine in Problem 6.8.

6.16 A 60 Hz, 1,740 rpm, single-phase motor is running at rated speed. Sketch the torque-slip characteristics of this machine and label all important slips with numerical values.

6.17 What type of motor would be used in the following applications? Also give your reasons - vacuum cleaner, refrigerator, washing machine, household oil-burner, desk fan, sewing machine, bench grinder, clock, food mixer, record player, portable electric drill, water pump.

6.18 For the elementary, dc machine in Example 6.7, calculate its instantaneous and average emfs and torques if it is driven at 1200 rpm.

6.19 The practical dc machine in Fig. 6.38 has a rotor, r = 6", ℓ = 12", g = 1/32", 2 turns/coil, and 8 coils. The pole pieces span 0.7π radians and the gap flux density is B = 1.0 T. If the machine is rotating at 1800 rpm with an armature current, $I_a = 10$ A,

a) How many field turns (N_f) are required to establish a gap flux density of 1.0T, if $I_f = 5$A?
b) What is Φ, average $- E_{cond}$, T_{cond}; total E_a and T?
c) Is this machine a motor or generator? Why?
d) What is the power flow through the coupling field, P_M and P_E. Calculate each independently.
e) If $R_{cond} = 0.01\Omega$, what is the terminal voltage, V_t?

6.20 A dc motor is rated 200 V, 50 A, 1,000 rpm, $R_a = 0.1\ \Omega$. Its magnetization curve is given at 1,000 rpm in Fig. 6.43. The machine is operated as shunt motor from an infinite bus at rated voltage. The rotational losses are measured at 1,540 watts and remain constant with speed. The field current is adjusted for rated speed at no-load.

a) What is the field current?
b) What is the speed regulation for this motor?
c) What is the horsepower output at full-load?
d) Sketch the speed-torque curve for this motor and numerically label known points on this curve.

6.21 The machine of Problem 6.20 is operated as a series motor at rated voltage.

a) How many series field turns are required to obtain rated speed at rated current?
b) What is the generated torque at rated speed?
c) What is the generated torque at start?
d) Sketch the speed-torque curve for this machine and numerically label known points on this curve.

6.22 It is desirable, for the shunt motor in Example 6.10, to obtain zero-speed regulation by adding a differentially-connected series field of negligible resistance to the armature.

a) What is the field current, I_f, adjusted to, to obtain this regulation?
b) How many series-field turns, N_s, are required to obtain this regulation?
c) Sketch your solution, with numerical values, on the volts-mmf plane.

6.23 A 10 hp, 200 V, 1,180 rpm, 47.1 A, dc shunt motor has an armature resistance of 0.50 ohm. The field current is set at 1.0 A for operation at full-load, rated terminal voltage and rated speed. Its magnetization curve is <u>not</u> Fig. 6.43.

a) What is the full-load efficiency?
b) At decreased load, the line current drops to 22 A. What is the motor speed?
c) In (b), what is the generated (Lorentz) torque?
d) What is the no-load speed?
e) Sketch solutions for (b) and (d) on the volt - mmf plane.
f) What is the speed regulation at FL?

6.24 A 10 hp, 200 volt, 50A, 2,000 rpm, dc series motor has an armature resistance of 0.4 Ω. Its magnetization curve is given at 1,000 rpm in Fig. 6.43.

a) How many series-field turns are required to obtain rated speed at rated current?
b) What is the generated torque at rated speed?
c) What is the generated torque at start?
d) Sketch the speed-torque curve for this machine and numerically label known points on this curve.

6.25 A dc shunt motor is mechanically coupled to a 3-phase synchronous generator (same shaft). The dc motor is connected to a 230-volt dc line and the ac generator is connected to a 3-phase, 230 V, infinite bus. The 60 Hz, 4-pole, Y-connected synchronous machine is rated 25 kVA, 230 V, $X_s = 1.6\ \Omega/\phi$. The 6-pole, dc machine is rated 25 kW, 230 V, $R_a = 0.1\ \Omega$. The rotational losses of both machines are negligible.

a) The two machines act as a motor-generator set and are adjusted to receive power from the dc line and deliver power to the ac line.
   1. What is the excitation emf, E_f, (polar form) of the ac machine, if it delivers rated kVA, unity pf, to the infinite bus.
   2. What is I_a (polar form) of the ac machine?
   3. What is I_a of the dc machine?
   4. Sketch the dc machine solution on the E_a vs I_f plane. Sketch the ac machine phasor diagram. Label both completely.

b) If the field current of the ac machine remains constant as in part (a),
   1. What adjustment is made to reduce the power flow between the dc and ac machines to zero?
   2. What is I_a (polar form) of the ac machine?
   3. What is I_a of the dc machine?
   4. Repeat part (a-4)

c) If the field current of the ac machine remains constant as in part (a)
   1. What adjustment is made to cause 25 kW to be taken from the ac line and delivered to the dc line?
   2. Repeat part (a-2,3,4).

d) If the field current of the ac machine remains constant as in part (a),
   1. What adjustment is made to cause maximum power to be taken from the ac line and delivered to the dc line?
   2. Repeat part (a-2,3,4).

CHAPTER 7

TRANSMISSION LINES

This chapter concludes the analysis of the major components of an elementary power system with the transmission line, as shown in Fig. 7.1.

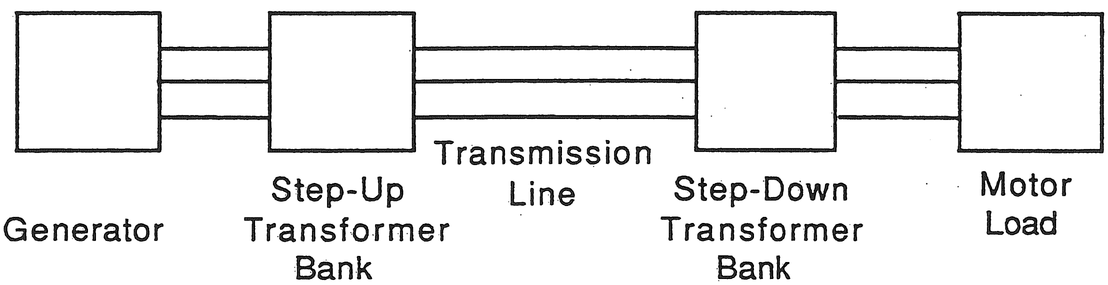


Figure 7.1 Elementary Power System

Since electric power is generated remotely from load centers, it is transmitted through the magnetic and electric fields of cables, at high voltage, where the cables are mounted on towers along a right of way, as shown in Fig. 7.2.

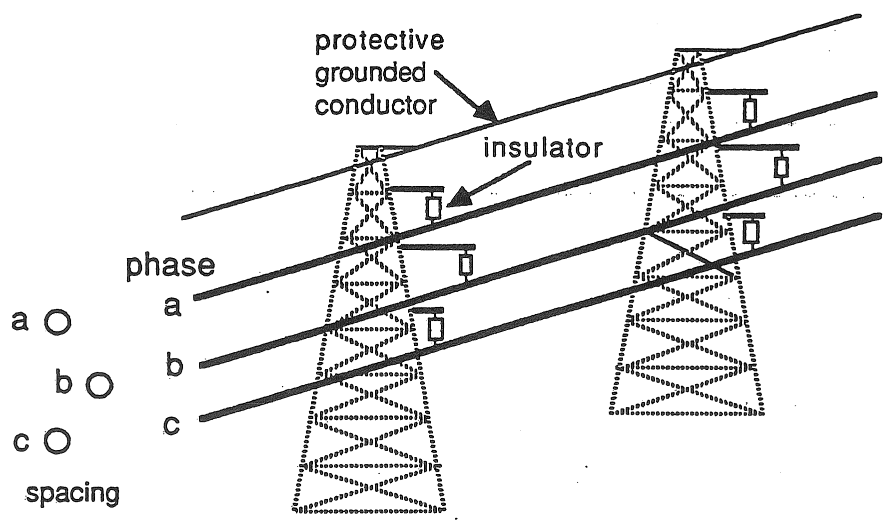


Figure 7.2 Transmission Towers and Cables

For a given power transmitted, the high transmission voltage results in relatively low line currents and, therefore, minimal, I^2R, line losses.

This chapter is concerned with the steadystate analysis of the high-voltage portion of power system transmission, whose cables are characterized by four parameters,

1. Resistance
2. Inductance
3. Capacitance
4. Conductance

These parameters, then, form the basis for transmission line models, which can be used to predict the effect of these parameters on power flow through the fields of transmission lines. Resistance characterizes the electric field within the cables; inductance, the magnetic field surrounding the cables; capacitance, the electric field terminating on the cables; and conductance, the leakage path between the cables and ground.

Transmission line cables may be run overhead, underground, or under water, which determines the type of materials and insulation with which they are made. The conductors used in cables are made of aluminum, copper, or their alloys, and since the resistivity of aluminum is nearly that of copper, but much more cost effective, aluminum conductors are most often used in cable construction.

Overhead, high-voltage, transmission lines are mounted on towers spaced approximately one-quarter mile apart, and while all-aluminum cables possess desirable electrical characteristics, they would fail mechanically, because of their own weight, or, with ice or wind load. For this reason most high-voltage overhead cables are made of ACSR type (aluminum conductor, steel reinforced), where aluminum strands are spirally wound around a steel core made of steel strands. The cross-section of ACSR cable is shown in Fig. 7.3.

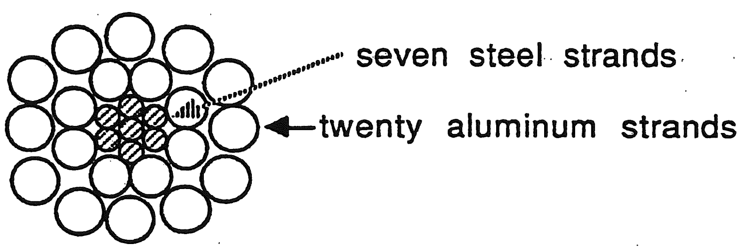
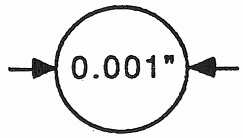


Figure 7.3 Cross-Section of ACSR Cable

The ACSR cable, then, provides the desirable mechanical and electrical characteristics at relatively low cost.

ACSR cables are manufactured in different aluminum cross-sections depending on the current they will carry.

The ampacity of a cable is primarily determined by the total cross-section area of the aluminum strands with which it is made. This cross-section area is measured in circular mils where a circular mil is defined,



$$1.0 \text{ CM} = 3.14 \times 10^{-6} \text{ in}^2$$

Table 7.1 is a list of some of the electrical characteristics of several ACSR cables.

Cable	Total Aluminum Cross-Section (CM)	Nominal Outside Diameter (in.)	R_{dc} 20°C Ω/1000'	R_{ac} (60 Hz) 20°C Ω/mile	R_{ac} (60 Hz) 50°C Ω/mile	GMR ft.
A	397,500	0.783	0.0430	0.2323	0.2551	0.0264
B	477,000	0.846	0.0359	0.1943	0.2134	0.0284
C	477,000	0.858	0.0357	0.1931	0.2120	0.0289
D	556,500	0.879	0.0309	0.1679	0.1843	0.0284
E	636,000	0.977	0.0269	0.1461	0.1603	0.0327
F	795,000	1.108	0.0215	0.1172	0.1284	0.0373
G	1,431,000	1.465	0.0120	0.0673	0.0735	0.0494

Table 7.1 ACSR Cable Electrical Characteristics

7-1 TRANSMISSION LINE RESISTANCE

The resistance of a cable is a function of frequency, temperature and spiraling. For a given temperature, the dc resistance of a conductor is given as,

$$R_{dc} = \frac{\rho \ell}{A} \quad (\Omega) \qquad \text{where, } \ell = \text{length, (m)} \tag{7.1}$$

A = cross-section, (m^2)

ρ = resistivity, (Ω-m)

Equation (7.1) is valid only for dc where the current density is uniform over the conductor cross-section. Under ac conditions, the current density is no longer uniform over the cross-section, because of the phenomenon called skin-effect, where, at increasing frequency, more and more of the current is forced to flow through the outer cross-section of the conductor. For this reason, ac resistance at 50-60 Hz is slightly higher than dc resistance,

$$R_{ac} \approx 1.03 \, R_{dc} \tag{7.2}$$

For normal temperature change, the resistance of a conductor also varies linearly with temperature,

$$R = R_o [1 + \alpha t] \qquad (\Omega) \qquad (7.3)$$

where, R_o = resistance at 0° C
α = temperature coefficient of conductor material, /C°
t = temperature, °C

Equation (7.3) is plotted,

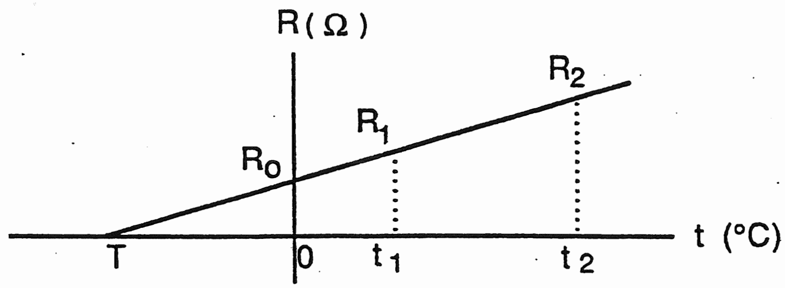


Figure 7.4 Conductor Resistance as a Function of Temperature

The zero-resistance intercept, T, is a function of the conductor material, and for hard-drawn aluminum,

$$T = 228 \text{ °C}$$

From Eqn. (7.3),

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1} = \frac{228 + t_2}{228 + t_1} \qquad (7.4)$$

The ac resistance of a cable, then, depends on its dc value as changed by skin-effect, temperature, and its increased length due to spiraling.

Example 7.1

The dc resistance of cable F in Table 7.1 is 0.0215 Ω/1000 ft., or, 0.1135 Ω/mile at 20°C.

The aluminum area of cable F is 795,000 CM and from Eqn. (7.1), if ρ for aluminum is 17 Ω-CM/ft at 20°C,

$$R_{dc} = \frac{\rho \ell}{A} = \frac{(17)(5,280)}{795,000} = 0.1129 \ \Omega/\text{mile @ } 20\text{°C}$$

The increase in length due to spiraling, then, according to the cable manufacturer, must be,

$$\frac{0.1135 - 0.1129}{0.1129} = 0.53\ \%$$

The ac resistance of cable F, from Table 7.1 is 0.1172 Ω/mile at 20 °C, so the increase in resistance from its dc value, due to skin effect, must be,

$$\frac{0.1172 - 0.1135}{0.1135} = 3.26\ \%,$$

$$R_{ac} = 1.0326\ R_{dc}$$

which is approximately Eqn. (7.2).

The ac resistance of cable F at 50 °C, from Table 7.1, is 0.1284 Ω/mile. From Eqn. (7.4),

$$\frac{R_{ac}\,(50\ °C)}{R_{ac}\,(20\ °C)} = \frac{228 + 50}{228 + 20}$$

$$R_{ac}\,(50\ °C) = (1.121)(0.1172) = 0.1314\ \ \Omega/\text{mile}$$

The 2.3% difference between the calculated and manufacturer's value is probably due to the aluminum used and the manufacturer's precision in measurement.

7-2 SINGLE-PHASE TRANSMISSION LINE INDUCTANCE

The inductance of a single-phase line will be derived and then extended to a three-phase transmission line. Consider first the magnetic field produced by a single-phase line.

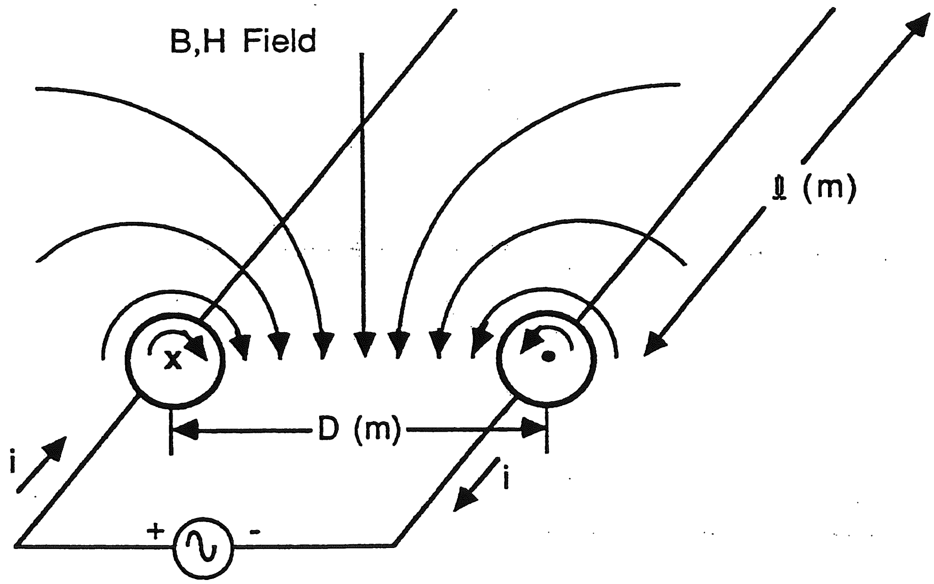


Figure 7.5 Magnetic Field of Single-Phase Line

The direction lines of the flux density and magnetic intensity, using the right-hand rule, are shown in Fig. 7.5. Observe that the magnetic field exists within the conductors as well as outside the conductors. Observe, also, that each weber of flux that pierces the area spanned by the conductors constitutes a flux linkage with the currents that produced it. The inductance, per meter, of the line is the ratio of the total flux linkages to the line current.

$$L = \frac{\lambda}{I} \qquad (H) \qquad (7.5)$$

The inductance, then, represents the magnetic field of the line, and its effect due to changing flux linkages with time.

Since a current distribution exists in space, the magnetic field produced by each conductor in Fig. 7.5, is given by Maxwell's mmf law,

$$\int_S \mathbf{J} \cdot d\mathbf{a} = \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} \qquad (A) \qquad (7.6)$$

Because the medium of the magnetic field is nonferrous, the fields problem is linear, and is, therefore, amenable to the principle of superposition. First the flux linkages of the transmission line created by the left conductor in Fig. 7.5 will be derived, and then summed with the flux linkages of the right conductor, to obtain the total flux linkages.

Consider the flux linkages produced by the left conductor.

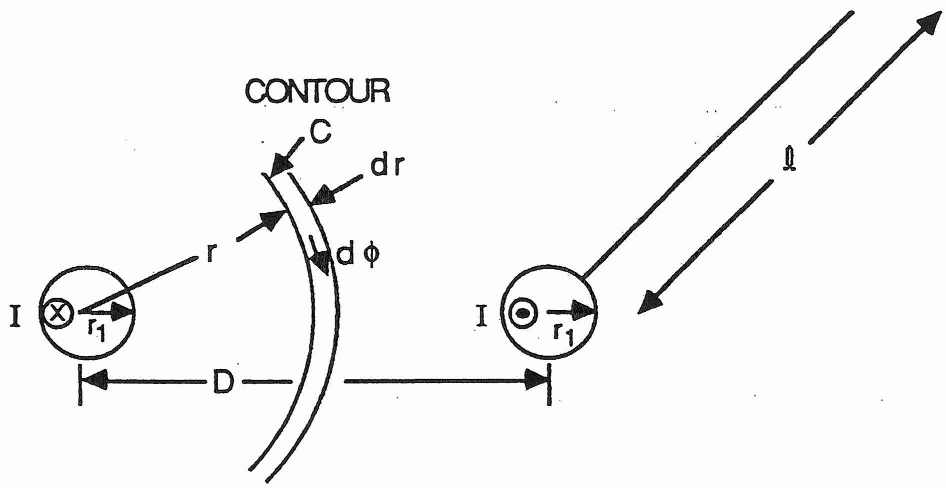


Figure 7.6 Flux Linkages Produced by Left Conductor

The line conductors are assumed circular with radius r_1 (m), separated, center to center, by a distance, D (m). With little error due to skin effect, the current density will be assumed uniform over the cross-section of each conductor. Taking advantage of symmetry, a circular contour, C, in Eqn. (7.6), will be chosen with radius, r (m), concentric with the left conductor. Maxwell's mmf law will be applied for contour radius less than r_1, in which case, the resultant flux linkages are internal to the left conductor, and for contour radius greater than r_1, the flux linkages are external to the left conductor.

For contours, $r \leq r_1$, Eqn. (7.6) becomes,

$$\frac{\pi r^2}{\pi r_1^2} I = 2\pi r H \quad ; \quad H = \frac{r}{2\pi r_1^2} I \qquad \text{(A/m)}$$

$$B = \frac{\mu_o r}{2\pi r_1^2} I \qquad \text{(T)}$$

then, $$d\phi = B\,da = B\,\ell\,dr = \frac{\mu_o \ell r I}{2\pi r_1^2} dr \qquad \text{(Wb)}$$

the incremental flux linkage is,

$$d\lambda = \left[\frac{\pi r^2}{\pi r_1^2}\right] d\phi = \frac{\mu_o \ell r^3 I}{2\pi r_1^4} dr \qquad \text{(Wb)} \qquad (7.7)$$

For contours, $r_1 \leq r \leq D$, Eqn. (7.6) becomes,

$$I = 2\pi r H \quad ; \quad H = \frac{I}{2\pi\ r} \qquad \text{(A/m)}$$

$$B = \frac{\mu_o I}{2\pi\ r} \qquad \text{(T)}$$

then, $$d\phi = B\,da = B\,\ell\,dr = \frac{\mu_o \ell I}{2\pi\ r} dr \qquad \text{(Wb)}$$

the incremental flux linkages are,

$$d\lambda = d\phi = \frac{\mu_o \ell I}{2\pi\ r} dr \qquad \text{(Wb)} \qquad (7.8)$$

For contour radius greater than D, the net current enclosed, by superposition of both conductors, is zero, and need not be considered. From Eqns. (7.7), (7.8), the <u>total</u> flux linkage with the left cable is,

$$\lambda = \underbrace{\int_0^{r_1} \frac{\mu_o\ \ell\ r^3\ I}{2\pi\ r_1{}^4} dr}_{\text{internal}} + \underbrace{\int_{r_1}^{D} \frac{\mu_o\ \ell\ I}{2\pi\ r} dr}_{\text{external}}$$

$$= \frac{\mu_o \ell I}{8\pi} + \frac{\mu_o \ell I}{2\pi} \ln \frac{D}{r_1}$$

$$\lambda = L I = \frac{\mu_o \ell I}{2\pi} \left(\frac{1}{4} + \ln \frac{D}{r_1}\right)$$

For, $\mu_o = 4\pi \times 10^{-7}$ (H/m), and, $\ell = 1.0$ m,

$$L = 2 \times 10^{-7} \left(\ln e^{1/4} + \ln \frac{D}{r_1} \right)$$

$$L = 2 \times 10^{-7} \ln \frac{D}{r_1\ e^{-1/4}} \qquad \text{(H/m/cable)} \qquad (7.9)$$

The geometric mean of n-discrete quantities is defined as,

$$\sqrt[n]{(x_1)(x_2) \ldots (x_n)}$$

and it can be shown that the geometric mean radius of a circular cross-section of radius, r_1, is,

$$\text{GMR} = r_1\ e^{-1/4} \qquad \text{(m)}$$

Equation (7.9) can then be written,

$$L = 2\times10^{-7} \ln \frac{D}{GMR} \qquad \text{(H/m/cable)} \qquad (7.10)$$

The inductance of a single-phase line is now extended to symmetrically and unsymmetrically-spaced, three-phase, transmission lines in the next section.

7-3 THREE-PHASE TRANSMISSION LINE INDUCTANCE

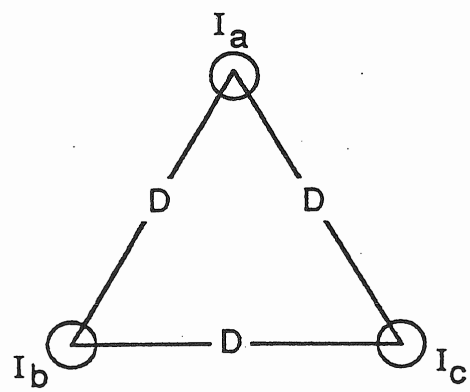


Figure 7.7 Three-Phase Line with Symmetrical Spacing

For balanced currents in Fig. 7.7, $I_a + I_b + I_c = 0$. The magnetic field configuration surrounding the three cables is, then, symmetrical, similar to the configuration of a single-phase line. It can be shown that the flux linkage with cable a or b or c, created by all three cables, is identical to the flux linkage of a single-phase line, i. e.,

$$\lambda_a = \lambda_b = \lambda_c = 2\times10^{-7} I \ln \frac{D}{GMR} = L I$$

$$L_a = L_b = L_c = 2\times10^{-7} \ln \frac{D}{GMR} \qquad \text{H/m/}\phi \qquad (7.11)$$

Since ACSR cables are used in the overhead, high voltage, transmission line of Fig. 7.7, the GMR of each cable is listed in Table 7.1.

In practice, the cables of Fig. 7.7 are not symmetrically spaced, the magnetic field configuration is no longer symmetrical, the flux linkage of each cable is not the same, and therefore, the cable inductances are not balanced.

Unsymmetrically-spaced cables, together with a transposed transmission line in space, are shown in Fig. 7.8.

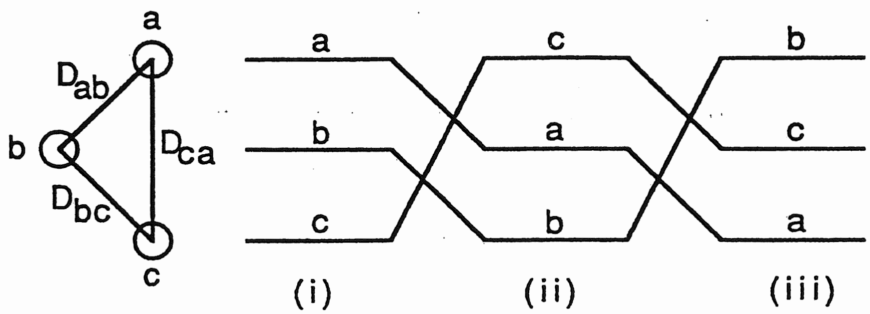


Figure 7.8 Transposed, Unsymmetrically Spaced Cables

In Fig. 7.8, the flux linkage of cable a or b or c is different for each of the space arrangements of the cables in i or ii or iii. It can be shown that the average flux linkage of each cable over the three space arrangements is the same, and therefore, the inductance of each cable is,

$$L_a = L_b = L_c = 2\text{x}10^{-7} \ln \frac{GMD}{GMR} \qquad H/m/\phi \qquad (7.12)$$

where,

$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$$

When we model the transmission line, per-phase, we are more concerned with the steady state, distributed, inductive reactance of the line at 60 Hz,

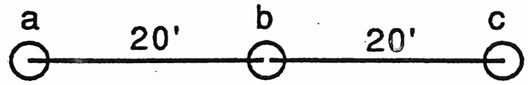
$$x_L = \omega L = 2\pi f L = 7.54\text{x}10^{-5} \ln \frac{GMD}{GMR} \qquad (\Omega/m/\phi)$$

and in English units,

$$x_L = 0.1213 \ln \frac{GMD}{GMR} \qquad (\Omega/mile/\phi) \quad (7.13)$$

Example 7.2

A three-phase, 60 Hz, transmission line is constructed of ACSR cable F, with a spacing,



a) Calculate the distributed line inductance and reactance.

$$GMD = \sqrt[3]{(20)(20)(40)} = 25.2 \text{ ft.}$$

From Table 7.1, $\quad GMR = 0.0373$ ft.

$$L = 2\text{x}10^{-7} \ln \frac{GMD}{GMR} = 2\text{x}10^{-7} \ln \frac{25.2}{0.0373} = 1.3 \ \mu H/m$$

$$x_L = 0.1213 \ln \frac{GMD}{GMR} = 0.1213 \ln \frac{25.2}{0.0373} = 0.79 \ \Omega/\text{mile}/\phi$$

b) What is the distributed resistance of this line at 50°C?

From Table 7.1, $\quad r = 0.1284 \quad (\Omega/\text{mile}/\phi)$

In part a), as a rule of thumb, the GMD-GMR ratio, regardless of cable spacing, is such, that for a three-cable transmission line,

$$x_L \approx 0.8 \ \ \Omega/\text{mile}/\phi$$

As the voltage of three-phase transmission lines is raised, to increase the power delivered along a given right of way, corona loss, due to the increased electric field intensity, becomes a problem. For this reason, three-phase lines, 500 kV and higher are bundled *, and to increase the power delivered along a given right of way, the lines are paralleled *, as shown in Fig. 7.9.

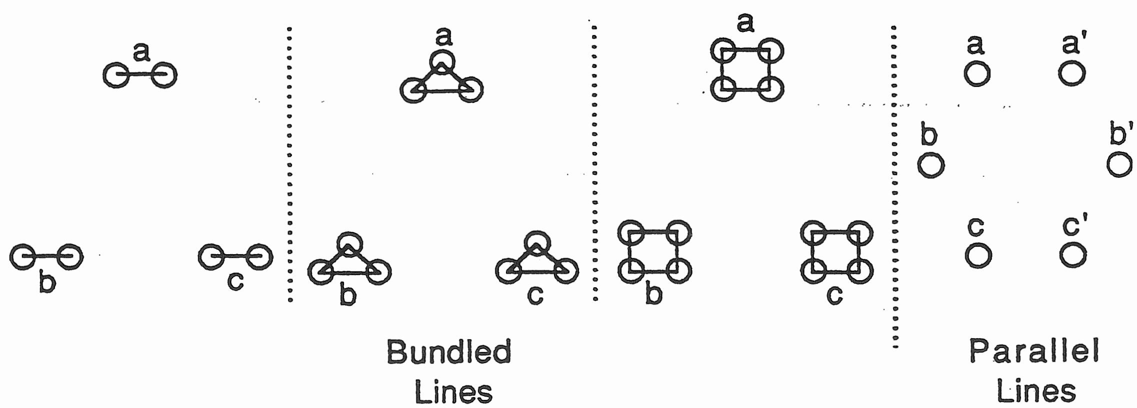


Figure 7.9 Bundled and Parallel Transmission Lines

For the remainder of this chapter, single-cable lines, only, will be considered.

* The analysis of single-cable, bundled and parallel lines is fully treated in W. D. Stevenson, Jr., "Elements of Power System Analysis", McGraw-Hill, New York, 1982.

7.4 SINGLE-PHASE TRANSMISSION LINE CAPACITANCE

The capacitance of a single-phase line will be derived, and then extended to a three-phase transmission line. Consider first, the electric field produced by a single-phase line far above earth.

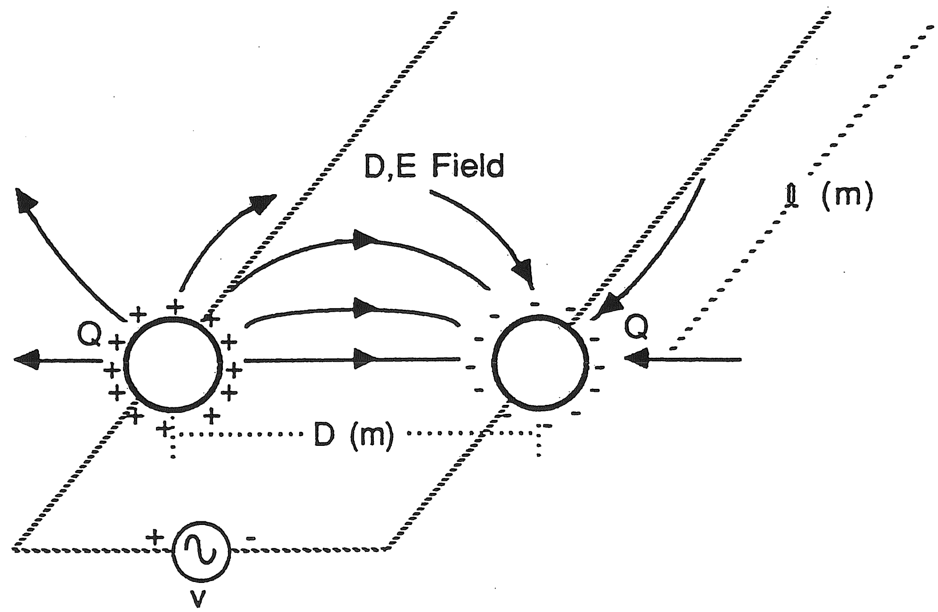


Figure 7.10 Electric Field of Single-Phase Line

The direction lines of the flux density and electric field intensity are shown in Fig. 7.10, where lines emanate from positive charge and terminate on negative charge. The capacitance of the line is the ratio of the charge to the difference in potential that produced it.

$$C = \frac{Q}{V} \qquad \text{(F)} \qquad (7.14)$$

The capacitance, then, represents the electric field of the line and its effect due to changing charge with time. Since a charge distribution exists in space, as in Fig. 7.10, the electric field produced by each conductor is given by Gauss's law,

$$\oint_S \mathbf{D} \cdot da = Q \qquad \text{(C)} \qquad (7.15)$$

where, D = Electric flux density, C/m^2
Q = Charge (C)

Gauss's law is a very general law, and states that if <u>any</u> closed surface in space encloses net charge, an electric field exists and is defined by Eqn. (7.15). More simply stated, the total electric flux emanating from a closed surface is identically equal to the total charge enclosed.

Consider the left cylindrical conductor in Fig. 7.10 with uniformly distributed charge Q, over its surface.

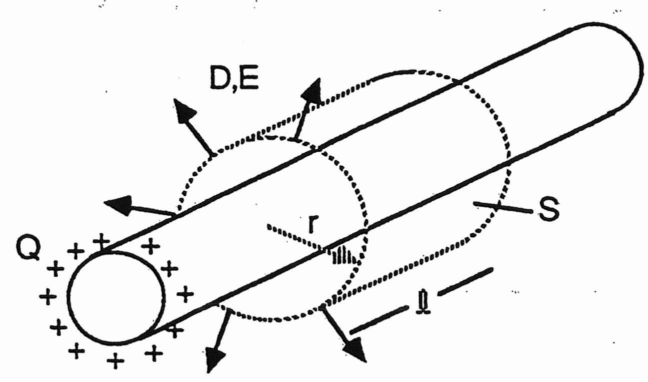


Figure 7.11 Gaussian Cylindrical Surface

Taking advantage of symmetry, a closed cylindrical surface, S, concentric with the conductor in Fig. 7.11, is chosen as the surface in Eqn. (7.15). Because of symmetry, D, is assumed radial, directed out of the cylinder from the positive charge. If a point on the cylinder were rotated through an angle θ, at constant radius, r, the electric flux density would not change because of the uniform charge distribution. Flux density, D, then, is not a function of θ, but varies with radius. Evaluating the integral in Eqn. (7.15),

$$D\,2\pi r \ell = Q \qquad (C)$$

$$D = \frac{Q}{2\pi r \ell} \qquad (C/m^2) \qquad (7.16)$$

where, Q = charge enclosed.

The electric field in intensity is related to the flux density,

$$E = \frac{D}{\varepsilon} \qquad (V/m) \qquad (7.17)$$

where ε is the permittivity of the medium and for free space or air,

$$\varepsilon_o = 8.85 \times 10^{-12} \qquad (F/m) \qquad (7.18)$$

The field intensity, then, is also radial and variable with radius,

$$E = \frac{Q}{2\pi r \varepsilon_o \ell} \qquad \text{(V/m)} \qquad (7.19)$$

Observe in Fig. 7.11, the cylindrical surface, S, is an equipotential surface and its potential, in this case, is lower than the potential of the conductor. The potential difference between any two equipotential, cylindrical surfaces is, then,

$$v_{12} = \int_{r_1}^{r_2} E dr \qquad \text{(V)} \qquad (7.20)$$

Since the medium surrounding the conductors is air, the fields problem is linear, and the principle of superposition can be used. Since the line capacitance is a function of the potential difference between the lines, the potential difference due to the charge on the left conductor will be determined, and then added to the potential difference due to the right conductor. Using a cylindrical, gaussian surface, Gauss's law is applied to the charge configuration in Fig. 7.10.

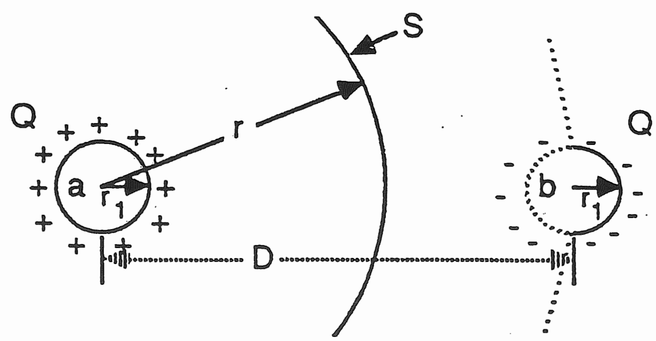


Figure 7.12 Potential Difference Produced by Left Conductor

From Fig. 7.12, the potential difference produced by the left conductor is,

$$V_{ab}\,(\text{left}) = \int_{r_1}^{D} \frac{Q}{2\pi r \varepsilon_o \ell} dr \qquad \text{(V)}$$

$$= \frac{Q}{2\pi \varepsilon_o \ell} \ln \frac{D}{r_1} \qquad \text{(V)}$$

and by the right conductor,

$$V_{ab}\,(\text{right}) = \frac{-Q}{2\pi\varepsilon_o \ell} \ln \frac{r_1}{D} \qquad (V)$$

By superposition,

$$V_{ab} = \frac{Q}{2\pi\varepsilon_o \ell}\left(\ln \frac{D}{r_1} - \ln \frac{r_1}{D} \right) \qquad (V)$$

$$= \frac{Q}{2\pi\varepsilon_o \ell} \ln \frac{D^2}{r_1^2} \qquad (V)$$

Since,

$$C_{ab} = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon_o \ell}{\ln \frac{D^2}{r_1^2}} \qquad (F)$$

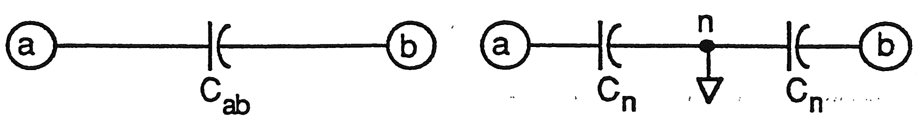
if,

$$\ell = 1.0 \text{ meter}$$

then,

$$C_{ab} = \frac{\pi\varepsilon_o}{\ln \frac{D}{r_1}} \qquad (F/m) \qquad (7.21)$$

Equation (7.21) is, then, the capacitance between two conductors located far above earth or neutral. It will be useful to define the capacitance to neutral as shown in Fig. 7.13.



$$C_{ab} = \frac{C_n C_n}{C_n + C_n} = \frac{C_n}{2} \quad ; \quad C_n = 2\,C_{ab}$$

Figure 7.13 Line Capacitance to Neutral

The line capacitance to neutral is, then, from Eqn. (7.21),

$$C_n = C_{an} = C_{bn} = \frac{2\pi\varepsilon_o}{\ln \frac{D}{r_1}} \qquad (F/m \text{ to neutral}) \qquad (7.22)$$

The capacitance of a single-phase line is now extended to symmetrically and unsymmetrically-spaced, three-phase lines in the next section.

7-5 THREE PHASE TRANSMISSION LINE CAPACITANCE

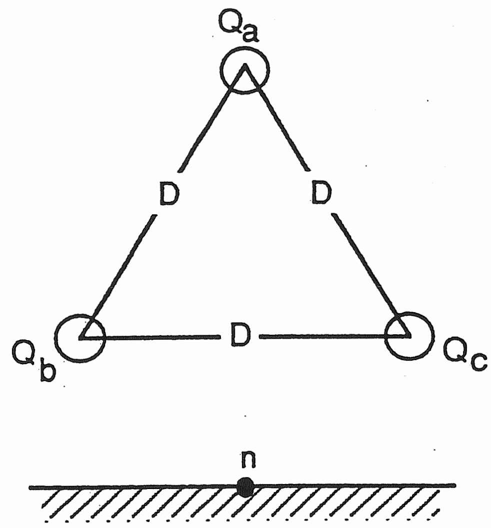


Figure 7.14 Three-Phase Line with Symmetrical Spacing

For balanced charges in Fig. 7.14, $Q_a + Q_b + Q_c = 0$. The electric field configuration surrounding the three cables is, then, symmetrical, similar to the configuration of a single-phase line. It can be shown that the total difference in potential between each cable and neutral, created by all three cables, is identical to the potential difference to ground of a single-phase line, i.e.,

$$V_{an} = V_{bn} = V_{cn} = \frac{Q}{2\pi\varepsilon_o} \ln\frac{D}{r_1} = \frac{Q}{C_n}$$

$$C_n = C_{an} = C_{bn} = C_{cn} = \frac{2\pi\varepsilon_o}{\ln\frac{D}{r_1}} \qquad \text{(F/m to neutral)} \qquad (7.23)$$

where, r_1, is the nominal radius of the ACSR cables in Fig. 7.14 and can be obtained from the nominal cable diameters listed in Table 7.1.

Since the cables, in Fig. 7.14, are not generally, symmetrically spaced, the potential difference to ground of each cable is not the same, and therefore, the capacitances to ground are not balanced. If, however, the lines are transposed in space, as in Fig. 7.8, it can be shown that the <u>average</u> potential difference to ground of <u>each</u> cable, over the three space arrangements is the <u>same</u>, and therefore, the capacitance to ground of each cable is,

$$C_{an} = C_{bn} = C_{cn} = \frac{2\pi\varepsilon_o}{\ln\frac{GMD}{r_1}} \qquad \text{(F/m to neutral)} \qquad (7.24)$$

where, $GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$; $\varepsilon_o = 8.85\times10^{-12}$ (F/m)

When we model the transmission lines, per-phase, we are more concerned with the steadystate, distributed, capacitive reactance of the line to neutral at 60 Hz,

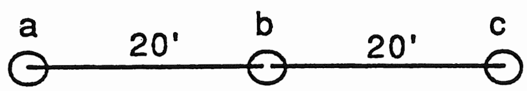
$$x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 4.77 \times 10^7 \ln \frac{GMD}{r_1} \qquad (\Omega\text{-m to neutral})$$

and in English units,

$$x_C = 0.02965 \ln \frac{GMD}{r_1} \qquad (M\Omega\text{-miles to neutral}) \qquad (7.25)$$

Example 7.3

At three-phase, 60 Hz, transmission line is constructed of ACSR cable F, with a spacing,



a) Calculate the distributed capacitance to neutral and the reactance to neutral.

b) List, in summary, the three, distributed parameters of this line.

a)

$$GMD = \sqrt[3]{(20)(20)(40)} = 25.2 \text{ ft.}$$

From Table 7.1, nominal diameter = 1.108 in.

$$r_1 = \frac{1.108}{(2)(12)} = 0.0462 \text{ ft.}$$

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{GMD}{r_1}} = \frac{(2\pi)(8.85 \times 10^{-12})}{\ln \frac{25.2}{0.0462}} = 8.82 \quad (\mu\mu F/m \text{ to neutral})$$

$$x_C = 0.02965 \ln \frac{GMD}{r_1} = 0.02965 \ln \frac{25.2}{0.0462}$$

$$= 0.1869 \quad (M\Omega\text{-miles to neutral})$$

b) Cable F, with flat spacing,

$$r = 0.1284 \quad \Omega/\text{mile}/\phi$$

$$x_L = 0.79 \quad \Omega/\text{mile}/\phi$$

$$x_C = 0.1869 \quad M\Omega\text{-miles to neutral}$$

The remaining parameter of the line, conductance to neutral, caused by line insulator leakage, is usually ignored by power engineers, since it is small, and in subsequent analysis it will be assumed zero.

In summary, the distributed parameters of a given, three-phase, single-cable transmission line can be calculated, using the ACSR tables, as,

$$r = \Omega/\text{mile}/\phi \ @ \ 20°\text{C or } 50°\text{C}$$

$$x_L = 0.1213 \ln \frac{GMD}{GMR} \quad \Omega/\text{mile}/\phi$$

$$x_C = 0.02965 \ln \frac{GMD}{r_1} \quad M\Omega\text{-mile to neutral}$$

7-6 TRANSMISSION LINE MODELS

Transmission-line networks, called power grids, exist all over the world and each grid usually consists of long transmission lines, that interconnect load centers in sparsely populated sections of the country, and short or medium - length transmission lines, that interconnect densely populated load centers. Long transmission lines are relatively few and are found in western United States, Africa, Sweden and other countries where mountainous or desert terrain exists. By far, the majority of transmission lines are short or medium in length. The relative length of a transmission line is determined by comparing it to a quarter-wavelength at 60 Hz. Electric power is propagated down transmission lines at a frequency of 60 Hz and because of mismatched loads, power is reflected, resulting in standing waves of voltage and current along long-transmission lines. For low-loss lines the velocity of propagation is essentially the speed of light and it is related to wavelength by the frequency of propagation,

$$u_p = f\,\lambda \approx 186{,}000 \ \text{(miles/sec)}$$

The wavelength at 60 Hz, is, then, approximately 3,000 miles, or $\lambda/4 = 750$ miles

A transmission line is,then, considered long when its length is in excess of 150 miles. Because of the variation in rms voltage and current along the line due to standing waves, distributed parameters r, x_L and x_c must be used to calculate voltage, current and power. When the line is less than 150 miles in length, the rms voltage and current along the line is essentially constant with length, and lumped parameters can be used. A line is considered short when its length is less than 50 miles, in which case, it accumulates negligible capacitance, which is neglected. This forms the basis for transmission line models and when lumped parameters can be used, the line is modeled as shown in Fig. 7.15.

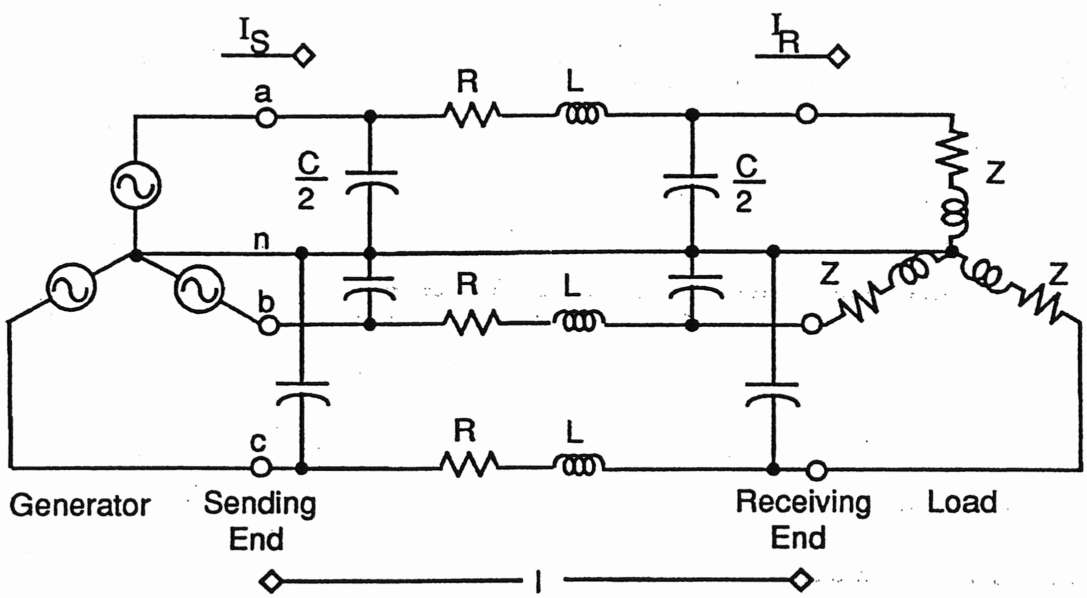


Figure 7.15 Transmission Line Model

Line resistance, R, is the lumped value of the distributed resistance along the line and represents the electric field within the cable or line losses. Line inductance,L, is the lumped value of the distributed inductance along the line and represents the magnetic field surrounding the line or the VARS required to set up this field. Line capacitance, C, is the lumped value of the distributed capacitance along the line and represents the electric field terminating on the line or the VARS required to set up this field. The capacitance is partitioned equally to more closely approximate a long line, which is our perogative, since this is a mathematical model. It must be emphasized that even though the model is an electric circuit, the power that is delivered to the load is propagated through the magnetic and electric fields that surround the transmission line. The three-phase loads of a transmission line are usually balanced, in which case, the three ports of the line, defined line to neutral, have three identical per-phase equivalent circuits, with the circuit of phase a usually selected. The source side of the line is the sending end, the load side is the receiving end, and the corresponding line voltages and currents are so indicated in Fig. 7.15. The per-phase equivalent circuit from Fig. 7.15 is then given,

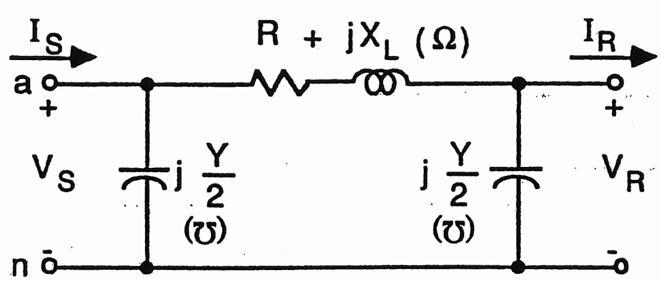


Figure 7.16 Per-Phase Equivalent Circuit With Lumped Parameters

The π-model in Fig. 7.16 is valid for lines of length, 150 miles or less, but gives excellent results for lines considerably longer than 150 miles. The line parameters are lumped since 150 miles is considerably shorter than $\lambda/4$, in which case, rms voltage and current along the line are essentially constant. The lumped parameters are determined,

$$R = r\ell \qquad (\Omega/\phi) \tag{7.26}$$

$$jX_L = j\,x_L\,\ell \qquad (\Omega/\phi) \tag{7.27}$$

$$-j\,X_c = -j\,x_c/\ell \qquad (\Omega \text{ to neutral})$$

$$j\,Y = \frac{1}{-j\,X_c} \qquad (\mho \text{ to neutral}) \tag{7.28}$$

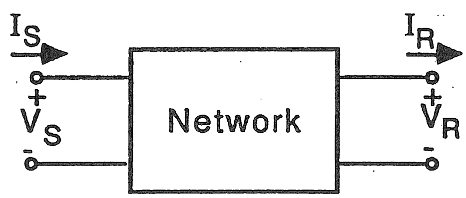
where, Y is the total capacitive susceptance to neutral.

For a given loading, V_R is usually chosen as the reference phasor and I_R is determined by the power factor and impedance of the load. With V_R and I_R known, the sending end voltage and current needed to achieve the load and line requirements can be determined using Kirchhoff's voltage and current laws

$$V_S = (V_R\frac{Y}{2} + I_R)\,Z + V_R = (1 + \frac{ZY}{2})\,V_R + Z\,I_R \qquad (V) \tag{7.29}$$

$$I_R = V_S\frac{Y}{2} + V_R\frac{Y}{2} + I_R = Y(1+\frac{ZY}{4})\,V_R + (1 + \frac{ZY}{2})\,I_R \qquad (A) \tag{7.30}$$

The transmission line can be considered as a two-port network,



There are several ways of expressing the relationship between the input and output voltages and currents of a two-port network, but the relationship most convenient for a transmission line is its hybrid-ABCD parameters,

$$V_S = A\,V_R + B\,I_R \qquad (V) \tag{7.31}$$

$$I_S = C\,V_R + D\,I_R \qquad (A) \tag{7.32}$$

By comparing Eqns. (7.29),(7.30) with Eqns. (7.31),(7.32), for a $\underline{\pi\text{-model}}$,

$$A = \left(1+\frac{ZY}{2}\right) \qquad B = Z \;(\Omega) \qquad (7.33)$$

$$C = \left(1+\frac{ZY}{4}\right)Y \;(\mho) \qquad D = \left(1+\frac{ZY}{2}\right) \qquad (7.34)$$

Once these hybrid parameters are known, any two input or output variables can be determined in terms of the remaining two variables. At this point it is important to emphasize that the voltages and currents in Eqns. (7.29) and (7.30) are sinusoidally varying quantities and their phasors are complex variables each with a magnitude and angle. The line parameters are also complex numbers each with a magnitude and angle. The terms in these equations cannot be added algebraicly but must be added vectorially.

Voltage regulation is an important figure of merit for the transmission line, since it is measure of its internal impedance,

$$V.R. = \frac{V_R\,(NL) - V_R\,(FL)}{V_R\,(FL)}$$

From Eqn. (7.29), at no-load, $I_R = 0$, then,

$$V_R\,(NL) = \left| \frac{V_S}{\frac{ZY}{2} + 1} \right| \qquad (V)$$

The π-model, then, is used for medium-length lines, ℓ < 150 miles, which approximates the input and output conditions for actual lines very well.

When the transmission line is shorter than 50 miles, the accumulated capacitance to neutral is negligibly small and its shunt susceptance can be omitted.

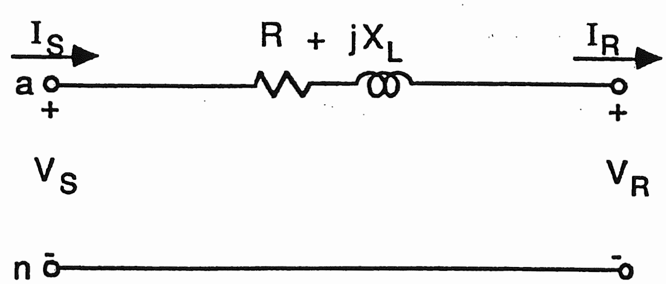


Figure 7.17 Short Transmission Line Model

For a given loading, the input-output voltage and current relationship is,

$$V_S = V_R + Z\,I_R \qquad (V) \qquad (7.35)$$

$$I_S = I_R \qquad (A) \qquad (7.36)$$

The hybrid parameters for a short line are, therefore,

$$A = 1 \qquad B = Z \ (\Omega) \tag{7.37}$$

$$C = 0 \ (\mho) \qquad D = 1 \tag{7.38}$$

The voltage regulation is,

$$\text{V.R.} = \frac{V_R\,(NL) - V_R\,(FL)}{V_R\,(FL)}$$

where,

$$V_R\,(NL) = |V_S|$$

Since the multimillion-dollar loads on a transmission line are usually designed with a limited tolerance in rated voltage, a regulation of less than ± 5% would be desirable.

Some insight into the voltage regulation of a short line can be obtained by considering its phasor diagrams for lagging, unity, and leading pf loads.

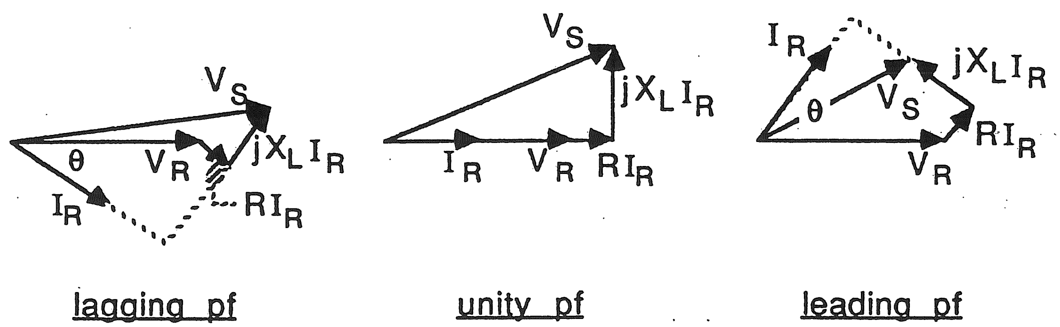


Figure 7.18 Short Line Phasor Diagrams

From Fig. 7.18, the magnitude of V_S compared to the magnitude of V_R shows that an inductive load has the greatest regulation followed by a resistive load with smaller regulation, and the capacitive load could have negative regulation if $|V_S| < |V_R|$. For a given load voltage and current, then, the type of load, and the internal impedance of the line determine the voltage regulation.

7-7 LONG TRANSMISSION LINE EQUATIONS AND MODEL

When the length of a transmission line is greater than 150 miles, the variation of rms voltage and current along the line is no longer negligible because of standing waves. Since the line is beginning to approach a quarter-wavelength, the use of lumped parameters is no longer justified. The use of distributed parameters now becomes important as indicated in the infinitesimal line-increment shown in Fig. 7.19.

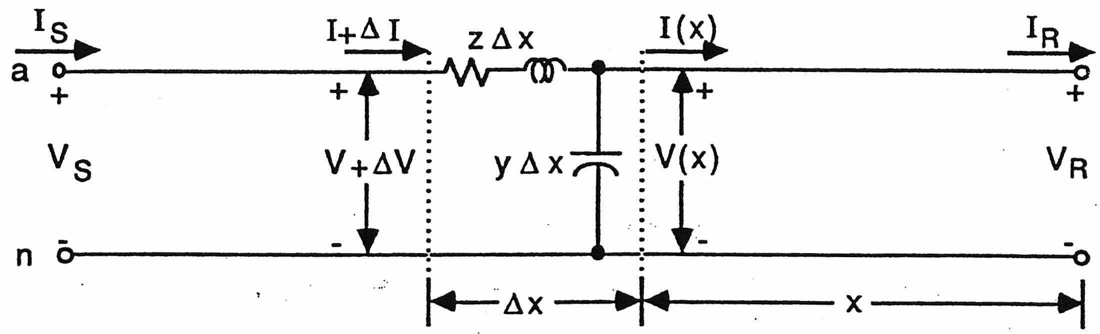


Figure 7.19 Infinitesimal Line Increment

As indicated in Fig. 7.19, the line has distributed parameters,

$$z = r + j\,x_L \quad \Omega/\text{mile}/\phi$$

$$y = \quad \mho \text{ per mile to neutral}$$

and distance, x, is measured from the receiving end of the line to the increment. The rms voltage and current at the receiving-end of the segment vary in magnitude and angle as x increases from the receiving end. Kirchhoff's voltage and current laws can now be written for the increment.

$$V(x) + \Delta V = z\,\Delta x\; I(x) + V(x)$$
$$I(x) + \Delta I = y\,\Delta x\, V(x) + I(x)$$

then,

$$\Delta V = z\,\Delta x\; I(x) \tag{7.39}$$
$$\Delta I = y\,\Delta x\, V(x) \tag{7.40}$$

In the limit as $\Delta x \rightarrow 0$, Eqns. (7.39) and (7.40) become,

$$\lim_{\Delta x \to 0} \left(\frac{\Delta V}{\Delta x}\right) = \frac{dV}{dx} = z\,I(x) \tag{7.41}$$
$$\lim_{\Delta x \to 0} \left(\frac{\Delta I}{\Delta x}\right) = \frac{dI}{dx} = y\,V(x) \tag{7.42}$$

Differentiating Eqns. (7.41) and (7.42) with respect to x,

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx} \tag{7.43}$$
$$\frac{d^2I}{dx^2} = y\frac{dV}{dx} \tag{7.44}$$

Substituting Eqns. (7.41), (7.42) in Eqns. (7.43), (7.44),

$$\frac{d^2V(x)}{dx^2} = zy\,V(x) = \gamma^2 V(x) \tag{7.45}$$

$$\frac{d^2I(x)}{dx^2} = zy\,I(x) = \gamma^2 I(x) \tag{7.46}$$

where, $\gamma = \sqrt{zy} = \alpha + j\beta$ (/mile) = propagation constant

The propagation constant is a complex number where,

$$\alpha = \frac{\text{nepers}}{\text{mile}} = \text{attenuation constant}$$

$$\beta = \frac{\text{radians}}{\text{mile}} = \text{phase constant}$$

The propagation constant will become important in the solutions to Eqns. (7.45) and (7.46). The classical solution to Eqns. (7.45) and (7.46) is of exponential form,

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \tag{7.47}$$

and from Eqn. (7.41),

$$I(x) = \frac{1}{z}\frac{dV(x)}{dx} = \frac{1}{z}(A_1 \gamma e^{\gamma x} - A_2 \gamma\, e^{-\gamma x})$$

$$= \frac{1}{\sqrt{z/y}}(A_1\, e^{\gamma x} - A_2\, e^{-\gamma x})$$

$$I(x) = \frac{1}{Z_c}(A_1\, e^{\gamma x} - A_2\, e^{-\gamma x}) \tag{7.48}$$

where, $Z_c = \sqrt{z/y}$ (Ω) = characteristic impedance of the line.

The constants A1, A2 are determined from the boundary conditions of the line,

$$V(0) = V_R \qquad\qquad I(0) = I_R$$

therefore, from Eqns. (7.47) and (7.48)

$$V_R = (A_1 + A_2)$$

$$I_R = \frac{1}{Z_c}(A_1 - A_2)$$

Solving for A_1 and A_2,

$$A_1 = \frac{V_R + Z_c I_R}{2} \qquad A_2 = \frac{V_R - Z_c I_R}{2}$$

Substituting A1 and A2 into Eqns. (7.47) and (7.48),

$$V(x) = \frac{V_R + Z_c I_R}{2}\, e^{\alpha x}\, \underline{/\beta x} + \frac{V_R - Z_c I_R}{2}\, e^{-\alpha x}\, \underline{/-\beta x} \tag{7.49}$$

$$I(x) = \frac{V_R/Z_c + I_R}{2}\, e^{\alpha x}\, \underline{/\beta x} - \frac{V_R/Z_c - I_R}{2}\, e^{-\alpha x}\, \underline{/-\beta x} \tag{7.50}$$

Equations (7.49) and (7.50), then, are the solutions to the differential equations (7.43) and (7.44). These equations give the rms-voltage and current, in magnitude and angle, of the <u>standing waves</u> on the line.

The first terms of both equations are the <u>incident</u> waves as the voltage and current <u>increase</u> in magnitude and angle with respect to the receiving-end voltage.

The second terms of both equations are the waves <u>reflected</u> from the load as they <u>decrease</u> in magnitude and angle with respect to the receiving-end voltage.

Observe the significance of the attenuation constant, α , and the phase constant, β, as the waves are propagated down the line. If the line is terminated in a load equal to its characteristic impedance, then $V_R = Z_c I_R$, and there is no reflected wave in either Eqn. (7.49) or (7.50). Under these conditions the line is said to be <u>flat</u>, which is desirable in many communication circuits but is never attained in power circuits because power loads are not so accommodating.

In hyperbolic form, Eqns. (7.49) and (7.50) can be written,

$$V(x) = V_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + Z_c I_R \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$I(x) = \frac{V_R}{Z_c} \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) + I_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)$$

or,

$$V(x) = V_R \cosh \gamma x + Z_c I_R \sinh \gamma x \tag{7.51}$$

$$I(x) = \frac{V_R}{Z_c} \sinh \gamma x + I_R \cosh \gamma x \tag{7.52}$$

The terminal conditions of the line can be established by letting, $x = \ell$, in Eqns. (7.51) and (7.52),

$$V_S = \cosh \gamma\ell \; V_R + Z_c \sinh \gamma\ell \; I_R \qquad (7.53)$$

$$I_R = \frac{1}{Z_c} \sinh \gamma\ell \; V_R + \cosh \gamma\ell \; I_R \qquad (7.54)$$

where, $\gamma\ell$, is a complex argument

and, $e^{\gamma\ell} = e^{\alpha\ell} \underline{/\beta\ell}$

As defined earlier, the hybrid parameters of the long line are,

$$A = \cosh \gamma\ell \qquad B = Z_c \sinh \gamma\ell \quad (\Omega) \qquad (7.55)$$

$$C = \frac{1}{Z_c} \sinh \gamma\ell \quad (\mho) \qquad D = \cosh \gamma\ell \qquad (7.56)$$

and, from Eqns. (7.33) and (7.34), the parameters of the π- model are,

$$A = (1 + \frac{ZY}{2}) \qquad B = Z \quad (\Omega) \qquad (7.57)$$

$$C = (1 + \frac{ZY}{4}) Y \quad (\mho) \qquad D = (1 + \frac{ZY}{2}) \qquad (7.58)$$

If Eqns. (7.55), (7.56) and (7.57), (7.58) are compared, the equivalent π-model for the long line is,

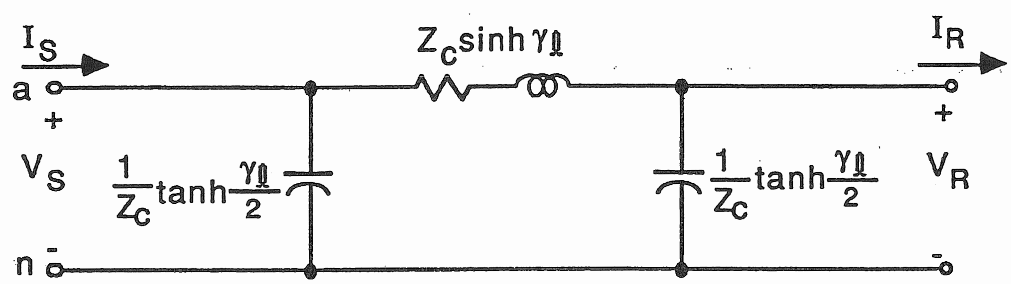


Figure 7.20 Long-Line Equivalent, Π- Model

The equivalent, π-model in Fig. 7.20 can be verified by using trignometric identity,

$$\tanh \frac{\gamma\ell}{2} = \frac{\cosh \gamma\ell - 1}{\sinh \gamma\ell}$$

Example 7.4

The transmission line of Example 7.3 has distributed parameters,

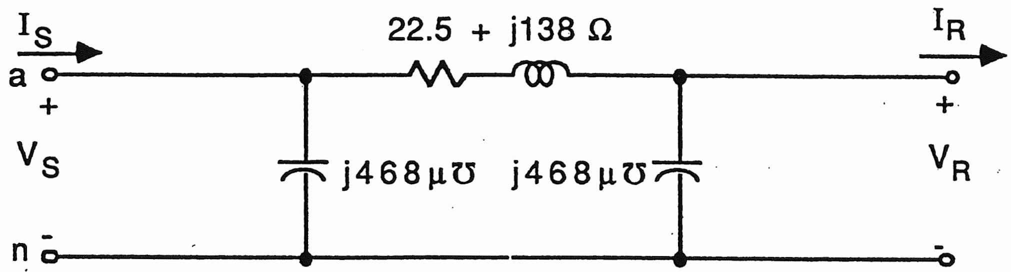
$$\begin{aligned} r &= 0.1284 && \Omega/\text{mile}/\phi \text{ @ } 50\ ^\circ\text{C} \\ x_L &= 0.79 && \Omega/\text{mile}/\phi \\ x_C &= 0.1869 && \text{M}\Omega\text{-miles to neutral} \end{aligned}$$

If the line has a length, 175 miles, calculate the π-model and compare it with the equivalent π-model of a long line.

a) π-model

$$Z = (0.1284 + j\,0.79)(175) = 22.5 + j\,138\ \ \Omega/\phi$$

$$Y = \frac{(175)(10^{-6})}{0.1869} = 936\ \mu\mho \text{ to neutral}$$



Π - Model

b) Equivalent π-model

$$z = 0.1284 + j\,0.79 = 0.8\ \underline{/80.8^\circ}\ \ \Omega/\text{mile}/\phi$$

$$y = \frac{1}{x_C} = \frac{10^{-6}}{0.1869\ \underline{/-90^\circ}} = 5.35 \times 10^{-6}\ \underline{/90^\circ}\ \ \mho \text{ per mile to neutral}$$

$$Z_c = \sqrt{z/y} = \sqrt{\frac{0.8\ \underline{/80.8^\circ}}{5.36\text{x}10^{-6}\ \underline{/90^\circ}}} = 386\ \underline{/-4.6^\circ}\ \ \Omega$$

$$\gamma\ell = \sqrt{zy}\,\ell = 175\sqrt{(0.8\ \underline{/80.8^\circ})(5.35\text{x}10^{-6}\ \underline{/90^\circ})}$$

$$= 0.362\ \underline{/85.4^\circ} = 0.029 + j0.361$$

$$\alpha\ell = 0.029 \text{ nepers} \qquad \beta\ell = 0.361 \text{ rad.} = 20.7^\circ$$

$$\cosh \gamma\ell = \frac{e^{0.029}\,\angle 20.7^\circ + e^{-0.029}\,\angle -20.7^\circ}{2} = 0.936\,\angle 0.628^\circ$$

$$\sinh \gamma\ell = \frac{e^{0.029}\,\angle 20.7^\circ - e^{-0.029}\,\angle -20.7^\circ}{2} = 0.355\,\angle 85.6^\circ$$

$$Z' = Z_c \sinh \gamma\ell = (386\,\angle -4.6^\circ)(0.355\,\angle 85.6^\circ) = 137\,\angle 81^\circ = 21.4 + j135\ \Omega$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\frac{\cosh \gamma\ell - 1}{\sinh \gamma\ell} = \frac{0.936\,\angle 0.623^\circ - 1}{(386\,\angle -4.6^\circ)(0.355\,\angle 85.6^\circ)} = 473 \times 10^{-6}\,\angle 90^\circ\ ℧$$

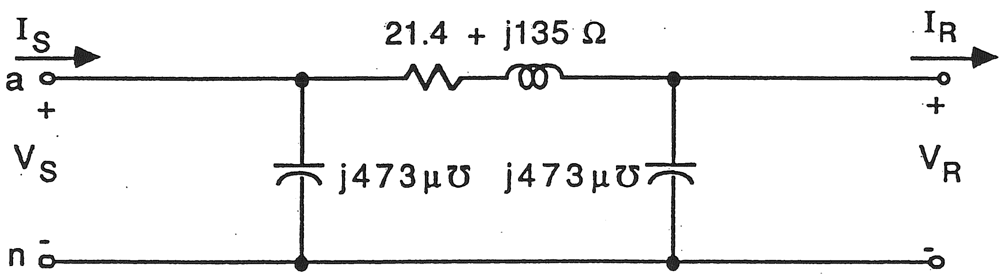


Figure 7.21 Long-Line Equivalent, Π - Model

The lumped parameters of the π-model differ from those of the long-line model by only a few percent, indicating that lumped parameters can be used, with little error, for lines reasonably longer than 150 miles.

If the Cable F, transmission line in Example 7.3 is rated 230 kV, 47 MVA and is connected to a 230 kV, 40 MW, 0.85 pf lagging, load, the interested reader should verify that the sending-end voltage must be,

a) Using the short-line model,

$$V_S = Z I_R + V_R \qquad = 144.2\,\angle 5^\circ\ \text{kV}(\phi) \quad ; \quad 249.7\ \text{kV(line)}$$

b) Using the lumped-parameter, π-model,

$$V_S = \left(1 + \frac{ZY}{2}\right) V_R + Z I_R \qquad = 140\,\angle 5.4^\circ\ \text{kV}(\phi) \quad ; \quad 242.4\ \text{kV(line)}$$

c) Using the long-line equations,

$$V_S = V_R \cosh \gamma\ell + Z_c I_R \sinh \gamma\ell \qquad = 135.5\,\angle 5.8^\circ\ \text{kV}(\phi) \quad ; \quad 234.8\ \text{kV(line)}$$

The long-line solution is, of course, accurate with the lumped parameter solution having approximately 3% error.

7-8 TRANSMISSION LINE AND CHARGING MVAR RATING

The long transmission line in Example 7.4, Fig. 7.21 is completely specified when its design voltage, MVA and parameters are given,

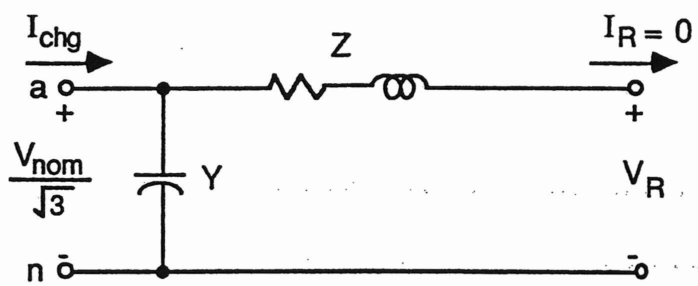
230 kV, 47 MVA, 175 miles

$$Z = 136.7\,\underline{/81^\circ}\ \Omega$$

$$Y = 946\,\underline{/90^\circ}\ \mu\mho$$

Power engineers, however, seldom specify, explicitly, the line susceptance; rather, they specify, Y, in terms of Charging MVAR.

Charging MVAR is a constant for the line and is defined as follows,



where, $V_{nom} = V_{rated}$

Figure 7.22 Charging MVAR Model

For the purposes of this definition, the capacitive susceptance, Y, is placed across the input of an unloaded line driven by nominal or rated voltage. The line current, is, then, the charging current and is given as,

$$I_{chg} = \frac{V_{nom.}}{\sqrt{3}}\, Y \qquad \text{(A)}$$

where, $V_{nominal}$ is the rated line voltage

The total charging vars are, then,

$$\text{Chg } \underline{\text{VAR}} = 3\left[\frac{V_{nom}}{\sqrt{3}}\right](I_{chg}) = 3\left[\frac{V_{nom}}{\sqrt{3}}\right]^2 Y = (kV_{nom} \times 1000)^2\, Y \qquad (7.59)$$

or,

$$\text{Chg } \underline{\text{MVAR}} = (kV_{nom})^2\, Y \qquad (7.60)$$

and, Y, can be found,

$$Y = \frac{\text{Chg MVAR}}{(kV_{nom})^2} \qquad (\mho)$$

The charging MVAR rating of the line, in Eqn. (7.60) is, then, a constant for <u>any</u> line, since kV_{nom} is the rated <u>line</u> voltage of the transmission line. The line could have also been rated,

$$\text{Chg MVAR} = (230)^2\,(946 \times 10^{-6}) = 50$$

from which, Y, can be found.

7-9 TRANSMISSION LINE POWER FLOW

Power flow through a transmission line is very important in power system analysis, since not all of the power delivered to a line reaches the load. The line losses and reactive power stored in the magnetic and electric fields surrounding the line, then, must be known in a watt-var power balance of the line, as indicated in Fig. 7.23.

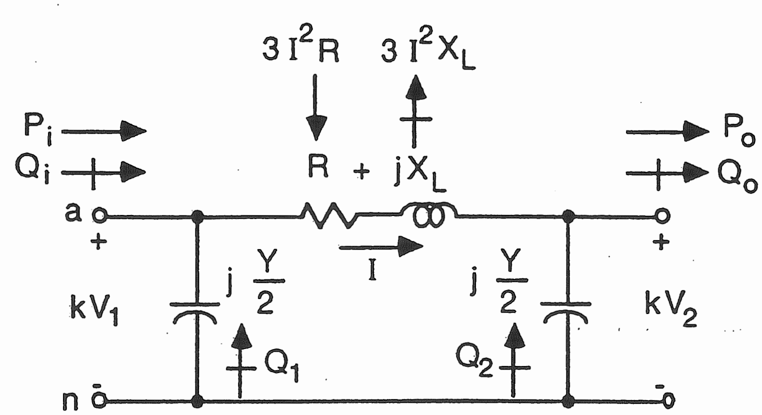


Figure 7.23 Power Flow Diagram

The power flow, indicated in the per-phase diagram of Fig. 7.23, is always total in the subsequent analysis.

The watt balance of Fig. 7.23 is, then,

$$P_i = 3\,I^2R + P_o \qquad (W) \qquad (7.61)$$

The var balance, remembering that inductance absorbs vars, and capacitance delivers vars, is,

$$Q_i = 3\,I^2 X_L + Q_o - Q_1 - Q_2 \qquad (VAR) \qquad (7.62)$$

where, $$\frac{Q_1}{\text{Chg MVAR}} = \frac{(kV_1)^2 \frac{Y}{2}}{(kV_{nom})^2\,Y} \quad ; \quad \frac{Q_2}{\text{Chg MVAR}} = \frac{(kV_2)^2 \frac{Y}{2}}{(kV_{nom})^2\,Y}$$

or, $$Q_1 = \left(\frac{kV_1}{kV_{nom}}\right)^2 \frac{\text{Chg MVAR}}{2} \quad ; \quad Q_2 = \left(\frac{kV_2}{kV_{nom}}\right)^2 \frac{\text{Chg MVAR}}{2} \qquad (7.63)$$

From Eqn. (7.63), it is seen why power engineers find the charging MVAR rating of the line very convenient.

Example 7.5

Using the long-line, equivalent π-model of the transmission line in Fig. 7.21, calculate, draw, and label the power flow diagram for this line, when terminated with a load, 40 MW, 0.85 pf lagging at 230 kV.

From Figs. 7.21 and 7.23,

$$\text{Chg MVAR} = (230)^2(946 \times 10^{-6}) = 50$$

$$V_R = \frac{230{,}000\,\underline{/0^\circ}}{\sqrt{3}} = 132{,}800\,\underline{/0^\circ} \quad (V)$$

$$I_R = \frac{40{,}000}{\sqrt{3}(230)(0.85)}\,\underline{/-31.8^\circ} = 118\,\underline{/-31.8^\circ} \quad (A)$$

$$I = \frac{Y}{2}V_R + I_R = (473 \times 10^{-6}\,\underline{/90^\circ})(132{,}800\,\underline{/0^\circ}) + 118\,\underline{/-31.8^\circ} = 100\,\underline{/0.343^\circ} \ (A)$$

$$P_i = 3\,I^2R + P_o = (3)(100)^2(21.4)(10^{-6}) + 40$$

$$= 0.642 + 40 = 40.642 \quad (MW)$$

$$V_S = V_R \cosh \gamma l + Z_c I_R \sinh \gamma l \quad = 135.5 \underline{/5.8^\circ} \text{ kV}(\phi) \quad ; \quad 234.8 \text{ kV(line)}$$

$$Q_1 = \left(\frac{kV_S}{kV_{nom}}\right)^2 \frac{\text{Chg MVAR}}{2} = \left[\frac{234.8}{230}\right]^2 \left[\frac{50}{2}\right] = 26.1 \quad \text{(MVAR)}$$

$$Q_2 = \left(\frac{kV_R}{kV_{nom}}\right)^2 \frac{\text{Chg MVAR}}{2} = \left[\frac{230}{230}\right]^2 \left[\frac{50}{2}\right] = 25 \quad \text{(MVAR)}$$

$$Q_i = 3 I^2 X_L + Q_o - Q_1 - Q_2 = (3)(100)^2(135)(10^{-6}) + 24.8 - 26.1 - 25$$

$$= 4.05 + 24.8 - 26.1 - 25 = -22.3 \quad \text{(MVAR)}$$

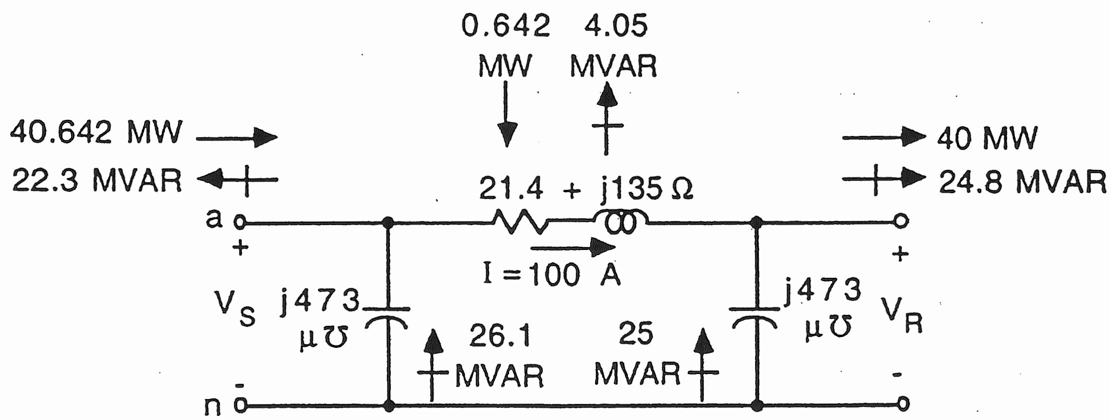


Figure 7.24 Long-Line Power Flow with 230 kV, 40 MW, 0.85 lagging Termination

As seen from the power-flow diagram, the line capacitance delivers more MVAR than that required by the line magnetic field and the load, resulting in net VAR flow from the input of the line.

At midnight, when the load on a power system is light, system generation must be adjusted to absorb vars delivered by the capacitance of connected, long lines. The power-flow diagram will, then, become important in later power system analysis.

7-10 SUMMARY

Since the distributed line parameters are important in the analysis of the line, they can be found from the cable-table for a given spacing,

$r =$ Ω/mile/φ @ °C
$x_L = 0.1213$ Ω/mile/φ
$x_c = 0.02965$ MΩ-miles to neutral

and, $z = r + j\,x_L \quad \Omega/\text{mile}/\phi$

$y = 1/x_C \quad \mho$ per mile to neutral

The long-line equations are valid for any length line,

$$V_S = \cosh \gamma l \; V_R + Z_c \sinh \gamma l \; I_R \qquad (V)$$

$$I_R = \frac{1}{Z_c} \sinh \gamma l \; V_R + \cosh \gamma l \; I_R \qquad (A)$$

where, $\gamma = \sqrt{zy} = \alpha + j\,\beta$ (/mile) and $Z_c = \sqrt{z/y}$ (Ω)

For lines less than 150 miles, the lumped-parameter, π-model can be used,

$$V_S = \left(1 + \frac{ZY}{2}\right) V_R + Z \; I_R \qquad (V)$$

$$I_S = Y\left(1 + \frac{ZY}{4}\right) V_R + \left(1 + \frac{ZY}{4}\right) I_R \qquad (A)$$

For lines less than 50 miles, the lumped-parameter, π-model can be used,

$$V_S = V_R + Z\,I_R \qquad (V)$$

$$I_S = I_R \qquad (A)$$

The transmission-line susceptance can be given as a charging MVAR rating,

$$\text{Chg MVAR} = (kV_{nom})^2 \; Y$$

where, $kV_{nom} = kV_{rating}$ (line)

This rating is very useful in the transmission-line power-flow diagram, where,

$$P_i = 3\,I^2 R + P_o \qquad (W)$$

$$Q_i = 3\,I^2 X_L + Q_o - Q_1 - Q_2 \qquad (VAR)$$

and, $Q_1 = \left(\frac{kV_1}{kV_{nom}}\right)^2 \frac{\text{Chg MVAR}}{2}$; $Q_2 = \left(\frac{kV_2}{kV_{nom}}\right)^2 \frac{\text{Chg MVAR}}{2}$

PROBLEMS

7.1 A transmission line is 100-miles long, with each cable made of 20 strands of hard-drawn aluminum. Each strand is 0.2" in diameter, and at 20°C has resistivity of 2.83×10^{-8} Ω-m and a temperature coefficient of resistance of 4.38×10^{-3} /°C. Find the resistance of each cable at 20°C and 50°C.

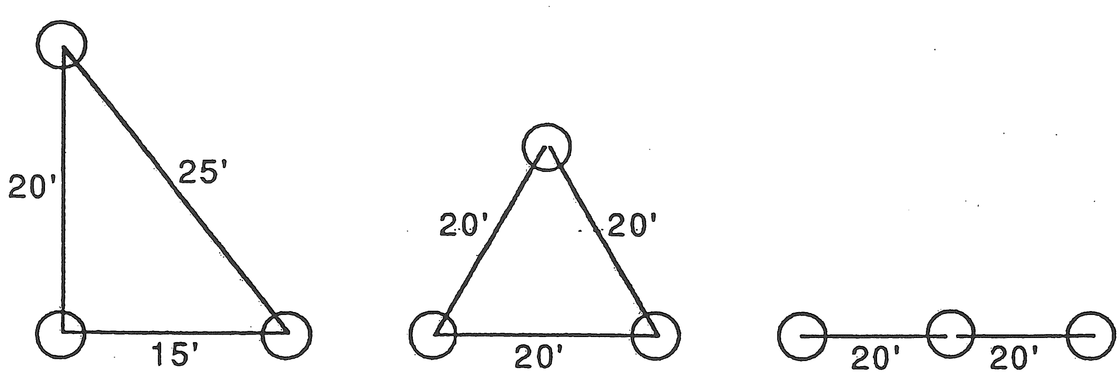
7.2 From Table 7.1, find the distributed resistance of the following ACSR-cables at 60 Hz for both 20°C and 50°C,
(a) Cable A (b) Cable B (c) Cable G

7.3 A single-phase line is made of two copper conductors spaced 15' between centers. If the diameter of each conductor is 0.1", and the line is 4 miles long, find the total inductance of the circuit and the inductive reactance at 60 Hz.

7.4 In Problem 7.3, find the capacitance, line to line and line to neutral. Find the capacitive reactance and susceptance from line to neutral at 60 Hz.

7.5 A three-phase, 60 Hz transmission line is made of ACSR-cable E, which has an equilateral spacing of 10-feet between the cables. Find the distributed resistance, inductive reactance and capacitive susceptance to neutral, per mile, at 50°C.

7.6 For each spacing below, find the distributed resistance at 50 °C, inductive reactance and capacitive susceptance to neutral for a 345 kV, 200-mile transmission line with ACSR-Cable C cables.



(a) (b) (c)

7.7 A three-phase, 120 kV, 100 MVA, 60 Hz, 100-mile, transmission line has the following parameters,

$$z = 0.04 + j\,1.2 \ \ \Omega/\text{mile}/\phi$$

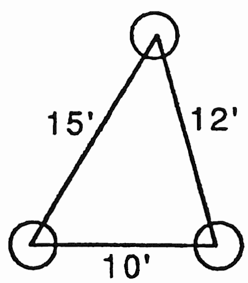
$$y = 3 \times 10^{-6} \ \ \mho \ \ /\text{mile to neutral}$$

If the line is loaded with an impedance $145 \angle 53.1^\circ$ Ω/ϕ, at rated voltage, find, in pu, using the line rating as a base,

(a) receiving-end current, pu.
(b) sending-end voltage and current, pu.
(c) receiving-end real and reactive power, pu.
(d) sending-end real and reactive power, pu.
(e) line efficiency
(f) line voltage regulation

7.8 Do Prob. 7.7 if the line is 175 miles long, and is delivering rated MVA, at rated voltage, 0.8 pf leading.

7.9 A three-phase, 220 kV, 60 Hz, 50-mile, transmission line has spacing,



If the cables are ACSR-Cable D, find the lumped resistance at 50°C, inductive reactance and capacitive susceptance to neutral . What is the capacitive charging current? What is the charging MVAR line rating?

7.10 A transmission line has lumped parameters,

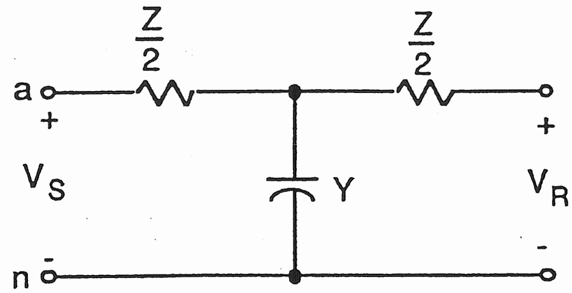
$$Z = j\,50 \ \ \Omega/\phi$$

$$Y = j\,0.02 \ \ \mho \ \ \text{to neutral}$$

With a sending-end voltage of 161 kV (line) and load impedance, $100 \angle 30^\circ$ Ω/ϕ, calculate the following for,

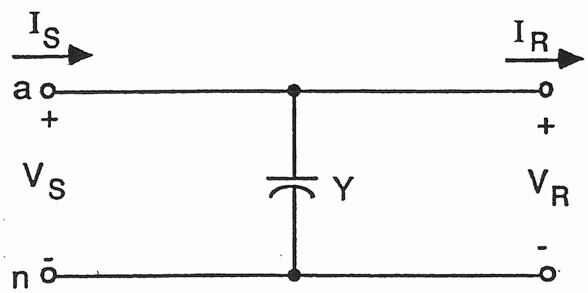
(i) the π-model

(ii) a T-model,

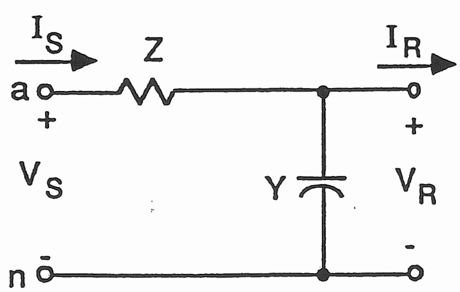


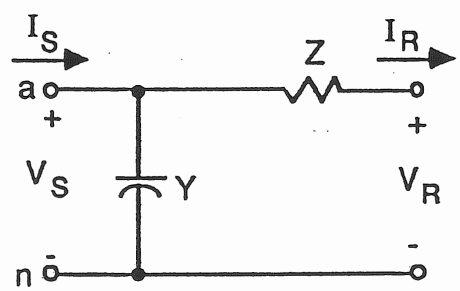
(a) the receiving-end voltage and current.
(b) real and reactive power at the sending-end.
(c) line efficiency.
(d) line regulation.
Comment on your answers.

7.11 Find the hybrid ABCD parameters for the line model,



7.12 Find the hybrid ABCD parameters for the medium-length line, H-models, shown below,





7.13 A three-phase, 138 kV, 200 mile, transmission line has the following parameters,

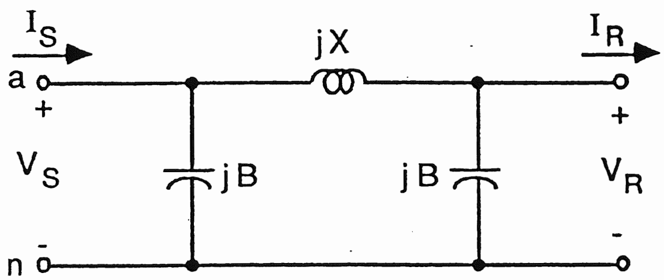
$$z = 0.2 + j\,0.8\ \Omega/\text{mile}/\phi$$

$$y = 5 \times 10^{-6}\ \mho\ /\text{mile to neutral}$$

The line is loaded with a load, 80 MW, unity pf, at rated voltage.

(a) Find the line constants, γ and Z_c.
(b) Find the load impedance per phase.
(c) Find the line efficiency.
(d) Find the line regulation.

7.14 A transmission line is represented by the π-model,



where, $V_S = \sqrt{2}\,V \cos \omega t$
$V_R = \sqrt{2}\,V \cos (\omega t - \phi)$

Show that the real and reactive power delivered to the line is,

$$P = \frac{3V^2}{X} \sin \phi$$
$$Q = \frac{3V^2}{X} (1 - \cos \phi) - 3V^2B$$

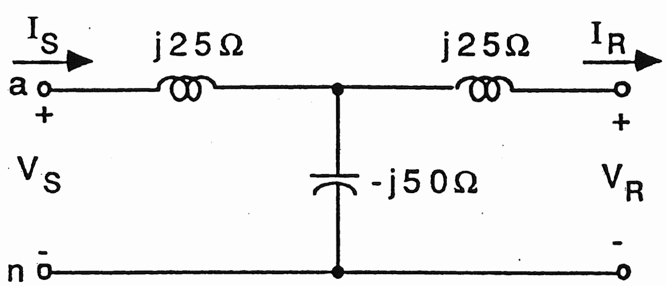
7.15 The medium-length, π-model has parameters,

$$Z = j\,100\ \Omega \quad ; \quad Y = j\,5 \times 10^{-4}\ \mho$$

$$V_S = 130 \angle 22.6^\circ\ \text{kV}(\phi) \quad ; \quad V_R = 120 \angle 0^\circ\ \text{kV}(\phi)$$

Draw and numerically label the power-flow-diagram for this line.

7.16 The medium-length, T-model, has parameters,



$$I_S = 130 \angle 100^\circ\ (A) \quad ; \quad I_R = 140 \angle 0^\circ\ (A)$$

Draw and numerically label the power-flow diagram for this line.

CHAPTER 8

POWER SYSTEM ANALYSIS

To this point we have modeled and analyzed the major components of an electric power system. Power flow has been emphasized throughout this text, since large amounts of power are generated at power plants, usually located near a source of water, and then transmitted many miles away to load centers. It is also clear that power must be generated on demand, since it cannot be stored, and a watt-var balance must be maintained continually, if system frequency is to be kept constant.

Modern power systems generate power at a low voltage level, transmit it at a high voltage level (Fig. 8.1), and then use it at a low voltage level (Fig. 8.2).

Power systems, then, consist of a high-voltage portion, which is loop-structured for reliability, (more than one transmission line feeds a bus), as shown in Fig.8.1,and a low-voltage portion, which distributes power radially (for practicality) and is shown in Fig. 8.2.

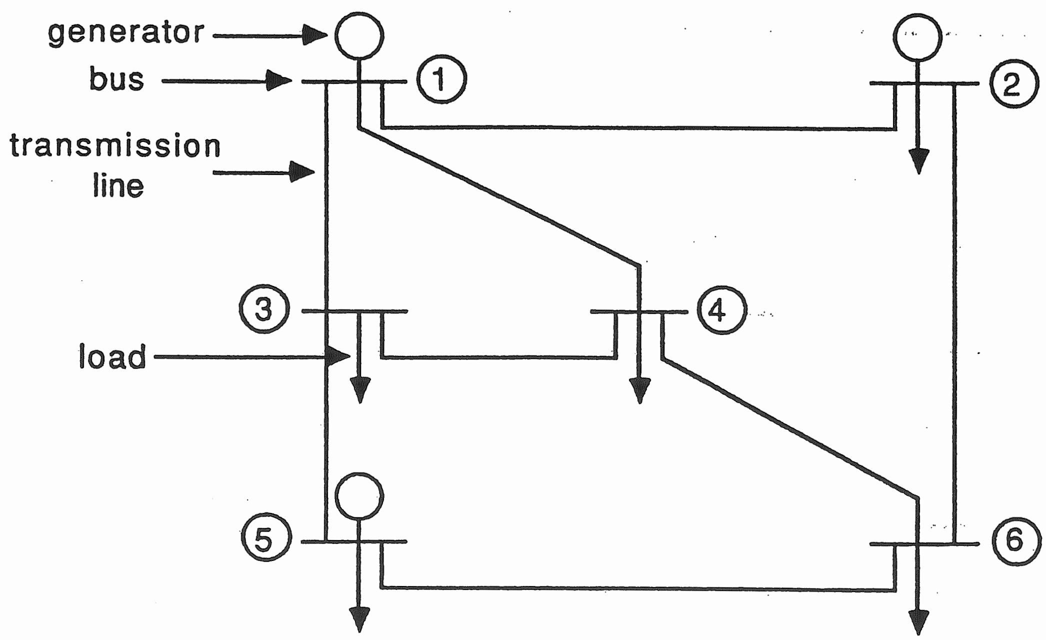


Figure 8.1 Transmission High-Voltage Loop Configuration

Each of the load arrows shown in the loop configuration of Fig. 8.1 might be the radial configuration shown in Fig. 8.2.

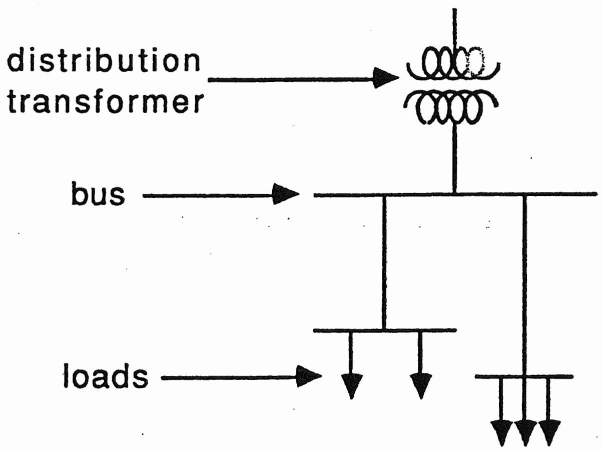
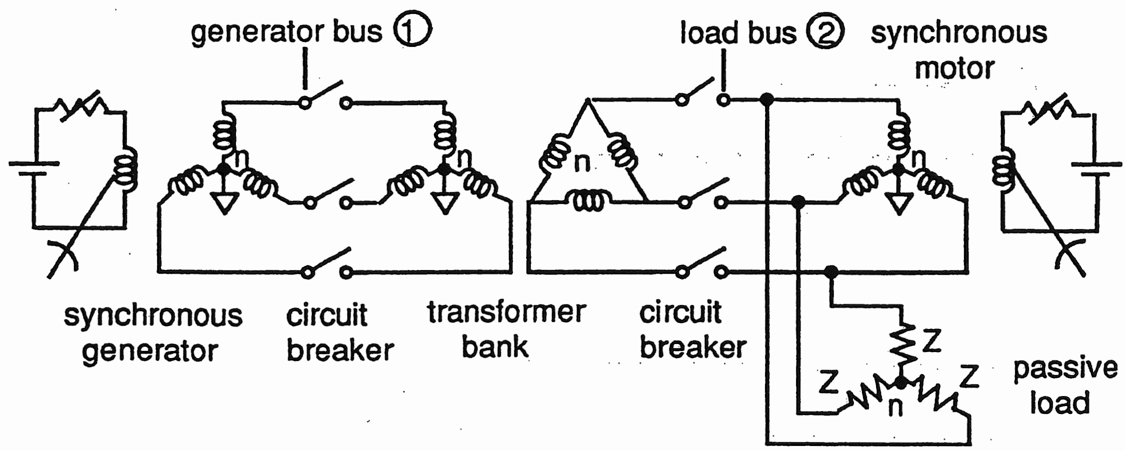


Figure 8.2 Distribution Low-Voltage Radial Configuration

Observe, in Figs. 8.1 and 8.2, for simplicity, three-phase machines, transmission lines, transformers and loads are shown as <u>single lines</u> and <u>symbols</u> (to be described later). The one-line diagram of a power system, then, is a very efficient method for showing a complex electric power system for the purposes of analysis.

8-1 ONE-LINE DIAGRAM AND IMPEDANCE DIAGRAM



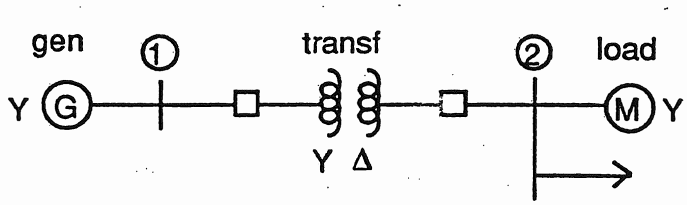


Figure 8.3 One-Line Diagram

A relatively simple power system is shown in Fig. 8.3 with its one-line diagram. As the figure indicates, one-line diagrams greatly simplify three-phase component connections. Machines are represented by circles together with their stator connection; buses are represented by straight lines together with the bus number or designation; transformer banks are represented by the symbol indicated, together with the bank connection; passive loads are represented by a straight arrow and circuit breakers by a small box. All components in a power system have circuit breakers on either component side so that in case a fault (short-circuit between lines or to ground) develops, the faulted section of the power system can be removed by opening appropriate breakers. In addition to the symbols shown, component ratings are usually given for system analysis.

For the purpose of calculating bus voltages, line currents and power flow in a loaded power system, the impedance diagram corresponding to the one-line diagram is usually drawn. The power system in Fig. 8.3 consists of three-phase components; each component has three ports, each defined from line to neutral. The impedance diagram, then, is found by tracking phase a throughout the system, however complicated the system may be. The per-phase model of each component, as seen from line to neutral, is then connected corresponding to the one-line diagram. All delta-connected components are replaced by their wye-equivalents so that neutral is defined throughout the system. This is done for Fig. 8.3 where the per-phase models of the generator, transformer bank, synchronous motor and passive load, as seen from phase a to neutral, are arranged according to the one-line diagram and are shown in Fig. 8.4. The per-phase equivalent diagrams for phases b and c are identical to phase a-circuit for normal system operation.

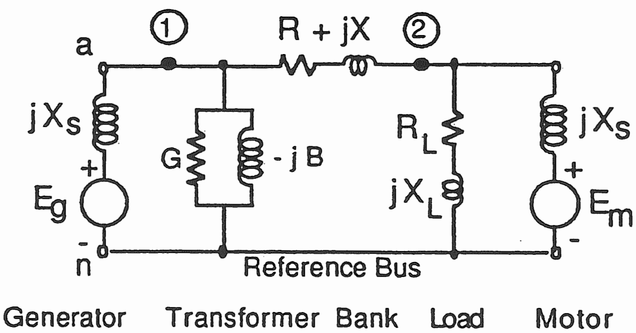


Figure 8.4 Impedance Diagram

The synchronous machines in Fig. 8.4 are represented by their synchronous reactances and excitation emfs, the transformer bank by its magnetizing branch and series winding impedance, and the load by its per-phase impedance. Ground is the reference bus, which, under normal system operation, is at the same potential as neutral, and from which all bus voltages are measured. The question arises in Fig. 8.4 as to the omission of the ideal transformer in the equivalent circuit for the transformer bank. Subsequently, the impedance diagram of Fig. 8.4 will be placed in the per-unit system on system base, and in anticipation of this, the ideal transformer is 1:1 per-unit and is therefore omitted.

8-2 SYSTEM BASE AND THE SHIFTING THEOREM

The one-line diagram and its corresponding impedance diagram are extremely important in system analysis because, for a given system loading, the bus voltages must lie between the limits 0.95-1.05 per-unit so as not to jeopardize the multimillion dollar loads connected to the buses, since loads are usually designed with +5% voltage tolerance. Circuit breakers must be sized to carry normal currents and be able to withstand fault currents. Line currents and power flow must be known so that transmission lines are not overloaded under worst-case conditions. To this end, the power system of Example 8.1 will be analyzed.

Example 8.1

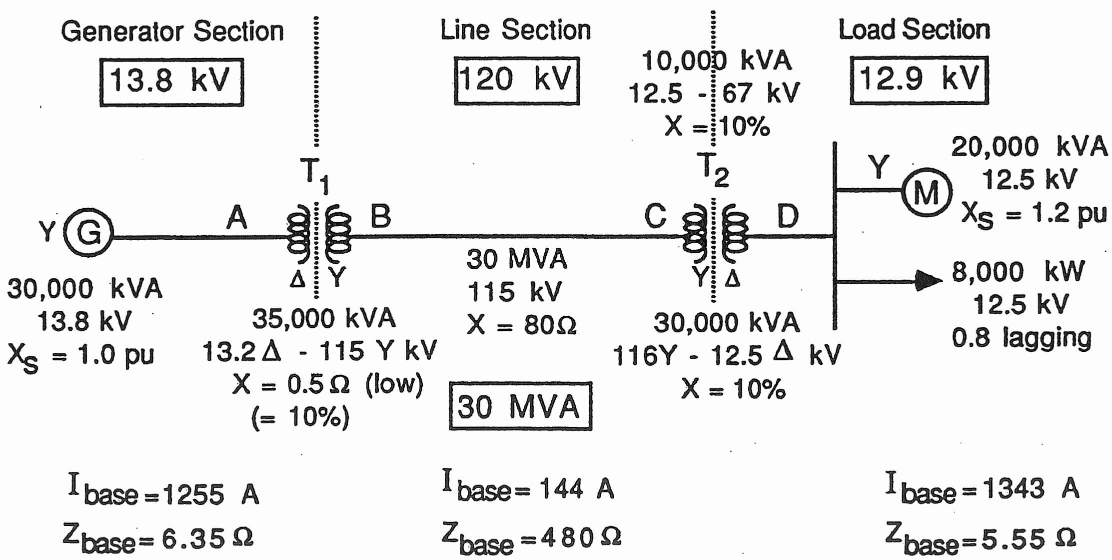


Figure 8.5 Elementary Power System

The power system in Fig. 8.5 has a three-phase generator driving a transmission line which, in turn, drives a motor and passive load.

The generator rating is given with its synchronous reactance on its own rating as a base.

Transformer bank, T_1, rating is given with its magnetizing branch assumed open and its equivalent winding resistance assumed negligible. Its equivalent leakage reactance is 0.5 ohm referred to the low side and on the bank rating as a base, it is 10%.

The transmission line rating is given with its line resistance assumed negligible and its line reactance is lumped at 80 ohms. The transmission line is short so that its charging-MVAR rating (admittance) is assumed negligible.

Transformer bank, T_2, rating is given above the symbol as its individual transformer rating since the bank connections are not indicated in this rating. The corresponding bank rating is then given below the symbol where it is recognized that the line side of the bank is always high voltage and the load side is always low voltage. The magnetizing branch is considered open and the winding resistance negligible. The leakage reactance, pu, on the individual transformer rating as a base is always equal to the leakage reactance, pu, on the bank rating as a base.

The motor rating is given with its synchronous reactance on its own rating as a base, and the passive-load rating is indicated.

Before analysis can proceed, it must be recognized that component impedances are all given on different bases, therefore they must all be shifted to an arbitrary base called system base.

System base is determined as follows,

1. Any power system, however complicated, consisting of n-transformer banks, can be divided into n+1 sections as indicated by the dashed lines in Fig. 8.5.

2. Base voltage in any one section is arbitrarily chosen, in which case, base voltage in the remaining sections is no longer arbitrary, but must be related to it by the effective turns-ratio of each transformer bank.

3. Base MVA is chosen arbitrarily, common to all sections. Electric power companies often use 100 MVA as a system base for their systems.

4. Base current and impedance are not arbitrary but must be derived in each section.

For the power system in Fig. 8.5, system base is arbitrarily chosen as the generator rating in the generator section. Base voltage in the generator section is, then, 13.8 kV and base volt-amperes is 30 MVA common to all sections. Base voltage in the line and load sections is not arbitrary,

$$\text{line base} = \left(\frac{115}{13.2}\right)(13.8) = 120 \text{ kV}$$

$$\text{load base} = \left(\frac{12.5}{116}\right)(120) = 12.9 \text{ kV}$$

System base values are constant normalizing values and must not be confused with actual system operating values. Therefore, to reduce error in system calculation, system base values are always shown blocked in each section of the power system, as indicated in Fig. 8.5. Base current and impedance can now be derived for each section,

generator section:

$$I_{base} = \frac{30{,}000}{\sqrt{3}\,(13.8)} = 1255 \text{ A}$$

$$Z_{base} = \frac{(13.8)^2}{30} = 6.35 \ \Omega$$

line section:

$$I_{base} = \frac{30{,}000}{\sqrt{3}\,(120)} = 144 \text{ A}$$

$$Z_{base} = \frac{(120)^2}{30} = 480 \ \Omega$$

load section:

$$I_{base} = \frac{30{,}000}{\sqrt{3}\,(12.9)} = 1343 \text{ A}$$

$$Z_{base} = \frac{(12.9)^2}{30} = 5.55 \ \Omega$$

The impedance diagram corresponding to the one-line diagram is shown in Fig. 8.6.

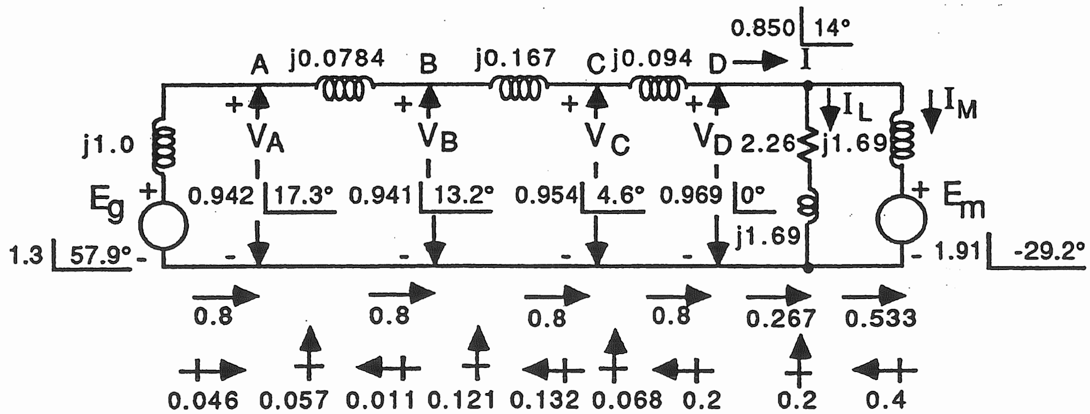


Figure 8.6 Per-Unit Bus Voltages, Line Currents and Power Flow

The impedance diagram in Fig. 8.6 includes reactances, only, for simplicity, but for load flow purposes, transformer bank winding resistance, transmission line resistance, and admittance would be included for more accuracy and completeness.

The component reactances are on different bases, and must be shifted to system base. The shifting theorem is derived as follows; a reactance of actual value, X (Ω), is placed on <u>base 1</u> and <u>base 2</u> as,

$$X_1\ (\text{pu}) = \frac{X\ (\Omega)}{\dfrac{(kV_{base1})^2}{MVA_{base1}}} \tag{8.1}$$

$$X_2\ (\text{pu}) = \frac{X\ (\Omega)}{\dfrac{(kV_{base2})^2}{MVA_{base2}}} \tag{8.2}$$

If Eqns. (8.1) and (8.2) are divided,

$$X_2\ (\text{pu}) = X_1\ (\text{pu}) \left(\frac{kV_{base1}}{kV_{base2}}\right)^2 \left(\frac{MVA_{base2}}{MVA_{base1}}\right) \tag{8.3}$$

The reactance theorem in Eqn. (8.3) simply shifts a reactance on a given base (base 1) to system base (base 2). The reactance of the generator is already on system base as is verified by the shifting theorem,

$$\text{G:} \quad X = j\,1.0 \left[\frac{13.8}{13.8}\right]^2 \left[\frac{30}{30}\right] = j\,1.0 \ \text{pu}$$

$$\text{T}_1\text{:} \quad X = j\,0.1 \left[\frac{13.2}{13.8}\right]^2 \left[\frac{30}{35}\right] = j\,0.0784 \ \text{pu}$$

or,

$$= j\,0.1 \left[\frac{115}{120}\right]^2 \left[\frac{30}{35}\right] = j\,0.0784 \ \text{pu}$$

$$\text{Line:} \quad X = \frac{j80}{Z_{base}} = \frac{j80}{480} = j\,0.167 \ \text{pu}$$

$$\text{T}_2\text{:} \quad X = j\,0.1 \left[\frac{116}{120}\right]^2 \left[\frac{30}{30}\right] = j\,0.094 \ \text{pu}$$

or,

$$= j\,0.1 \left[\frac{12.5}{12.9}\right]^2 \left[\frac{30}{30}\right] = j\,0.094 \ \text{pu}$$

$$\text{Load:} \ Z_L = \frac{\dfrac{(12.5)^2}{8/0.8} \angle 36.9^\circ}{Z_{base}} = \frac{15.63 \angle 36.9^\circ}{5.55} = 2.82 \angle 36.9^\circ = 2.26 + j1.69 \ \text{pu}$$

or,

$$V_L = \frac{12{,}500}{\sqrt{3}} \angle 0^\circ = 7{,}217 \angle 0^\circ \ \text{V} \ ; \quad I_L = \frac{8{,}000}{\sqrt{3}\,(12.5)(0.8)} \angle -36.9^\circ = 462 \angle -36.9^\circ \ \text{A}$$

$$Z_L = \frac{V_L}{I_L} = \frac{7{,}217 \angle 0^\circ}{462 \angle -36.9^\circ} = \frac{15.63 \angle 36.9^\circ\ \Omega}{Z_{base}} = 2.26 + j1.69 \ \text{pu}$$

$$\text{M:} \quad X = j1.2 \left[\frac{12.5}{12.9}\right]^2 \left[\frac{30}{20}\right] = j1.69 \ \text{pu}$$

The reactances of Fig. 8.5 are now on a common system base as shown in Fig. 8.6 and the system can now be analyzed under actual operating conditions :

The passive load is operating at rated voltage, rated MVA, 0.8 pf lagging and the motor is torqued and its field current is adjusted so that it is drawing rated MVA, 0.8 pf leading at rated voltage from the line.

a) Calculate the excitation emf of the generator that would be required to deliver the above specified loading with rated voltage at load bus, D.

b) Calculate the resulting bus voltages, line currents and power flow throughout the system.

a) Voltages and Currents -

$$V_D\,(pu) = \frac{V_D}{V_{base}} = \frac{12{,}500/\sqrt{3}\,\angle 0^\circ}{12{,}900/\sqrt{3}} = 0.969\,\angle 0^\circ \text{ pu}$$

$$I_L\,(pu) = \frac{I_L}{I_{base}} = \frac{\dfrac{8{,}000}{\sqrt{3}\,(12.5)(0.8)}\angle -36.9^\circ}{1343} = \frac{462\,\angle -36.9^\circ}{1343} = 0.344\,\angle -36.9^\circ \text{ pu}$$

$$I_M\,(pu) = \frac{I_M}{I_{base}} = \frac{\dfrac{20{,}000}{\sqrt{3}\,(12.5)}\angle 36.9^\circ}{1343} = \frac{924\,\angle 36.9^\circ}{1343} = 0.688\,\angle 36.9^\circ \text{ pu}$$

$$I\,(pu) = I_L + I_M = 0.275 - j0.207 + 0.550 + j0.413 = 0.850\,\angle 14^\circ \text{ pu}$$

$$V_C\,(pu) = X_2 I + V_D = (0.094\,\angle 90^\circ)(0.850\,\angle 14^\circ) + 0.969\,\angle 0^\circ = 0.954\,\angle 4.6^\circ \text{ pu}$$

$$V_B\,(pu) = (X_{line} + X_2)\, I + V_D = (0.261\angle 90^\circ)(0.850\,\angle 14^\circ) + 0.969\angle 0^\circ = 0.941\,\angle 13.2^\circ \text{ pu}$$

$$V_A\,(pu) = (X_1 + X_{line} + X_2)\, I + V_D = (0.339\angle 90^\circ)(0.850\,\angle 14^\circ) + 0.969\angle 0^\circ = 0.942\,\angle 17.3^\circ \text{ pu}$$

$$E_G\,(pu) = (X_G + X_1 + X_{line} + X_2)\, I + V_D = (1.339\angle 90^\circ)(0.850\,\angle 14^\circ) + 0.969\angle 0^\circ = 1.3\,\angle 57.9^\circ \text{ pu}$$

$$E_M\,(pu) = V_D - X_M I_M = 0.969\,\angle 0^\circ - (1.69\,\angle 90^\circ)(0.688\,\angle 36.9^\circ) = 1.91\,\angle -29.2^\circ \text{ pu}$$

b) Power flow -

$$S_A\,(pu) = V_A I^* = (0.942\,\angle 17.3^\circ)(0.850\,\angle -14^\circ) = 0.801\,\angle 3.3^\circ = 0.8 + j\,0.046 \text{ pu}$$

$$S_B\,(pu) = V_B I^* = (0.941\,\angle 13.2^\circ)(0.850\,\angle -14^\circ) = 0.8\,\angle -0.8^\circ = 0.8 - j\,0.011 \text{ pu}$$

$$S_C\,(pu) = V_C I^* = (0.954\,\angle 4.6^\circ)(0.850\,\angle -14^\circ) = 0.811\,\angle -9.4^\circ = 0.8 - j\,0.132 \text{ pu}$$

$$S_D\,(pu) = V_D I^* = (0.969\,\angle 0^\circ)(0.850\,\angle -14^\circ) = 0.824\,\angle -14^\circ = 0.8 - j\,2.0 \text{ pu}$$

$$S_L\,(pu) = V_D I_L^* = (0.969\,\angle 0^\circ)(0.344\,\angle 36.9^\circ) = 0.333\,\angle 36.9^\circ = 0.267 + j\,0.2 \text{ pu}$$

$$S_M\,(pu) = V_D I_M^* = (0.969\,\angle 0^\circ)(0.688\,\angle -36.9^\circ) = 0.667\,\angle -36.9^\circ = 0.533 - j\,0.4 \text{ pu}$$

$$Q_{T1} = I^2 X_1 = (0.850)^2 (0.074) = 0.057 \text{ pu}$$

$$Q_{line} = I^2 X_{line} = (0.850)^2 (0.167) = 0.121 \text{ pu}$$

$$Q_{T2} = I^2 X_2 = (0.850)^2 (0.094) = 0.068 \text{ pu}$$

As Fig. 8.6 indicates, the generator supplies 0.8 per-unit MW to the system which is exactly the real power converted usefully at the load, since no real power is dissipated by the line and the transformer banks.

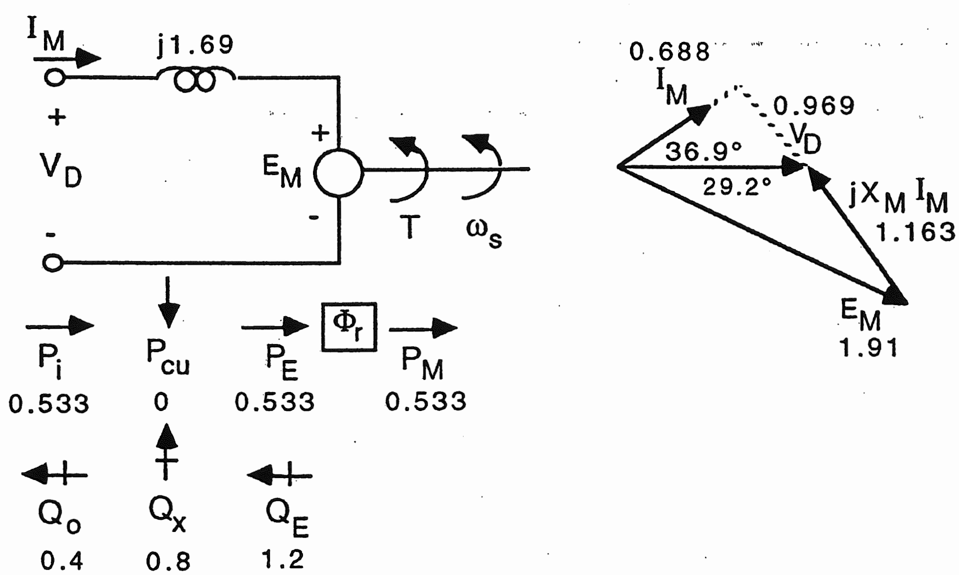
The reactive power flow is interesting in that the motor and the generator supply 0.446 per-unit MVAR which is exactly the reactive power required by the line (0.121 pu), the transformer banks (0.057 and 0.068 pu) and the passive load (0.2 pu).

The role of the synchronous motor and the synchronous generator, as set forth by the principles of the previous chapters, can now be emphasized in this power system, as shown in Example 8.2.

Example 8.2

Calculate the power flow (pu) through the synchronous motor and the synchronous generator in Example 8.1; draw and numerically label the corresponding phasor diagrams.

a) Synchronous motor -



$V_D = 0.969\,\angle 0^\circ$ pu, $\quad I_M = 0.688\,\angle 36.9^\circ$ pu

$E_M = V_D - jX_M I_M = 1.91\,\angle{-29.2^\circ}$ pu

$P_i = V_D I_M \cos\theta = (0.969)(0.688)\cos 36.9^\circ = 0.533$ pu

$P_{cu} = I_M^2 R = 0$ pu

$P_E = P_M = E_M I_M \cos\beta = (1.91)(0.688)\cos 66.1^\circ = 0.533$ pu

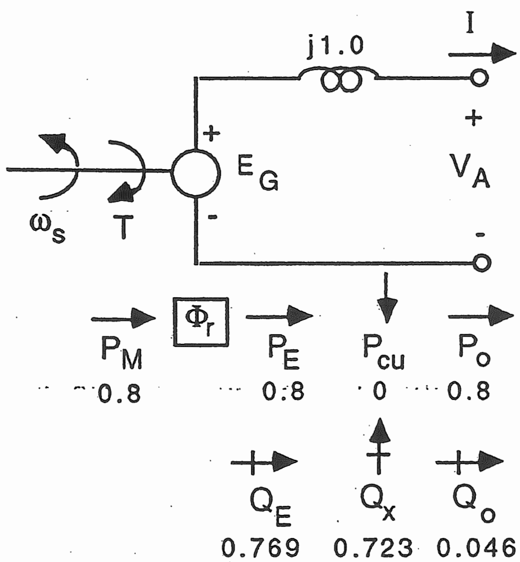
$Q_E = E_M I_M \sin\beta = (1.91)(0.688)\sin 66.1^\circ = 1.2$ pu

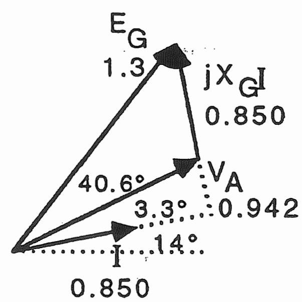
$Q_X = I_M^2 X_M = (0.688)^2(1.69) = 0.8$ pu pu

$Q_o = V_D I_M \sin\theta = (0.969)(0.688)\sin 36.9^\circ = 0.4$ pu

Observe from the above diagrams that the motor is torqued until 0.533 pu MW is drawn from the line. The field current is adjusted so that the motor is overexcited with an excitation emf, $E_M = 1.91$ pu kV. The motor, then, delivers 0.4 pu MVAR to the line, which helps the generator to supply the MVAR required by the passive load, transformer banks and transmission line.

b) Synchronous generator -





$$V_A = 0.942\angle 17.3^\circ \text{ pu}, \quad I = 0.850\angle 14^\circ \text{ pu}$$

$$E_G = jX_G I + V_A = 1.3\angle 57.9^\circ \text{ pu}$$

$$P_M = P_E = E_G I \cos\beta = (1.3)(0.850)\cos 43.9^\circ = 0.8 \text{ pu}$$

$$P_{cu} = I^2R = 0 \text{ pu}$$

$$P_o = V_A I \cos\theta = (0.942)(0.850)\cos 3.3^\circ = 0.8 \text{ pu}$$

$$Q_E = E_G I \sin\beta = (1.3)(0.850)\sin 43.9^\circ = 0.769 \text{ pu}$$

$$Q_X = I^2 X_G = (0.850)^2(1.0) = 0.723 \text{ pu}$$

$$Q_o = V_A I \sin\theta = (0.942)(0.850)\sin 3.3^\circ = 0.046 \text{ pu}$$

Observe from the above diagrams that the generator is driven by a prime mover that delivers 0.8 pu MW to the system, which is required by the passive load and the motor.

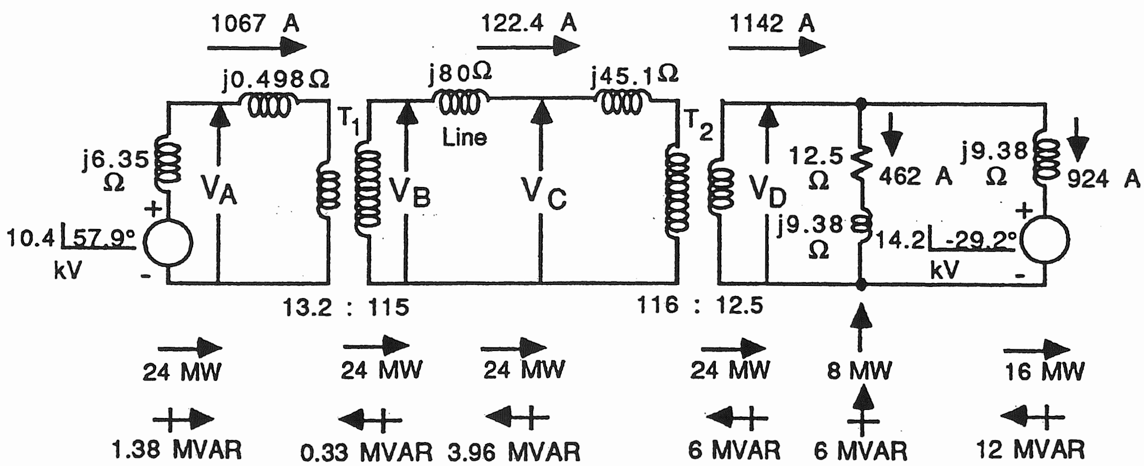
The field current is adjusted so that the generator is overexcited with an excitation emf, E_G = 1.3 pu kV. The generator then delivers 0.046 pu MVAR which, when added to 0.4 pu MVAR supplied by the motor, equals the MVAR requirement of the passive load, transformer banks and transmission line.

Note that once the power system is placed in the per-unit system, the factors 3 and $\sqrt{3}$ are never used to calculate voltages and power flow, since these factors are already part of the base values used to convert actual values to per-unit.

The powerful advantage of the per-unit system applied to a complicated power system on system base will now become evident, where it may be instructive to calculate the inverse quantities in Fig. 8.6.

Example 8.3

Redraw Fig. 8.6 and numerically label all voltages, currents, impedances and power-flow with actual values. Refer to Fig. 8.5 for appropriate base values.



$V_A = (0.942)(13.8) = 13$ kV (line)

$V_B = (0.941)(120) = 112.9$ kV (line)

$V_C = (0.954)(120) = 114.5$ kV (line)

$V_D = (0.969)(12.9) = 12.5$ kV (line)

$E_G = (1.3 \angle 57.9°)\left(\frac{13.8}{\sqrt{3}}\right) = 10.4 \angle 57.9°$ kV (ϕ)

$E_M = (1.91 \angle -29.2°)\left(\frac{12.9}{\sqrt{3}}\right) = 14.2 \angle -29.2°$ kV(ϕ)

$I_{gen} = (0.850)(1255) = 1067$ A

$I_{line} = (0.850)(144) = 122.4$ A

$I = (0.850)(1343) = 1142$ A

$I_L = (0.344)(1343) = 462$ A

$I_M = (0.688)(1343) = 924$ A

$X_{gen} = (j\,1.0)(6.35) = j\,6.35\ \Omega$

$X_{T1} = (j\,0.0784)(6.35) = j\,0.498\ \Omega$ (low)

$\quad = (j\,0.0784)(480) = j\,37.6\ \Omega$ (high)

$X_{line} = (j\,0.167)(480) = j\,80\ \Omega$

$X_{T2} = (j\,0.094)(480) = j\,45.1\ \Omega$ (high)

$\quad = (j\,0.094)(5.55) = j\,0.522\ \Omega$ (low)

$Z_L = (2.26 + j\,1.69)(5.55) = 12.5 + j\,9.38\ \Omega$

$X_{mot} = (j\,1.69)(5.55) = j\,9.38\ \Omega$

$$P_A = P_B = P_C = P_D = (0.8)(30) \qquad = 24 \text{ MW}$$

$$P_L = (0.267)(30) \qquad = 8 \text{ MW}$$

$$P_M = (0.533)(30) \qquad = 16 \text{ MW}$$

$$Q_A = (0.046)(30) \qquad = 1.38 \text{ MVAR}$$

$$Q_B = (0.011)(30) \qquad = 0.33 \text{ MVAR}$$

$$Q_C = (0.132)(30) \qquad = 3.96 \text{ MVAR}$$

$$Q_D = Q_L = (0.2)(30) \qquad = 6 \text{ MVAR}$$

$$Q_M = (0.4)(30) \qquad = 12 \text{ MVAR}$$

The above quantities were all obtained by using system-base values. It might also be instructive to check these values by using the principles of previous chapters:

$$I_M = \frac{20{,}000}{\sqrt{3}\,(12.5)} \qquad = 924 \text{ A}$$

$$I_L = \frac{8{,}000}{\sqrt{3}\,(12.5)(0.8)} \qquad = 462 \text{ A}$$

$$I = |\,924\,\underline{/36.8^\circ} + 462\,\underline{/-36.8^\circ}\,| \qquad = 1142 \text{ A}$$

$$I_{line} = \left(\frac{12.5}{116}\right)(1142) \qquad = 122.4 \text{ A}$$

$$I_{gen} = \left(\frac{115}{13.2}\right)(122.4) \qquad = 1067 \text{ A}$$

$$X_{T2}\,(\text{low}) = \left(\frac{12.5}{116}\right)^2 (45.1) \qquad = 0.522\ \Omega$$

$$X_{T1}\,(\text{high}) = \left(\frac{115}{13.2}\right)^2 (0.498) \qquad = 37.6\ \Omega$$

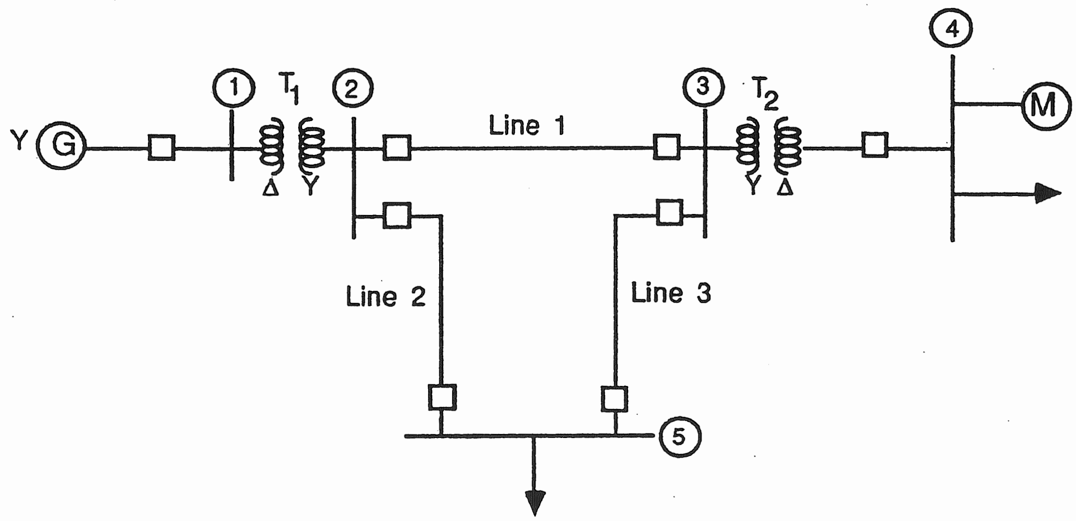
In summary, Examples 8.1, 8.2 and 8.3 graphically show how a complicated power system can be placed on system base, analyzed in per-unit, under actual operating conditions, without concern for high or low transformer-bank equivalent circuits, and then checked for bus voltage, current and power-flow tolerances.

The powerful advantage of the per-unit system will become even more evident as more complex power systems are analyzed.

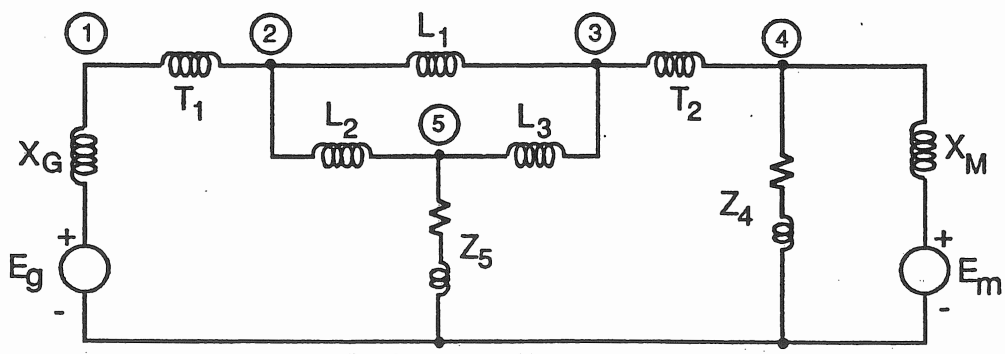
Consider now, the more complex power system in Example 8.4,

Example 8.4

In the power system below, a three-phase generator drives two transformer banks, three transmission lines, two passive loads and a synchronous motor. For simplicity, the machines, transmission lines, and transformer banks are represented by reactances and emfs where applicable and the passive loads by impedances. Draw the impedance diagram for this power system.



By tracking phase a to neutral throughout this system,



a multinode network-results that can be placed on system base and analyzed, procedurally, by converting all emf sources to current sources and writing the nodal equation at each power system node. This method of analysis will not be emphasized in this introductory text, but is developed in more advanced power-system analysis texts.

8-3 POWER FLOW FUNDAMENTALS

As is evident from Example 8.1, when a power system is placed on system base and system loading is specified, after analysis, all the bus-voltage <u>magnitudes</u> and <u>angles</u> are known, in per unit, from which the power flow in all the transmission lines can be calculated.

Power engineers are extremely interested in not only the magnitude and direction of this power flow but how it can be controlled, especially if one or more lines are overloaded.

This section is intended to give great insight into the fundamentals of power flow in a transmission line and at the same time emphasize the role of generators and motors from which this power originates, and then is used.

Consider a transmission line that is part of a large power system whose impedance and bus voltages, in per-unit, are known in magnitude and angle.

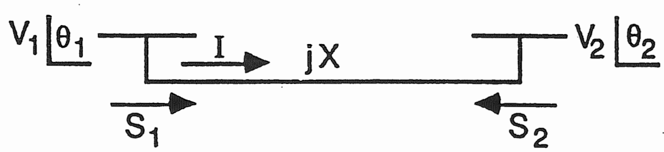


Figure 8.7 Transmission Line

The power flow in this line is quantified by calculating the complex power delivered to the line by each bus.

$$I = \frac{V_1 \angle\theta_1 - V_2 \angle\theta_2}{jX} \quad \text{(pu)} \tag{8.4}$$

$$I^* = \frac{V_1 \angle{-\theta_1} - V_2 \angle{-\theta_2}}{-jX} \quad \text{(pu)} \tag{8.5}$$

$$S_1 = V_1 I^* = \frac{V_1 \angle\theta_1\, V_1 \angle{-\theta_1} - V_1 \angle\theta_1\, V_2 \angle{-\theta_2}}{-jX}$$

$$= \frac{V_1^2 - V_1 V_2 \angle\theta_1 - \theta_2}{-jX}$$

$$= \frac{V_1^2 - V_1 V_2 \cos(\theta_1 - \theta_2) - jV_1 V_2 \sin(\theta_1 - \theta_2)}{-jX}$$

$$S_1 = \frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2) + j\left[\frac{V_1^2}{X} - \frac{V_1 V_2}{X} \cos(\theta_1 - \theta_2)\right] \tag{8.6}$$

$$S_2 = V_2(-I^*) = \frac{-V_2 \underline{/\theta_2}\, V_1 \underline{/-\theta_1} + V_2 \underline{/\theta_2}\, V_2 \underline{/-\theta_2}}{-jX}$$

$$= \frac{V_2^2 - V_1 V_2 \underline{/\theta_2 - \theta_1}}{-jX}$$

$$= \frac{V_2^2 - V_1 V_2 \cos(\theta_1 - \theta_2) + jV_1 V_2 \sin(\theta_1 - \theta_2)}{-jX}$$

$$S_2 = -\frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2) + j\left[\frac{V_2^2}{X} - \frac{V_1 V_2}{X}\cos(\theta_1 - \theta_2)\right] \qquad (8.7)$$

From Eqns. (8.6) and (8.7)

$$P_1 = \frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2) \qquad P_2 = -\frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2)$$

$$(8.8)$$

$$Q_1 = \frac{V_1^2}{X} - \frac{V_1 V_2}{X}\cos(\theta_1 - \theta_2) \qquad Q_2 = \frac{V_2^2}{X} - \frac{V_1 V_2}{X}\cos(\theta_1 - \theta_2)$$

Equations (8.8) show that the real power delivered by each bus varies as the <u>sine</u> of the <u>difference</u> in bus voltage angles and the reactive power varies as the <u>cosine</u> of the <u>difference</u> in bus voltage angles.

In practical, large power systems, the difference in bus angles is usually small, i. e., the difference is less than approximately 8°. Consider the sine and cosine functions in Fig. 8.8.

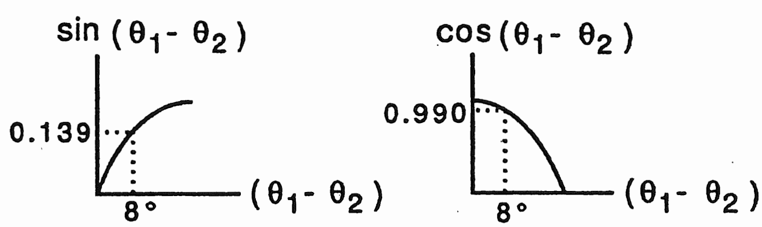


Figure 8.8 Sine and Cosine of $(\theta_1 - \theta_2)$

From Fig. 8.8, for small angle differences, the sine changes proportionately with angle difference, whereas the cosine remains approximately equal to one.

With this in mind, the real power flow in Eqns. (8.8) delivered to and received from the line is characterized as follows,

1. For small angles, the magnitude of the real power is approximately proportional to the difference in bus angles, i.e., $P \approx K_1 (\theta_1 - \theta_2)$.

2. The direction of the real power is determined by the $\sin(\theta_1 - \theta_2)$, i.e., if $\theta_1 > \theta_2$, real power flows to the right in Fig. 8.7, and if $\theta_1 < \theta_2$, real power flows to the left in this figure.

We conclude, for small bus angle differences, MW flow is always towards decreasing (more lagging) bus angle, and a change in bus angle difference drastically affects the MW flow.

Keeping Fig. 8.8 in mind, the reactive power flow in Eqns. (8.8) can be rewritten,

$$Q_1 = \frac{V_1}{X}\left[V_1 - V_2 \overbrace{\cancel{\cos(\theta_1-\theta_2)}}^{\approx 1}\right] \qquad Q_2 = \frac{V_2}{X}\left[V_2 - V_1 \overbrace{\cancel{\cos(\theta_1-\theta_2)}}^{\approx 1}\right]$$

$$Q_1 \approx (V_1 - V_2)\frac{V_1}{X} \qquad Q_2 \approx -(V_1 - V_2)\frac{V_2}{X} \qquad (8.9)$$

The reactive power flow delivered to and received from the line is characterized as follows,

1. For small angles, the magnitude of the reactive power is approximately proportional to the difference in bus voltage magnitudes, i.e, $Q \approx K_2 (V_1 - V_2)$.

2. The direction of the reactive power is determined by $(V_1 - V_2)$, i.e., if $V_1 > V_2$, reactive power flows to the right in Fig. 8.7 and if $V_1 < V_2$, reactive power flows to the left in this figure.

We conclude, for small bus angle differences, MVAR flow is always towards decreasing bus voltage magnitude and a change in bus voltage magnitude drastically affects MVAR flow.

We also conclude, by comparing Eqns. (8.8) and (8.9), that for small bus angle differences, changes in MW and MVAR flow are essentially independent of each other, since each equation is a function of voltage magnitude or angle difference only.

The question now arises as to how the bus voltage magnitudes and angles are established, how they can be changed, and what are the consequent power flows. This question is considered in Example 8.5.

Example 8.5

A simple power system consists of a three-phase synchronous generator driving a transmission line and a synchronous motor load.

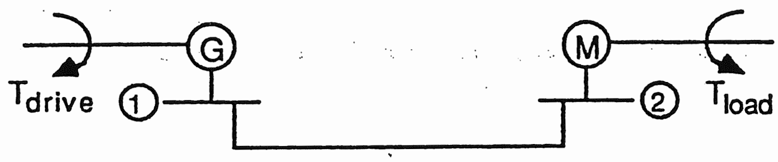


Figure 8.9 Simple Power System

The corresponding impedance diagram, with reactances on system base, is shown in Fig. 8.10.

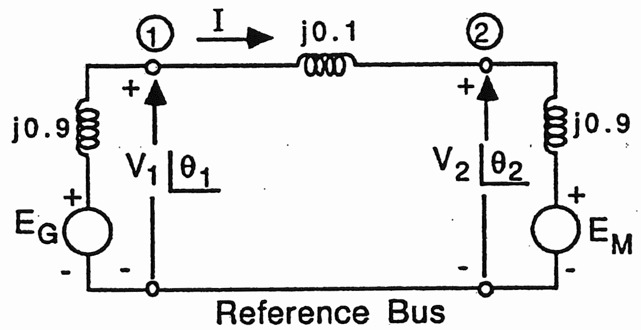


Figure 8.10 Impedance Diagram

The transmission line reactance is 0.1 per unit ; the machine reactances include the transformer banks, and are 0.9 per unit each. The machines in Fig. 8.9 are large with negligible rotational and electrical losses, therefore,

$$P_i = P_o = K_1 (\theta_1 - \theta_2)$$

$$T_{drive}\,\omega_s = T_{load}\,\omega_s = K_1 (\theta_1 - \theta_2)$$

or,
$$T_{drive} = T_{load} = K (\theta_1 - \theta_2) \qquad (8.10)$$

The machines are brought up to speed and the field current of the generator is adjusted so that V_1 is rated, i.e., $V_1 = 1.0$ pu. The generator prime-mover throttle is opened slightly to supply the small losses of both machines at synchronous speed, and the field current of the motor is adjusted so that V_2 is rated, i.e., $V_2 = 1.0$ pu. The motor is unloaded so that from Eqn. (8.10),

$$T_{drive} \approx T_{load} = 0$$

The machines are floating on the line with phasor diagrams,

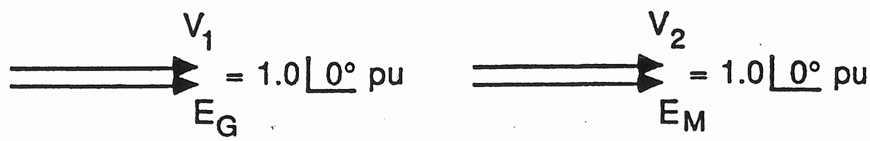


Figure 8.11 Floating on the Line

Transmission line current is small since there is negligible power or var flow. Power, var flow, bus voltages and angles, are now established as follows,

1. The excitation of each machine is adjusted to establish each bus-voltage magnitude.

2. The throttle of the prime mover of the generator is opened, increasing its drive torque (Fig. 5.15), which advances the generator bus voltage angle, θ_1.

3. At the same time the motor is loaded which retards (Fig. 5.15) its bus voltage angle, θ_2, which is taken as the reference angle.

Whether the generator or motor is directly connected to the buses as in Fig. 8.9 or whether they are remotely connected via the loop configuration in Fig. 8.1, the end result is similar to the procedure described above; the consequent power flow must result in a MW and MVAR balance as described in the following three cases.

Case 1: The motor is loaded and the prime mover throttle is opened such that $(\theta_1 - \theta_2) = 5°$. The generator excitation is decreased, and the motor excitation is increased, such that $V_1 = 0.95$ pu and $V_2 = 1.0$ pu.

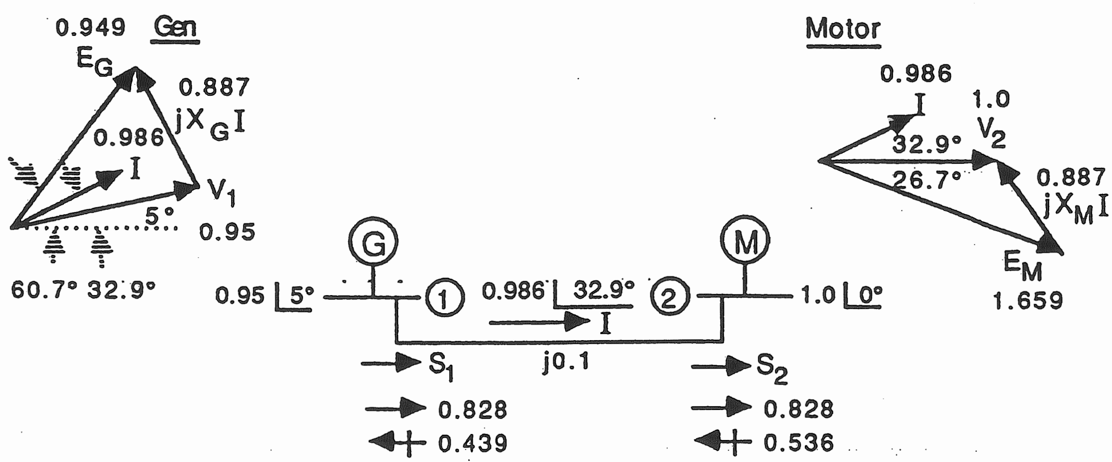


Figure 8.12 Case 1, $V_1 = 0.95\angle 5°$

$$I = \frac{0.95\angle 5^\circ - 1.0\angle 0^\circ}{j0.1} = 0.986\angle 32.9^\circ$$

$$S_1 = V_1 I^* = (0.95\angle 5^\circ)(0.986\angle -32.9^\circ) = 0.828 - j0.439 \text{ pu}$$

$$S_2 = V_2 I^* = (1.0\angle 0^\circ)(0.986\angle -32.9^\circ) = 0.828 - j0.536 \text{ pu}$$

$$E_G = jX_G I + V_1 = (0.9\angle 90^\circ)(0.986\angle 32.9^\circ) + 0.95\angle 5^\circ = 0.949\angle 60.7^\circ \text{ pu}$$

$$E_M = V_2 - jX_M I = 1.0\angle 0^\circ - (0.9\angle 90^\circ)(0.986\angle 32.9^\circ) = 1.659\angle -26.7^\circ$$

Observe in Fig. 8.12, MW flow is towards decreasing (more lagging) phase and MVAR flow is towards decreasing voltage magnitude. By comparing Figs. 8.11 and 8.12, the phasor diagrams, above, confirm the system adjustments described in case 1. The motor is overexcited delivering MVAR to, and receiving MW from, the line. The generator is underexcited, delivering MW required by the motor, and receiving MVAR, remaining from those supplied to the line by the motor.

Case 2 : The motor is loaded and the prime mover throttle is opened such that $(\theta_1 - \theta_2) = 5°$. The generator excitation is further decreased and the motor excitation is further increased such that $V_1 = 0.9$ pu and, $V_2 = 1.0$ pu.

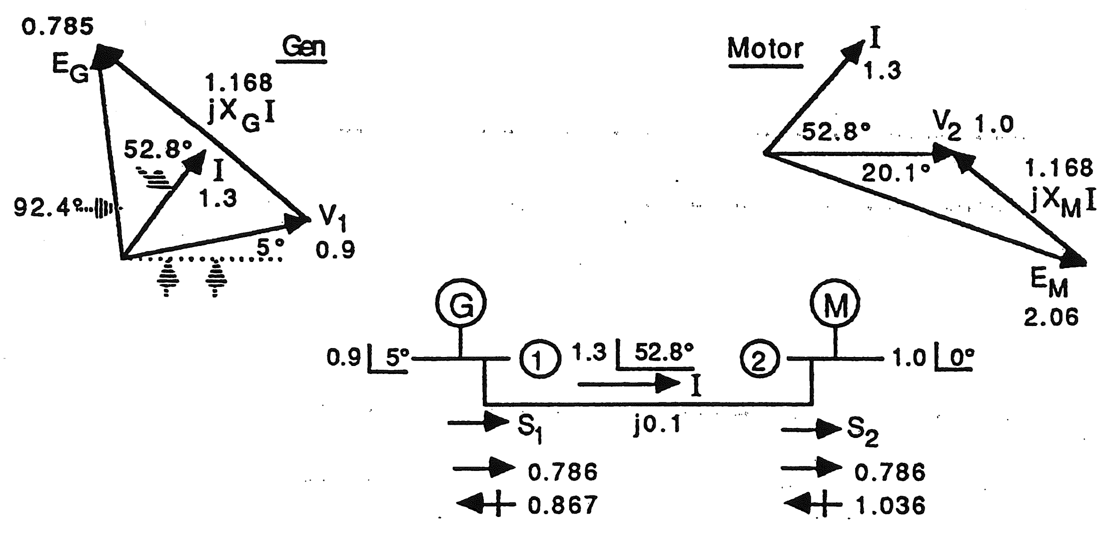


Figure 8.13 Case 2, $V_1 = 0.9\angle 5°$

If Cases 1and 2 are compared, the bus angle difference is the same so the MW flow remains essentially unchanged, however, a small change in bus voltage magnitude results in a large change in MVAR flow.

Case 3 : The motor is loaded and the prime mover throttle is opened such that $(\theta_1 - \theta_2) = 7°$. From Case 2, the generator excitation is now increased and the motor excitation is now decreased such that $V_1 = 0.95$ pu and $V_2 = 1.0$ pu.

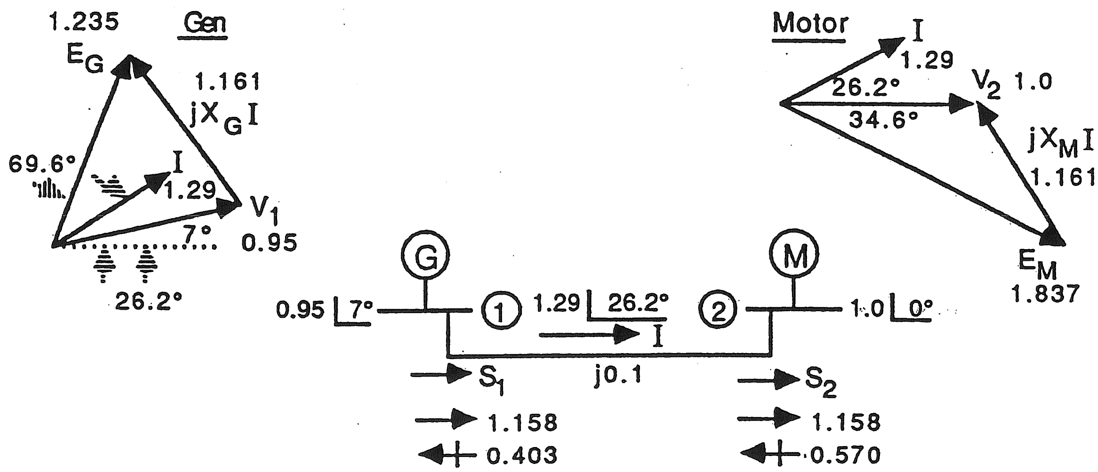


Figure 8.14 Case 3, $V_1 = 0.95\angle 7°$

If Cases 1 and 3 are compared, the bus voltage magnitudes are the same so the MVAR flow remains essentially unchanged, however, a small change in bus angle difference results in a large change in MW flow.

In summary, when a power system is loaded with either motors and/or passive loads, the drive torques of all the generators must increase to achieve a MW balance. The MW flow, in each transmission line, is then essentially determined by bus angle difference for small angle. When the MVAR requirement of a power system is increased, the excitation of all the generators must change to achieve a MVAR balance. The MVAR flow, in each transmission line, is then essentially determined by the difference in bus voltage magnitude for small angle. As Cases 1, 2 and 3 indicate, for small bus angle difference, the MW flow can be controlled essentially independently of MVAR flow.

8-4 SUMMARY

Power systems, in general, consist of a high-voltage portion with a loop configuration that has more than one transmission line at each bus so that if a line is faulted because of ice build-up, insulator failure, etc., the faulted line can be removed and alternate lines at each bus can carry increased load to insure reliability. The low-voltage portion of the system is of radial configuration which distributes electric power to urban and rural residences and small industrial plants, and is radial because of practicality.

The one-line diagram of a power system considerably simplifies its connection pattern and its corresponding impedance diagram allows a procedural analysis. The impedance diagram is placed on an arbitrary system base so that all component impedances can be shifted from their given base to a common system base using the shifting theorem,

$$X_2\,(\mathrm{pu}) = X_1\,(\mathrm{pu}) \left(\frac{kV_{base1}}{kV_{base2}}\right)^2 \left(\frac{MVA_{base2}}{MVA_{base1}}\right)$$

Once the reactance diagram is on system base, then, for actual operating load conditions, all bus voltages, line currents and power flow can be calculated in per unit and checked for rated tolerances. The question then arises as to what can be done if bus voltages are low and if line currents and power flows are excessive. There are many ways that these variables can be controlled, which is outside of the scope of this introductory text, but they all depend on basic power flow fundamentals. For simplicity, all component impedances are replaced by reactances, and with this assumption, great insight can be obtained into simplified power flow in a transmission line,

$$P_1 = \frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2) \approx K_1 (\theta_1 - \theta_2)$$

$$Q_1 = \frac{V_1^2}{X} - \frac{V_1 V_2}{X} \cos(\theta_1 - \theta_2) \approx K_2 (V_1 - V_2)$$

From these equations it is seen that the magnitude and direction of <u>real</u> power flow is essentially determined by the difference in <u>bus angle</u>, and the magnitude and direction of <u>reactive</u> power flow is essentially determined by the difference in bus <u>voltage magnitude</u> ; the two equations are essentially independent of each other.

Since real power flow depends on voltage angle and reactive power depends on voltage magnitude, the question arises as to how bus voltage magnitude and angle can be changed. Here, the principles of the synchronous machine become important. Regardless of where the synchronous generators are placed in a power system,

1. The <u>excitation</u> of each machine essentially establishes the voltage <u>magnitudes</u> of the buses in its vicinity.

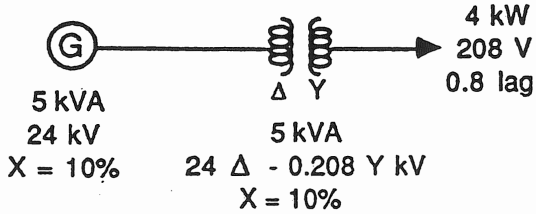
2. The <u>drive torque</u> of each machine essentially establishes the voltage <u>angle</u> of the buses in its vicinity.

In summary, the excitation and the drive or load torque are the only two variables of a synchronous machine that can be adjusted to achieve a MW-MVAR balance within the system.

Finally, a rigorous and in-depth study of transformers, transmission lines, and machines is pointless, at this introductory level, unless their characteristics and role in a complete power system can be shown, which is why this chapter is included in this text.

PROBLEMS

8.1

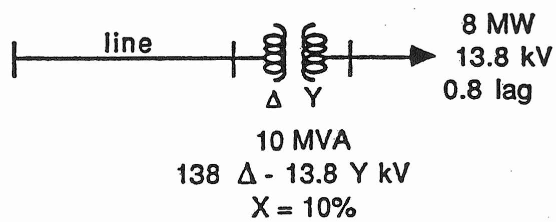


For the power system shown, what impedance (Ω) does the generator see, line to neutral,

a) If the load is connected in delta?

b) If the load is connected in wye?

8.2



For the power system shown,

a) What is the rating of each individual transformer that makes up the transformer bank?

b) What is the load impedance (pu) if system base in the transmission line section is 200 kV, 25 MVA?

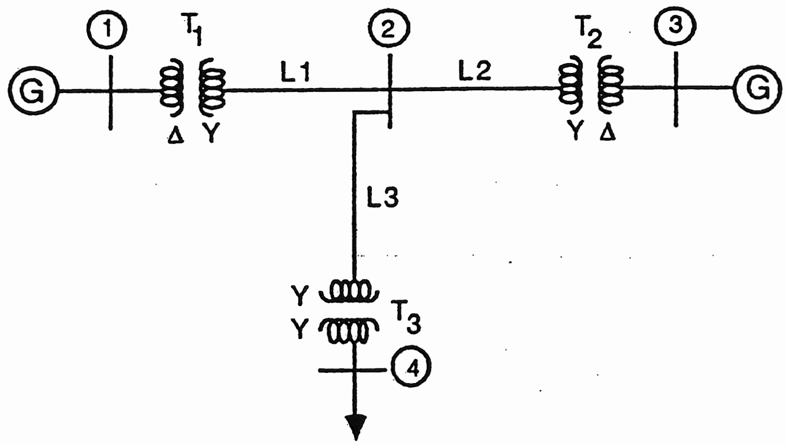
8.3 The components of the power system in Example 8.4 have the following ratings,

G :	100 MVA, 13.8 kV, X = 120%
M :	10 MVA, 15 kV, X = 20%
Z_4 :	25 MVA, 15 kV, 0.85 lag
Z_5 :	50 MVA, 230 kV, 0.8 lag
T_1 :	120 MVA, 13.8 Δ–230 Y kV, X = 10%
T_2 :	40 MVA, 230 Y–18 Δ kV, X = 10%
L_1, L_2, L_3 :	j100 Ω

Calculate the impedances, in per unit, if system base in the generator section is the generator rating.

8.4 In Problem 8.3, if the motor draws rated MVA, 0.8 power factor leading at rated voltage from the line, what are the bus voltages and the generator excitation voltage? What are the transmission line currents and power flows in magnitude and direction?

8.5



For the power system shown, the component ratings are,

G_1 : 400 MVA, 15 kV, X = 120%

G_2 : 300 MVA, 15 kV, X = 100%

T_1 : 450 MVA, 371 Y–15 Δ kV, X = 10%

T_2 : Three individual trnsformers each rated 100 MVA, 200–15 kV, X = 10%

T_3 : Three individual transformers each rated 200 MVA, 200–10.4 kV, X =10%

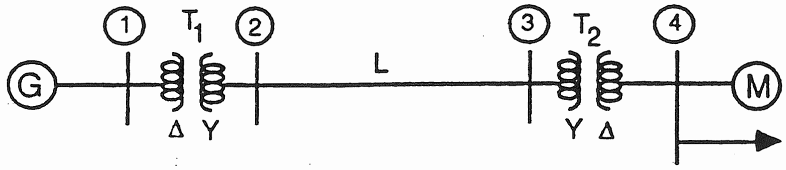
L_1 : j 36 Ω
L_2 : j 40 Ω
L_3 : j 24 Ω

Z_4 : 400 MW, 18 kV, 0.8 lag

Draw the impedance diagram and numerically label all impedances, in per unit, if system base in the load section is the load rating.

8.6 In Problem 8.5, the load is drawing rated MW, 0.8 pf lagging, at rated voltage, from bus 4. If the generators share the load equally, what are the bus voltages and generator excitation emfs? What are the transmission line current and power flows in magnitude and direction?

8.7



For the power system shown, the component ratings are,

G : 30 MVA, 13.8 kV, X = 100%

M : 20 MVA, 12.5 kV, X = 120%

T_1 : 35 MVA, 13.2 Δ–115 Y kV, Z = 0.012 + j 0.1 pu

T_2 : 30 MVA, 116 Y–12.5 Δ kV, Z = 0.015 + j 0.1 pu

L : 30 MVA, 115 kV, 6.61 Chg. MVAR, Z = 20 + j 80 Ω

Z_4 : 10 MVA, 12.5 kV, 0.8 pf lagging

Draw the impedance diagram and numerically label all impedances, in per-unit, if system base in the generator section is the generator rating.

8.8 The two machines of Example 8.5 are floating on the line as in Fig. 8.11. Then each machine is adjusted so that $V_1 = 1.05\,\underline{/0^\circ}$ and, $V_2 = 1.0\,\underline{/0^\circ}$. Describe the adjustments that must be made on each machine to obtain the above bus voltages and calculate the transmission line power flow in magnitude and direction. Draw and numerically label the phasor diagram for each machine. Calculate the power flow through each machine and briefly describe the fundamental conclusions reached concerning this problem.

8.9 Do Problem 8.8 when the machines are adjusted so that $V_1 = 1.05\,\underline{/5^\circ}$ and $V_2 = 1.0\,\underline{/0^\circ}$.

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